Lecture 17: Investment
(Chapter 17)

Topics: 
- Business fixed
- Residential
- Inventory

Intro:

Recall are three categories of investment:

Business fixed: equipment and structures that businesses buy to use in production

Residential: new housing, for households to live and landlords to rent out.

Inventories: goods business put aside in storage, that not able to sell.

Have model for each of these. Focus mostly on business fixed.

Develop more detailed understanding of investment function we have been using: I = I( r). Why is a function of the interest rate.

1) Business Fixed Investment
   a) Intro

   Standard model is call neoclassical model of investment. The details behind story gave in first part of course.

   Are benefits and costs of firm for owning capital goods. Decide how much investment to undertake.

   The parts of story are clearer if pretend is distinction between two kinds of firms: Production firms (produce goods and rent capital) and Rental firms (buy capital and rent it out to other firms)

   This separation is not necessary, but makes elements of the story clearer.
b) Rental price of capital - focus on production firms

(Recall neoclassical model from ch.3. Here give more details.)

Consider a typical firm, maybe a bakery. (Café Roma) Decides how much capital (space and equipment) to rent by comparing cost and benefit of each unit.

Cost is nominal “rental price” of capital, $R$ (distinct from interest rates $r$ & $i$.)

Sells its output at (outrageous) price $P$. ($P$ is price of typical good; represents price level in the economy)

$MPK = F_K(K,L)$, is extra units of output for one extra unit of capital input.

the extra profit from renting an additional machine is the extra revenue from selling the output that machine makes minus the cost of renting that machine:

$$\Delta \text{profit} = \Delta \text{revenue} - \Delta \text{cost} = (P \times MPK) - R$$

To maximize profit, the firm continues to rent more capital until the MPK falls to equal the real rental price:

$$\text{MPK} = R/P$$

Can regard this as demand for capital to rent. Slopes down because the MPK is low when level of capital is high.

Supply of capital fixed at a point in time, so supply curve is vertical.

Real rental price adjusts to make demand of capital equal supply.

Example: Cobb-Douglas production function

$$Y = AK^\alpha L^{1-\alpha}$$

so

$$\text{MPK} = \alpha A(L/K)^{1-\alpha}$$

so real rental price must be:

$$R/P = \alpha A(L/K)^{1-\alpha}$$

Shows things that affect the rental price of capital:

1) lower stock of capital, higher real rental price of capital ($\downarrow K \rightarrow \uparrow R/P$)
2) greater amount of labor, higher R/P ($\uparrow L \rightarrow \uparrow R/P$)
3) Better technology, higher R/P ($\uparrow \alpha A \rightarrow \uparrow R/P$)
c) Cost of Capital - Look at rental firms

Rent building or equipment (landlord of Café Roma)
Decision: how much capital to buy and rent out.

Benefit: real rental price R/P

Cost more complicated. Three costs:
1) Interest cost:
   If borrow money to buy capital, say
   nominal “purchase price of unit of capital” is $P_K$, and
   nominal interest rate is $i$.
   Then interest cost per unit of capital = $i P_K$

   Could also look at it this way. If rental firm has wealth, has choice: either buy capital and rent it out, or buy bond and collect nominal interest rate($i$). For each unit of capital buy at price $P_K$, are foregoing interest on bond worth $P_K$ could have bought, so opportunity costs is $i P_K$.

2) Depreciation:
   When in use, suffers wear and tear, which lowers its value. Called depreciation.
   Depreciation rate, $\delta$, is fraction of value lose per period because of wear and tear.
   So dollar cost off depreciation is $\delta P_K$.

3) capital loss
   The dollar price of the capital may change separate from wear and tear. Real estate prices went up in 1980s, and down in 1990s. Unit price of capital $P_K$ may up or down - gain or loss.
   Express cost as $-\Delta P_K$

Total cost of renting out unit of capital
   \[
   \text{total cost} = i P_K - \Delta P_K + \delta P_K \\
   = P_K (i - \Delta P_K / P_K + \delta)
   \]

Says cost of capital depends on price of capital, interest rate, the rate at which capital prices are changing, and depreciation rate.
example: if Roma Landlord: building cost 1m million,
   Say nominal interest rate = 0.10, so interest cost = $100,000 per year.
   Building prices are rising 6% a year, so excluding wear and tear, capital gain
   of $60,000.
   Say depreciation rate is 20%, so depreciation cost is 200,000 a year.
   So cost of capital = 100,000 - 60,000 + 200,000 = $240,000

   To make expression simpler, assume price of capital goods $\Delta P_k / P_k$ rises with
   prices of other goods.
   Then equal rate of inflation. since nominal interest rate minus inflation equals
   real interest rate, we can write:

   \[ \text{Cost of Capital} = P_k(r + \delta) \]

   Express in real term:
   \[ \text{“Real cost of Capital”} = (P_k/P)(r + \delta) \]

   says real cost of capital depends on relative price of capital good, real interest
   rate and depreciation rate.
d) **Determinants of investment** - still looking at rental firm for moment

Decision of whether to buy more capital to rent out or not:

Rental firm will maximize its profit, which is function of revenue IT gets and costs IT faces for capital:
- extra revenue it gets for extra unit of capital it buys is rental price it can rent it out for.
- Extra costs for extra unit of capital is the cost of capital developed above.

\[ \Delta \text{profit} = \Delta \text{revenue} - \Delta \text{cost} \]

\[ = \frac{R}{P} - \left( \frac{P_g}{P} \right) (r + \delta) \]

But recall from production firm story that the market for rental capital will require that \( \text{MPK} = \frac{R}{P} \)

\[ \Delta \text{profit} = \text{MPK} - \left( \frac{P_g}{P} \right) (r + \delta) \]

Interpret:
- Rental firm will increase its capital stock if it profitable to do so -- if marginal product of capital is greater than the cost of capital.

