CHAPTER 7 Learning objectives

- Learn the closed economy Solow model
- See how a country’s standard of living depends on its saving and population growth rates
- Learn how to use the “Golden Rule” to find the optimal savings rate and capital stock

selected poverty statistics

In the poorest one-fifth of all countries,
- daily caloric intake is 1/3 lower than in the richest fifth
- the infant mortality rate is 200 per 1000 births, compared to 4 per 1000 births in the richest fifth.
- ...
Income and poverty in the world
selected countries, 2000

Huge effects from tiny differences

In rich countries like the U.S., if government policies or "shocks" have even a small impact on the long-run growth rate, they will have a huge impact on our standard of living in the long run...

Huge effects from tiny differences

<table>
<thead>
<tr>
<th>annual growth rate</th>
<th>percentage increase in standard of living after...</th>
</tr>
</thead>
<tbody>
<tr>
<td>of income per capita</td>
<td>...25 years</td>
</tr>
<tr>
<td>2.0%</td>
<td>64.0%</td>
</tr>
<tr>
<td>2.5%</td>
<td>85.4%</td>
</tr>
</tbody>
</table>
Huge effects from tiny differences

If the annual growth rate of U.S. real GDP per capita had been just one-tenth of one percent higher during the 1990s, the U.S. would have generated an additional $449 billion of income during that decade.

The lessons of growth theory

...can make a positive difference in the lives of hundreds of millions of people.

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and our government's policies

The Solow Model

- due to Robert Solow, won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policy making
  - benchmark against which most recent growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run
How Solow model is different from Chapter 3’s model

1. ________________
   investment causes it to grow, 
depreciation causes it to shrink.

2. ________________
   population growth causes it to grow.

3. The consumption function is simpler.

How Solow model is different from Chapter 3’s model

4. No $G$ or $T$
   (only to simplify presentation; 
   we can still do fiscal policy experiments)

5. Cosmetic differences.

The production function

- In aggregate terms: $Y = F(K, L)$
- Define: $y = \quad$ ________________
  $k = \quad$ ________________
- Assume ________________:
  $zY = F(zK, zL)$ for any $z > 0$
- Pick $z = 1/L$. Then
  $Y/L = F(K/L, 1)$
  $y = F(k, 1)$
  $y = f(k)$ where $f(k) = F(k, 1)$
The production function

Output per worker, \( y \)

Capital per worker, \( k \)

\[ f(k) \]

\[ MPK = \quad \]

Note: this production function exhibits ________ MPK.

The national income identity

\[ Y = C + I \]  (remember, no \( G \))

In “per worker” terms:

\[ \quad \]

where \( c = \quad \) and \( i = \quad \)

The consumption function

\( s = \) the saving rate,

\( s \) is an exogenous parameter

Note: \( s \) is the only lowercase variable that is not equal to its uppercase version divided by \( L \)

\[ \quad \]

(\( s \) is an exogenous parameter)

Consumption function: \[ \quad \] (per worker)
Saving and investment

- Saving (per worker) = \( y - c \)
  
  = 
  
  = 

- National income identity is \( y = c + i \)

Rearrange to get: 

(Investment = saving, like in chap. 3!)

- Using the results above,

Output, consumption, and investment

Output per worker, \( y \)

Capital per worker, \( k \)

Depreciation

\( \delta = \) the rate of depreciation

= 

\( \delta k \)

Capital per worker, \( k \)
**Capital accumulation**

The basic idea:
Investment makes the capital stock bigger, depreciation makes it smaller.

