CHAPTER 3 National Income:
Where it Comes From and Where it Goes
(chapter 3) revised 9/21/09

Introduction

- In the last lecture we defined and measured some key macroeconomic variables.
- Now we start building theories about what determines these key variables.
- In the next couple lectures we will build up theories that we think hold in the long run, when_________________.
- Called __________________ theory.

The Neoclassical model

Is a general equilibrium model:
- Involves multiple _____.
- each with own supply and demand
- _____ in each market adjusts to make quantity demanded equal quantity supplied.
Neoclassical model
The macroeconomy involves three types of markets:
1. 
2. needed to produce goods and services
3. 

Are also three types of agents in an economy:
1. Households
2. Firms
3. Government

Three Markets – Three agents

Neoclassical model
Agents interact in markets, where they may be demander in one market and supplier in another

1) Goods market:
Supply:
Demand: by households for consumption, government spending, and other firms demand them for investment
Neoclassical model

2) Labor market (factors of production)
Supply:
Demand:

3) Financial market
Supply:
Demand: firms borrow funds for investment; government borrows funds to finance expenditures.

Neoclassical model

- We will develop a set of equations to characterize supply and demand in these markets

- Then use algebra to solve these equations together, and see how they interact to establish a general equilibrium.

- Start with production...

Part 1: Supply in goods market:

Production

Supply in the goods market depends on a production function:

\[ Y = F(K, L) \]

Where

\( K = \)

\( L = \)
The production function

- shows how much output (\(Y\)) the economy can produce from \(K\) units of capital and \(L\) units of labor.
- reflects the economy’s level of technology.
- Generally, we will assume it exhibits \textit{constant returns to scale}.

Returns to scale

Initially \(Y_1 = F(K_1, L_1)\)

Scale all inputs by the same factor \(z\):
\[
K_2 = zK_1 \quad \text{and} \quad L_2 = zL_1 \quad \text{for} \ z > 1
\]
(If \(z = 1.25\), then all inputs increase by 25%)

What happens to output, \(Y_2 = F(K_2, L_2)\) ?

- If _____ returns to scale, \(Y_2 = zY_1\)
- If _____ returns to scale, \(Y_2 > zY_1\)
- If _____ returns to scale, \(Y_2 < zY_1\)

Exercise: determine returns to scale

Determine whether each the following production function has constant, increasing, or decreasing returns to scale:

\[
F(K, L) = 2K + 15L
\]
Assumptions of the model

1. Technology is fixed.
2. The economy’s supplies of capital and labor are fixed at

\[ K = \bar{K} \quad \text{and} \quad L = \bar{L} \]

Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

So we have a simple initial theory of supply in the goods market:

\[ \bar{Y} = F(\bar{K}, \bar{L}) \]

Part 2: Equilibrium in the factors market

- Equilibrium is where factor supply equals factor demand.
- Recall: Supply of factors is fixed.
- Demand for factors comes from firms.
Demand in factors market

Analyze the decision of a typical firm.

- It buys labor in the labor market, where ____________.
- It rents capital in the factors market, ____________.
- It uses labor and capital to produce the good, which it sells in the goods market, at price P.

Assume the market is competitive:

Each firm is small relative to the market, so its actions do not affect the market prices.

It takes ________________________________

It then chooses the optimal quantity of Labor and capital to maximize its profit.

How write profit:

Profit = revenue - labor costs - capital costs

=
Demand in the factors market

- Increasing hiring of L will have two effects:
  1) Benefit:
  2) Cost:

- To see how much output rises, we need the marginal product of labor (MPL)

Marginal product of labor (MPL)

An approximate definition (used in text):
The extra output the firm can produce using one additional labor (holding other inputs fixed):

\[ MPL = F(K, L + 1) - F(K, L) \]

The MPL and the production function

As more labor is added:

Slope...
Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).

- Intuition:
  - ↑\(L\) while holding \(K\) fixed
  - \(\Rightarrow\) fewer machines per worker
  - \(\Rightarrow\) lower productivity

MPL with calculus

We can give a more precise definition of MPL:

\[
\text{MPL} = \lim_{\Delta L \to 0} \frac{F(K, L + \Delta L) - F(K, L)}{\Delta L}
\]

where \(\Delta\) is 'delta' and represents change

- Earlier definition assumed that \(\Delta L = 1\).
  \[F(K, L + 1) - F(K, L)\]
- We can consider smaller change in labor.

MPL as a derivative

As we take the limit for small change in \(L\):

\[
\text{MPL} = \lim_{\Delta L \to 0} \frac{F(K, L + \Delta L) - F(K, L)}{\Delta L} = f_L(K, L)
\]

Which is the definition of the (partial) derivative of the production function with respect to \(L\), treating \(K\) as a constant.

This shows the slope of the production function at any particular point, which is what we want.
The MPL and the production function

![Diagram showing production function and MPL]

Derivative as marginal product

2) \( Y = F(L) = 3L^2 = 3\sqrt{L} \)

\[ \frac{\partial Y}{\partial L} = f_L = \frac{6}{3L} \]

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<th>L</th>
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<tbody>
<tr>
<td>( F(L) ):</td>
<td>6</td>
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<td>( f_L ):</td>
<td>0.75</td>
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Return to firm problem: hiring \( L \)

Firm chooses \( L \) to maximize its profit. How will increasing \( L \) change profit?

