Lecture 10: IS
(chapter 10, first half)

topics: Keynesian Cross
       Multipliers
       IS Curve

1a) Intro:
   Great Depression caused rethinking of Classical theory - not explain why output drop 30% from 1929 to 1933, and unemployment rise to 25%. This theory said supply determines the amount of output. How change so much?
   1936 Keynes come up with new theory: the General Theory of Employment, Interest and Money. Idea was that prices are sticky, and demand for output determines equilibrium level more than supply.
   Here develop IS-LM theory of AD - gives story behind AD curve. Takes price level as given, then shows how output is determined.
   Two parts to model:
   IS: Investment equals saving, equilibrium in goods market
   LM: Liquidity and money: equilibrium in money market.

b. IS curve - Keynesian Cross

   First building block is Keynesian Cross
   Recall \( Y^d = C + I + G \) where \( Y^d \) is demand for goods, or expenditure on goods
   or if \( C = C(Y-T) \), and take other components as fixed
   \( Y^d = C(Y-T) + I-bar + G-bar \)

   Distinguish between two parts of investment, planned and unplanned part. What happens if make goods but not sell? Becomes inventory accumulation, which we classify as a type of investment.
   Subtract out unplanned inventory accumulation from investment.

   Then call this planned expenditure:
   \( E = C(Y-T) + I-bar + G-bar \) where \( E \) is planned expenditure, \( I \) is planned investment

   Draw graph: planned expenditure as function of income.
   Notice axes: planned expenditure, income
   Slopes up because of rising consumption. Slope is MPC.

\[ E = C(Y-T) + I-bar + G-bar \]
Now assume goods market is in equilibrium when actual expenditure equals planned expenditure. Recall that Y equals not only total income but also actual expenditure. so can write equilibrium as:

**equilibrium:** \( \text{Actual Expenditure} = \text{Planned Expenditure} \)

\[ Y = E \]

Draw this as a 45-degree line:

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How reach equilibrium: unplanned inventories play central role

If start at output too high. Not all bought, so part becomes inventory, unplanned inventory accumulation.

Firm will produce less. Output fall. Keep going until reach equilibrium. Similar if start at level too low.

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### 2) Multipliers

#### a) Government expenditure multiplier:

If government expenditure rises by \( \Delta G \), shift up E line, planned expenditure line by \( \Delta G \).

Equilibrium moves from point A to B. How much does income increase.

Graphically see income increases more than rise in G, this is because E line is positively sloped.

**Def:** Government-purchases multiplier: \( \Delta Y / \Delta G \).

Keynesian cross says multiplier greater than one.

Intuition: When Gov spend, becomes income for someone doing the selling. But then this person will spend part of the income as consumption, which becomes income for someone else. What fraction of new income gets re-spent: MPC.
Math: So total change in income:

\[
\Delta Y = \Delta G + (MPC \times \Delta G) + (MPC^2 \times \Delta G) + \ldots
\]

is geometric series

\[
= \frac{1}{1-(1-MPC)} \times \Delta G
\]
or \[
\frac{\Delta Y}{\Delta G} = \frac{1}{1-(1-MPC)}
\]

Show example:
if \(C = a + b(Y-T)\), \(I = I_{bar}\), \(T = T_{bar}\), \(G = G_{bar}\)

\[
Y = C + I + G
\]

\[
Y = a + b(Y-T) + I_{bar} + G_{bar}
\]

\[
Y - bY = a - bT + I_{bar} + G_{bar}
\]

\[
(1-b)Y = \ldots
\]

\[
Y = \frac{1}{1-(1-b)}I_{bar} + \frac{1}{1-(1-b)}G_{bar} + \frac{a}{1-(1-b)} - \frac{b}{1-(1-b)}T
\]

(keep this equation)

So if \(G\) increases 1$, increase by \(1/(1-b)\) dollars.

\(b = MPC\). If \(b = .75\), the gov purch mult is 4.

So if \(G\) increases 1$, GDP will rise by 4$.

Or calculus derivation: multiplier is \(dY / dG\)

\[
Y = C(Y-T) + I + G
\]

differentiate

\[
dY = MPC \, dY + dG
\]

\[
dY /dG = \frac{1}{1-(1-MPC)}
\]

So size of multiplier depends on how much of income people spend. If spend a lot of
it, then will make many rounds through economy, so multiplier will be large. If
save a lot, and not spend much, then multiplier will be small.

See role of saving in homework.