**Capital accumulation**

Change in capital stock = investment – depreciation

\[ \Delta k = \_ \_ - \_ \_ k \]  

Since \( i = sf(k) \), this becomes:

\[ \Delta k = \_ \_ \_ \_ \_ \_ \]

**The equation of motion for \( k \)**

\[ \Delta k = sf(k) - \delta k \]

- the Solow model’s central equation
- Determines behavior of capital over time...
- ...which, in turn, determines behavior of all of the other endogenous variables because they all depend on \( k \). E.g.,
  - income per person: \( y = \_ \_ \_ \_ \_ \_ \_ \)
  - consump. per person: \( c = \_ \_ \_ \_ \_ \_ \)
The steady state

\[ \Delta k = sf(k) - \delta k \]

If investment is just enough to cover depreciation \([sf(k) = \delta k]\), then capital per worker will remain constant:

\[ \underline{\text{___________}}. \]

This constant value, denoted \(k^*\), is called the \(\underline{\text{_______________________}}\).

Moving toward the steady state

\[ \Delta k = sf(k) - \delta k \]

A numerical example

Production function (aggregate):

\[ Y = F(K, L) = \sqrt{K \times L} = K^{1/2}L^{1/2} \]

To derive the per-worker production function, divide through by \(L\):

\[ \underline{Y \over L} = \underline{\text{______________}} \]

Then substitute \(y = Y/L\) and \(k = K/L\) to get

\[ Y = f(k) = \underline{\text{______}} \]
A numerical example, cont.

Assume:
- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

Approaching the Steady State: A Numerical Example

<table>
<thead>
<tr>
<th>Year</th>
<th>$k$</th>
<th>$y$</th>
<th>$c$</th>
<th>$i$</th>
<th>$\delta k$</th>
<th>$\Delta k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>2.00</td>
<td>1.40</td>
<td>0.60</td>
<td>0.400</td>
<td>0.200</td>
</tr>
<tr>
<td>2</td>
<td>4.20</td>
<td>2.04</td>
<td>1.43</td>
<td>0.61</td>
<td>0.420</td>
<td>0.195</td>
</tr>
<tr>
<td>3</td>
<td>4.39</td>
<td>2.09</td>
<td>1.47</td>
<td>0.62</td>
<td>0.440</td>
<td>0.189</td>
</tr>
<tr>
<td>4</td>
<td>4.58</td>
<td>2.14</td>
<td>1.49</td>
<td>0.64</td>
<td>0.458</td>
<td>0.184</td>
</tr>
<tr>
<td>10</td>
<td>5.60</td>
<td>2.37</td>
<td>1.66</td>
<td>0.71</td>
<td>0.560</td>
<td>0.150</td>
</tr>
<tr>
<td>25</td>
<td>7.35</td>
<td>2.70</td>
<td>1.89</td>
<td>0.81</td>
<td>0.732</td>
<td>0.080</td>
</tr>
<tr>
<td>100</td>
<td>8.96</td>
<td>2.99</td>
<td>2.09</td>
<td>0.89</td>
<td>0.896</td>
<td>0.002</td>
</tr>
<tr>
<td>$\infty$</td>
<td>9.00</td>
<td>3.00</td>
<td>2.10</td>
<td>0.90</td>
<td>0.900</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Exercise: solve for the steady state

Continue to assume $s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

Use the equation of motion

$$\Delta k = sf(k) - \delta k$$

to solve for the steady-state values of $k$, $y$, and $c$. 
Solution to exercise:

\[ \Delta k = 0 \] def. of steady state

\[ sf(k^*) = \delta k^* \] eq'n of motion with \( \Delta k = 0 \)

Solve to get: \( k^* = 9 \) and \( y^* = \) _______

Finally, \( c^* = (1 - s)y^* = 0.7 \times 3 = 2.1 \)

An increase in the saving rate

An increase in the saving rate raises investment...
...causing the capital stock to grow toward a new steady state:

Prediction:

- Higher \( s \) \( \Rightarrow \) _______
- And since \( y = f(k) \), higher \( k^* \) \( \Rightarrow \) _______
- Thus, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.
The Golden Rule: introduction

• Different values of $s$ lead to different steady states. How do we know which is the "best" steady state?
• Economic well-being depends on consumption, so the "best" steady state has the highest possible value of consumption per person: $c^* = (1-s)f(k^*)$
• An increase in $s$:
  • ______________________________________
  • ______________________________________
• So, how do we find the $s$ and $k^*$ that maximize $c^*$?