Note: We can see we do not need make distinction between production firms and rental firms. One step story rather than two, with same result:

If firm buys its own capital, not rent, this condition will determine how much it buys - it looks directly at MPK, rather than at \( \frac{R}{P} \), which market equilibrium will make equal to MPK.

So if \( \Delta \text{profit} > 0 \), then \( \Delta K > 0 \). Call this positive net investment:

**Net investment:** change in capital stock (excludes investment expenditure to replace depreciating capital)

This is a function of gap between benefit and cost of capital:

\[ I_n = \Delta K = I_n [\text{MPK} - \left( \frac{P_g}{P} \right) (r + \delta)] \]

Can also define more usual investment expenditure, gross, which adds in expenditure on replacing depreciated capital.

**Gross investment:** combined expenditure on new capital and replacement of depreciating capital
\[ I = \ln + \delta K \]
\[ = \ln \left[ MPK - \left( \frac{P_k}{P} \right) (r + \delta) \right] + \delta K \]

Conclude:
- Investment expenditure is a negative function of the real interest rate
- Investment also depends on:
  1) \( MPK \), which must equal \( R/P \), the real rental price
  2) the cost of capital, which is a function of the relative purchase price of a unit of capital \( (P_k/P) \)
  3) depreciation.

Model shows why investment expenditure depends on interest rate: rise in \( r \) raises the cost of capital. So it reduces the amount of profit from owning capital and reduces the incentive to accumulate more capital.

Can represent in curve: investment function slopes down:

Investment may also depend on current income, if there are borrowing constraints the firms face. If firm want to buy new capital, like new factory, it may need to borrow financing to purchases it, because costly. - loans from bank or sell bonds to public.

Sometimes firms face borrowing constraints, like consumers. As a result, when income rises in economy and revenues rise for firm, firm has extra cash it can use to carry out investment projects it had wanted to do.

Evidence: Study of Keiretsu in Japan, big and well connected to banks. No trouble getting financing for investment projects. Other firms small. Tend to carry out investment projects when have more liquidity: cash flowing through firm because big order paid for, etc.
2) Residential Investment

Simple model of housing market: supply and demand for houses. When supply increases, this creating of new houses is residential investment.

First part: Show how relative price of housing $P_H/P$ is determined by supply and demand for existing stock of houses.

We take supply of housing at a point in time as fixed. Can’t create more housing instantly, takes time.

Demand curve slopes down because high prices cause people to live in smaller houses, etc

Second part: shows how relative price of housing determines supply of new houses.

Construction firms buy materials and hire labor to build houses, then sell houses at market price. Their costs depend on overall price level $P$, and revenue depends on price of houses $P_H$. The higher the relative price of housing, more will build.

This increase in supply, building of new houses, is residential investment.

Effect of income:

Rise in income increases demand for housing. Shift in demand curve raises relative price of housing. Suppliers will start to build more.

Residential investment is positive function of income.

Effect of interest rate:

Rise in interest rate makes it more costly to pay interest payments on mortgage on house. Can afford to buy less house. Often means bank will only let you borrow a smaller amount up front, because fear interest costs higher than can afford.

Residential investment is negative function of interest rate.

Note: tax treatment odd: allowed to deduct nominal interest payments from taxes. So is like a subsidy for borrowing and living in home. Some say this has caused over investment in housing relative to business capital.
3) Inventory investment

Small part of investment (1% of GDP), but is important for study of short-run fluctuations because fluctuates the most.
In typical recession, more than half of fall in sending comes from decline in inventory investment.
Increase in inventories is inventory investment. If not add to inventories, but allow sales to draw inventories down, is negative inventory investment.

4 reasons for holding inventories:
1) Production smoothing: produce ahead for Christmas rush
2) as factors of production: need spare parts
3) avoid stock-outs: if demand is higher than expected, not want to risk running out.
4) work in progress: if between steps in production process

Affect of income:
If economy in boom, want more inventories: more spare parts, more in work progress, greater risk of running out...

Affect of interest rate:
If put things in inventory to sell tomorrow rather than a sale today, are forgoing income today. Will get sales income tomorrow, but if sell today, this income could have been put in bank and earned interest. so interest rate shows opportunity cost of holding inventories.

Conclusions on Keynesian investment function: \[ I = I(r) \]

1) All three components are negative functions of the real interest rate.

2) May well be positive functions of income. This changes the multipliers:
So might be that \[ I = Ibar - hr + dY \].
Take \( r \) as fixed \( r = rbar \) (Keynesian cross)
\[ Y = C + I + G \]
\[ C = Cbar + bY \]
So: \[ Y = Cbar + bY + Ibar - h rbar + dY + Gbar \]
\[ Y = (b + d)Y + Cbar + Ibar + Gbar - h rbar \]
\[ Y = [1/(1-b-d)] x [Cbar + Ibar + Gbar] - h rbar \]

Government spending multiplier here is \([1/(1-b-d)]\), which is larger than the usual multiplier, since \( d \) is greater than zero.
The intuition is that any initial rise in income will give rise to extra rounds of investment expenditure as well as consumption expenditure.