Lecture notes: 101/105
(revised 10/13/00)

Lecture 5: Growth part 1: Solow Growth Model
(1st half of chapter 4)

1a) Definitions
Want to understand output per person.

Know what determines the amount of output:
\[ Y = F(K,L) \]

Can divide by L, for representation of output per worker:
\[ \frac{Y}{L} = F\left(\frac{K}{L},\frac{L}{L}\right) \]

Can do this if production function is constant returns to scale, where using 1/L as the Z we are multiplying by.

Rewrite with \( y = \frac{Y}{L}, k = \frac{K}{L} \):
\[ y = F(k,1) \text{ or } y = f(k) \]

example: cobb-douglas:
\[
\begin{align*}
Y &= A K^{\alpha} L^{1-\alpha} \\
\frac{Y}{L} &= A \left(\frac{K}{L}\right)^\alpha \left(\frac{L}{L}\right)^{1-\alpha} \\
y &= A k^\alpha
\end{align*}
\]

So to find out what determines output per worker, need know what determines level of capital per worker.

b) What determines capital stock?
 Increase capital stock by investing: creation of new capital.
 But over time old capital wear out: depreciation.

\[ \Delta k = \text{investment} - \text{depreciation} \]

c) Consider **Depreciation**: 
Will assume a certain fraction of capital stock wears out each year. Call \( \delta \) the depreciation rate. Example, if piece of capital lasts average of 25 years, then depreciation rate is 4% per year, \( \delta = 0.04 \).

Amount of capital that depreciates each year is: Total depreciation = \( \delta k \).
d) What determines level of investment?:

begin with national income accounts in per capita terms, where for moment abstract from government expenditure.

\( y = c + i \)

Solow model assumes that the consumption function takes a simple form:
A certain constant fraction of income is consumed. Remainder is saved, so write:

\[ c = (1 - s) y \]

(like consumption function in last lecture, but with no constant term)

where \( s \) = saving rate: fraction of income saved.

Put back into accounting identity:

\[ y = (1 - s)y + i \]

or \( i = sy \)

since investment equals saving, investment will also be a constant fraction of output, the fraction that is saved.

Recall from above that output depends on \( k \):

\[ y = f(k) \]

so \( i = s f(k) \)

e) Put these together:

\[ \Delta k = \text{investment - depreciation} = sf(k) - \delta k \]

When investment exceeds depreciation, then capital stock will on net be increasing.
shows are two forces at work: If starting off at low capital stock, k1, depreciation is very low. But investment is higher. So capital stock will on net be accumulating. Moving toward right on bottom axis year by year.

But if start of at high capital stock, k2, depreciation is very high, while investment is lower. So on net will be losing capital stock - not able to replace all that is depreciating. So will be moving to left on bottom axis year by year.

Why is it that if starting from a low capital stock, it tends to increase over time, while if starting at high, it tends to decrease over time?

At low capital stock, each unit of capital is very productive, so each unit produces much income, hence there is much saving and investment generated per unit of capital.

But at high levels of capital per labor, each unit of capital is contributing less to output, and hence generating less saving and hence investment. So then it is possible that depreciation exceeds new investment.

There is one level of capital stock at which is no tendency for k to change. Once reach it, will stay there.

**Steady state level of capital:** level of k where k neither increases nor decreases, that is level of k where will settle to in long run.

**f) Numerical example:**

suppose production function:  $Y = K^{1/2}L^{1/2}$

generate per worker $Y/L = K^{1/2}L^{1/2}/L$

or $y = k^{1/2}$

assume s = 0.3, and δ=0.1

suppose begin at k = 4

then we can compute that $I = 0.3*4^{1/2} = 0.6$

$\delta k = 0.1 \times 4 = .4$

so $\Delta k = 0.6 - 0.4 = 0.2$

So second year $k_2 = k_1 + \Delta k = 4.2$

If do it again, get $k_3 = 4.395, k_4 = 4.584, k_5 = 4.768, \ldots$ eventually converges to $k^* = 9$.

**Algebraic way to compute the steady state:**

know $k^*$ defined by $\Delta k = 0$

where $\Delta k = sf(k^*) - \delta k^*$

so $0 = sf(k^*) - \delta k^*$

or $k^*/f(k^*) = s/\delta$

in our case $k^*/(k^{1/2}) = .3/.1$

$k^*/2 = 3$

$k^* = 9$

and steady state output: $y^* = k^{*1/2} = 3$
2) **Convergence:**

An implication of Solow growth model is:

- **convergence:** tendency of a poor economy to grow at a higher rate per capital than a rich economy and thereby to catch up to the rich economy.

Look at data on growth rates in regions and countries. (shown in class) Regions within the US converge, but countries in the world seem not to converge.

Reconsider theory: If two countries are different in these respects, will converge to different steady states. US states similar to each other in production technology and saving behavior. Europeans fairly similar. Countries of whole world show much more diversity. So refine our conclusion:

- **Conditional convergence:** convergence of two economies depends on whether their saving rates, production functions or other parameters are the same.

3) **Change in saving rate:**

Consider what happens if country raises its saving rate from $s_1$ to $s_2$. Then for each unit of output, more is being saved and invested in higher capital stock. *Curve $s^*f(k)$ shifts up:*

When saving rises, investment now above depreciation again, so capital stock begins to rise over time. Not in steady state any more, steady state has shifted to new point $k^*_2$.

But note when $k$ rises to $k^*_2$, it will stop growing again. **Implies 2 lessons in Solow growth model:**

1) Saving rate is a key determinant of the steady-state capital stock. If the saving rate is high, the economy will have a large capital stock and high output level in steady state.

2) An increase in the saving rate will raise the steady state levels of $k$ and output, but it will only generate growth temporarily, while the economy is converging to the new steady state.