Saw shifting \(G\), what about shifting level of \(C\), or level of \(I\)?

b) Tax multiplier

If cut taxes: not affect expenditure directly, because not component like \(G\) was.

Affects disposable income \(Y-T\), which in turn affects consumption.

So curve shifts up by \(MPC \times \Delta T\). Show graph above.

This has multiplier effect also

Story: expenditure becomes income for someone else…. But first round is smaller
because only indirect effect by raising disposable income.

Guess what tax multiplier must be: \(MPC/(1-MPC)\)

See because initial round is only fraction consumed.

See in math: equation above.

See in calculus:

\[
Y = C(Y-T) + I + G
\]

differentiate

\[
dY = MPC \, dY - MPC \, dT
\]

\[
(1-MPC) \, dY = -MPC \, dT
\]

\[
dY /dT = -MPC/(1-MPC)
\]

(for a rise in \(T\), or \(-dT\) for a fall in \(T\))
3a) **IS curve:**

Have been ignoring interest rate. Recall think that investment is a negative function of the interest rate, because becomes more expensive to borrow.

Consider investment function \( I = I(\ r) \). And how a change in \( r \) affects \( I \), then how this affects Keynesian cross diagram and hence \( Y \).

**Story**
1) Start at a given \( r \). Leads to a certain \( I \). Leads to a certain \( Y \). Draw \( Y-r \) combination.
2) Rise in \( r \) leads to fall in \( I \): move along Investment function.
3) Fall in \( I \) shifts \( E \) line down and eqm \( Y \) falls.
4) draw new \( r-Y \) combination.
5) can trace out all such combinations. Call this IS curve.

**Summarize:**

Rise in \( R \) makes investment fall. This has a multiplied effect on income (falls)
So when \( Y \) rises because of a fall in \( r \), this is a movement along the IS curve.
b) Fiscal policy shifting IS:

Can show more about increase in G here.
Saw what did to Keynesian Cross: shift up E, and increase Y.
How represent this in IS:

With higher G, then Y will be higher, for any value of r. For any r, Y will be higher. This means whole curve has shifted t right.
What was r, say rbar. New G means for that same rbar, Y will be higher.

***important:
So if see Increase in Y need distinguish between movement along IS and shift in IS.
If change in income is caused by change in r, then can show by movement up and down along curve; if due to something else, so that Y changes for a given r, only way can show it is by shifting whole curve.

IS tells story: given a certain interest rate, then I must be certain level and so Y will be certain level. Describes goods market.

c) Do math to derive IS:

Say 

\[ C = 95 + .75(Y-T) \]
\[ I = 100 - 100r \]
\[ G = 20, \ T = 20 \]

so 

\[ Y = 215 + .75(Y-20) - 100r \]
\[ .25Y = 200 - 100r \]
\[ IS: Y = 800 - 400r \]
\[ or \ IS: r = 2 - .0025Y \]
Recall IS tells story of effect of change in \( r \) on I and Y.

If Investment is very responsive to \( r \), then a drop in \( r \) raises I more and hence raises Y more. So IS will be flatter. See also in math.

Want to know effect of increase G. Will shift IS.

Say G = 50 not 20.

so \[ Y = 245 + .75(Y-20) - 100r \]

.25Y = 230 - 100r

IS: \[ Y = 920 - 400r \]

or IS: \[ r = 2.3 - .0025Y \]

But can’t compute new \( r \) and Y, because only have one curve - many possible combinations. Need another curve to pin down particular values.

d) **Loanable Funds interpretation of IS Curve**

Recall when studied market for goods in Chapter 3, noted that equilibriumm in goods market implies equilibrium in a market for loanable funds.

Start with \[ Y = C + I + G \]

or \[ Y - C - G = I \]

plug in consumption function for C and Investment function for I

\[ Y - C(Y-T) -G = I( r) \]

Had said Y fixed in Neoclassical economy. Now Y and r both can change – will be jointly determined by a more complicated process.

Left side is saving, supply of loanable funds. Right side is investment or demand for loanable funds. Is responsive to the interest rate. Give level of saving, r will adjust to make I equal supply of loanable funds.

What happens if Y increases: spend part, save rest. So total level of saving rises. Extra loanable funds for investment, r falls to stimulate investment.

Can say this traces out the Is curve: A rise in Y raises saving, needs to stimulate I so lowers r.

Different from earlier interpretations: fall in r stimulates I, so raises Y by multiplier).