\( \Delta \) profit = \( \Delta \) revenue - \( \Delta \) cost

= ________

If this is: > 0 should ________
< 0 should ________
= 0 ________
Firm problem continued

So the firm’s demand for labor is determined by the condition:

Hires more and more $L$, until $MPL$ falls enough to satisfy the condition.

Also may be written:

where $W/P$ is the 'real wage'

Real wage

Think about units:

- $W = ____$
- $P = _____$
- $W/P = ($/hour) / ($/good) = _____$

The amount of purchasing power, measured in units of goods, that firms pay per unit of work

Example: deriving labor demand

- Suppose a production function for all firms in the economy:

  $Y = K^{0.5}L^{0.5}$

  $MPL = _____$

  Labor demand is where this equals real wage:
Labor demand continued

or rewrite with $L$ as a function of real wage

$$0.5K^{0.5}L^{0.5} = \frac{WP}{P}$$

$$\left(0.5K^{0.5}L^{0.5}\right)^2 = \left(\frac{W}{P}\right)^2$$

$$\frac{1}{0.25}K^{-1}L = \left(\frac{P}{W}\right)^2$$

$L_{demand} = \ldots$

So a rise in wage $\rightarrow$ ________________

rise in capital stock $\rightarrow$ ________________

Labor market equilibrium

Take this firm as representative, and sum over all firms to derive aggregate labor demand. Combine with labor supply to find equilibrium wage:

demand: $0.5K^{0.5}(L_{demand})^{0.5} = \frac{W}{P}$

supply: $L_{supply} = \bar{L}$

equilibrium: ________________

So rise in labor supply $\rightarrow$ ________________

MPL and the demand for labor

Each firm hires labor up to the point where $\frac{MPL}{MPL} = \frac{W}{P}$
Determining the rental rate

We have just seen that \( MPL = \frac{W}{P} \)

The same logic shows that \( MPK = \)______:

- diminishing returns to ______: \( MPK \downarrow \) as \( K \uparrow \)
- The \( MPK \) curve is the firm’s demand curve for renting capital.
- Firms maximize profits by choosing \( K \) such that \( MPK = \)______.

How income is distributed:

We found that if markets are competitive, then factors of production will be paid their marginal contribution to the production process.

- total labor income = ________________
- total capital income = ________________

Euler’s theorem:

Under our assumptions (constant returns to scale, profit maximization, and competitive markets)...

- total output is divided between the payments to capital and labor, depending on their marginal productivities, ________________.

\[
\bar{V} = MPL \times \bar{L} + MPK \times \bar{K}
\]
Mathematical example
Consider a production function with Cobb-Douglas form:
\[ Y = AK^\alpha L^{1-\alpha} \]
where \( A \) is a constant, representing technology

Show this has constant returns to scale:
multiply factors by \( Z \):
\[
F(ZK, ZL) = A (ZK)^\alpha (ZL)^{1-\alpha} \\
= Z \cdot A \cdot K^\alpha L^{1-\alpha} \\
= Z \cdot F(K, L)
\]

Mathematical example continued

• Compute marginal products:
  \[
  \text{MPL} = \\
  \text{MPK} = 
  \]

• Compute total factor payments:
  \[
  \text{MPL} \cdot L + \text{MPK} \cdot K = Y \\
  \]
  
  So total factor payments equals total production.

Outline of model
A closed economy, market-clearing model
Goods market:
\( \text{DONE} \) Supply side: production
\( \text{Next} \) Demand side: C, I, and G
Factors market
\( \text{DONE} \) Supply side
\( \text{DONE} \) Demand side
Loanable funds market
  □ Supply side: saving
  □ Demand side: borrowing
Demand for goods & services

Components of aggregate demand:

\[ C = I = G \]

\( G \) = government demand for g & s

(closed economy: no \( NX \))

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Consumption, \( C \)

- def: disposable income (\( Y-T \)) is:

- Consumption function: 
  Shows that

- def: The marginal propensity to consume (MPC) is

- So MPC = derivative of the consumption function with respect to disposable income
- MPC must be between 0 and 1.

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The consumption function

\[ C = C(Y-T) \]

The slope of the consumption function is the MPC.
Consumption function cont.

Suppose consumption function:

\[ C = 10 + 0.75Y \]

MPC = _____

For extra dollar of income, spend _____ dollars consumption

Marginal propensity to save = _____

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Investment, \( I \)

- The investment function is \( I = I(r) \), where \( r \) denotes the real interest rate, the nominal interest rate corrected for inflation.

- The real interest rate is:
  - the cost of borrowing
  - the opportunity cost of using one’s own funds to finance investment spending.