The Golden Rule Capital Stock

$k^*_{gold} \equiv \text{the Golden Rule level of capital},$

To find it, first express $c^*$ in terms of $k^*$:

$$c^* = y^* - i^*$$
$$= f(k^*) - i^*$$
$$= f(k^*) - \delta k^*$$
Then, graph $f(k^*)$ and $\delta k^*$, and look for the point where the gap between them is biggest.

The Golden Rule Capital Stock

$\delta k^*$

$\delta k^*$

$k^*_\text{gold}$

The Golden Rule Capital Stock

$c^* = f(k^*) - \delta k^*$ is biggest where

$\frac{dc^*}{dk^*} = \text{MPK} - \delta$

We want to maximize: $c^* = f(k^*) - \delta k^*$

From calculus, at the maximum we know the derivative equals zero.

Find derivative: $\frac{dc^*}{dk^*} = \text{MPK} - \delta$

Set equal to zero: $\text{MPK} - \delta = 0$ or $\text{MPK} = \delta$
The transition to the Golden Rule Steady State

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust $s$.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k > k_{gold}$ then increasing $c^*$ requires a __________.

In the transition to the Golden Rule, consumption is _______ at all points in time.

Starting with too little capital

If $k < k_{gold}$ then increasing $c^*$ requires an ________.

Future generations enjoy higher consumption, but the current one experiences __________.
Population Growth

- Assume that the population—labor force—grow at rate \( n \). (\( n \) is exogenous)
  \[
  \frac{\Delta L}{L} = n
  \]

- EX: Suppose \( L = 1000 \) in year 1 and the population is growing at 2%/year (\( n = 0.02 \)).
  Then \( \Delta L = n L = 0.02 \times 1000 = 20 \), so \( L = 1020 \) in year 2.

Break-even investment

\[(\delta + n)k = \text{break-even investment},\]

Break-even investment includes:
- \( \_\_\_ \) to replace capital as it wears out
- \( \_\_\_ \) to equip new workers with capital
  (otherwise, \( k \) would fall as the existing capital stock would be spread more thinly over a larger population of workers)

The equation of motion for \( k \)

- With population growth, the equation of motion for \( k \) is
  \[
  \Delta k = \text{actual investment} - \text{break-even investment}
  \]
The impact of population growth

An increase in \( n \) causes an increase in break-even investment, leading to a decrease in capital per worker, \( k \).

Prediction:

- Higher \( n \) \( \Rightarrow \) lower \( k^* \).
- And since \( y = f(k) \), lower \( k^* \) \( \Rightarrow \) lower \( y \).
- Thus, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

International Evidence on Population Growth and Income per Person

[Graph showing data points for different countries' income per person in 1992 and population growth rates from 1960 to 1992.]

- [Data points for various countries like Chad, Kenya, Zimbabwe, Cameroon, Pakistan, Uganda, India, Indonesia, Israel, Mexico, Brazil, Peru, Egypt, Singapore, U.S., U.K., Canada, France, Finland, Japan, Denmark, Ivory Coast, Germany, Italy, etc.]

Population growth (percent per year, average 1960-1992)
The Golden Rule with Population Growth

To find the Golden Rule capital stock, we again express \( c^* \) in terms of \( k^* \):

\[
c^* = y^* - i^* = f(k^*) - \frac{\text{\footnotesize MPK}}{\delta + n}
\]

\( c^* \) is maximized when \( \text{MPK} = \delta + n \)
or equivalently,

In the Golden Rule Steady State, the marginal product of capital equals the population growth rate.

Chapter Summary

1. The Solow growth model shows that, in the long run, a country’s standard of living depends
   - positively on its saving rate.
   - negatively on its population growth rate.
2. An increase in the saving rate leads to
   - higher output in the long run
   - faster growth temporarily
   - but not faster steady state growth.

Chapter Summary

3. If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.

   If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.