So, _______

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The investment function

![Graph](chart.png)

Spending on investment goods is a downward-sloping function of the real interest rate.
Government spending, $G$

- $G$ includes government spending on goods and services.
- $G$ excludes transfer payments
- Assume government spending and total taxes are exogenous:
  \[ G = \bar{G} \quad \text{and} \quad T = \bar{T} \]

The market for goods & services

- Agg. demand:
- Agg. supply:
- Equilibrium:
  \[ \text{The real interest rate} = \]

We can get more intuition for how this works by looking at the loanable funds market

The loanable funds market

A simple supply-demand model of the financial system.

One asset: "loanable funds"
- demand for funds: investment
- supply of funds: saving
- "price" of funds: real interest rate
Demand for funds: Investment

The demand for loanable funds:
- **comes from investment:**
  Firms borrow to finance spending on plant & equipment, new office buildings, etc.
  Consumers borrow to buy new houses.
- **depends negatively on** \( r \), the "price" of loanable funds (the cost of borrowing).

Supply of funds: Saving

The supply of loanable funds comes from saving:
- **Households** use their saving to make bank deposits, purchase bonds and other assets.
  These funds become available to firms to borrow to finance investment spending.
- **The government** may also contribute to saving if it does not spend all of the tax revenue it receives.
Types of saving

- private saving =
- public saving =
- national saving, $S$
  $= \text{private saving} + \text{public saving}
  = (Y - T) - C + T - G$

EXERCISE:
Calculate the change in saving
Suppose $\text{MPC} = 0.8$ and $\text{MPL} = 20$.
For each of the following, compute $\Delta S$:

a. $\Delta G = 100$
b. $\Delta T = 100$

Answers

$\Delta S = \Delta Y - \Delta C - \Delta G = ...$

a. $\Delta S =$
b. $\Delta S =$
digression:

Budget surpluses and deficits

- When $T > G$,
  \[ \text{budget} = (T - G) = \text{public saving} \]

- When $T < G$,
  \[ \text{budget} = (G - T) \]
  and public saving is negative.

- When $T = G$,
  budget is balanced and public saving = ___.

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Loanable funds supply curve

\[ S = Y - C(V - T) - \bar{G} \]

National saving does not depend on $r$, so the supply curve is vertical.

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Loanable funds market equilibrium

\[ I(r) \]

Equilibrium real interest rate

Equilibrium level of investment
## The special role of \( r \)

\( r \) adjusts to equilibrate the goods market and the loanable funds market simultaneously:

If L.F. market in equilibrium, then
\[
Y - C - G = I
\]

Add \((C + G)\) to both sides to get
\[
Y = C + I + G \quad \text{(goods market eq’m)}
\]

Thus, 

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<tr>
<th>Eq’n in L.F. market</th>
<th>Eq’n in goods market</th>
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## Algebra example

Suppose an economy characterized by:

- **Factors market supply:**
  - labor supply = **1000**
  - Capital stock supply = **1000**

- **Goods market supply:**
  - Production function: \( Y = 3K + 2L \)

- **Goods market demand:**
  - Consumption function: \( C = 250 + 0.75(Y - T) \)
  - Investment function: \( I = 1000 - 5000r \)
  - \( G = 1000, \ T = 1000 \)

## Algebra example continued

Given the *exogenous variables* \((Y, G, T)\), find the equilibrium values of the *endogenous variables* \((r, C, I)\)

Find \( r \) using the goods market equilibrium condition:

\[
Y = C + I + G
\]

\[
5000 = 250 + 0.75(5000 - 1000) + 1000 - 5000r + 1000
\]

\[
5000 = 5250 - 5000r
\]

So \( r = \boxed{0.5} \)

And
\[
I = 1000 - 5000 \times 0.05 = \boxed{900}
\]
\[
C = 250 + 0.75(5000 - 1000) = \boxed{2250}
\]
Mastering the loanable funds model

Things that shift the saving curve

a. ________
   i. ________: changes in $G$ or $T$

b. ________
   i. preferences
   ii. tax laws that affect saving (401(k), IRA)

CASE STUDY
The Reagan Deficits

- Reagan policies during early 1980s:
  - increases in defense spending: $\Delta G > 0$
  - big tax cuts: $\Delta T < 0$

- According to our model, both policies reduce national saving:

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{I}) - \bar{G}$$

$$\uparrow \bar{G} \Rightarrow \downarrow \bar{S} \quad \downarrow \bar{T} \Rightarrow \uparrow \bar{C} \Rightarrow \downarrow \bar{S}$$

1. The Reagan deficits, cont.

1. The increase in the deficit reduces saving...

2. …which causes the real interest rate to rise...

3. …which reduces the level of investment.
Chapter summary

1. Total output is determined by
   - how much capital and labor the economy has
   - the level of technology

2. Competitive firms hire each factor until its marginal product equals its price.

3. If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

Chapter summary

4. The economy's output is used for
   - consumption
     (which depends on disposable income)
   - investment
     (depends on the real interest rate)
   - government spending
     (exogenous)

5. The real interest rate adjusts to equate the demand for and supply of
   - goods and services
   - loanable funds

Chapter summary

6. A decrease in national saving causes the interest rate to rise and investment to fall.
An increase in investment demand causes the interest rate to rise, but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.