Question 1: Small Open Economy with Investment

a) Write the intertemporal budget constraint:

combine $Y_1 - I_1 - G_1 - C_1 = B_1$

with $Y_2 + (1+r_1)B_1 - G_2 - C_2 = 0$

$$Y_2 + (1+r_1)(Y_1 - I_1 - G_1 - C_1) - G_2 - C_2 = 0$$

or

$$A_2 K_2^{\alpha} + (1+r_1)(Y_1 - K_2 - G_1 - C_1) - G_2 - C_2 = 0$$

Rewrite problem:

$$\max_{C_1, C_2, K_2} \; U(C_1) + \beta U(C_2)$$

$$s.t. \; (Y_1 - K_2 - G_1 - C_1) + \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2 - C_2\right) = 0$$

FOCs:

$$U'_{C_1} = \lambda$$

$$\beta U'_{C_2} = \lambda \frac{1}{1+r_1}$$

$$\lambda\left(\frac{1}{1+r} A_2 \alpha K_2^{\alpha-1} - 1\right) = 0, \; A_2 \alpha K_2^{\alpha-1} = 1 + r_1$$

so

$$U'_{C_1} = \beta(1+r_1)U'_{C_2}, \quad C_2 = \beta(1+r_1)C_1$$

$$I_1 = K_2 = \left(\frac{\alpha A_2}{1+r_1}\right)^{\frac{1}{1-\alpha}}$$

b) The household wishes to smooth consumption, subject to the consumption-tilting incentive created by the interest rate. It will invest up to the point that the marginal product of capital equals the gross return on the bond.

A higher level of productivity term (*A*) raises the marginal product of capital, calling for more investment to lower marginal product back to the level of the return on bonds. A rise in the interest rate requires a rise in the marginal product of capital, which implies a higher level of investment.

c) Solve for C_1 using the long-run budget constraint and consumption Euler:

$$C_1 = Y_1 - K_2 - G_1 + \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2\right) - \frac{1}{1+r_1} C_2$$

$$C_1 = Y_1 - K_2 - G_1 + \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2\right) - \beta C_1$$

$$C_1 = \left[Y_1 - K_2 - G_1 + \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2\right) \right]\left(\frac{1}{1+\beta}\right)$$

Substitute this into the period 1 budget constraint (where $CA_1 = B_1$ in this case):

$$CA_1 = Y_1 - I_1 - G_1 - C_1 = Y_1 - I_1 - G_1 - \left[Y_1 - K_2 - G_1 + \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2\right) \right]\left(\frac{1}{1+\beta}\right)$$

$$CA_1 = \left[\beta Y_1 - \beta I_1 - \beta G_1 - \frac{1}{1+r_1}\left(A_2 K_2^{\alpha} - G_2\right) \right]\left(\frac{1}{1+\beta}\right)$$

Sub in investment function:

$$CA_1 = \left[\beta Y_1 - \beta G_1 + \frac{G_2}{1+r_1} - \beta\left(\frac{\alpha A_2}{1+r_1}\right)^{\frac{1}{1-\alpha}} - \frac{1}{1+r_1} A_2 \left(\frac{\alpha A_2}{1+r_1}\right)^{\frac{\alpha}{1-\alpha}} \right]\left(\frac{1}{1+\beta}\right)$$

$$CA_1 = \left[\beta Y_1 - \beta G_1 + \frac{G_2}{1+r_1} - \left(\frac{\alpha A_2}{1+r_1}\right)^{\frac{1}{1-\alpha}} \left(\beta + \alpha^{-1}\right) \right]\left(\frac{1}{1+\beta}\right) \qquad (1)$$

d) A higher level of G_1 lowers the CA, as the household smoothes consumption over the fall in net output in period 1. A higher level of G_2 raises the CA, as households save for the future obligation.

e) A higher level of A_2 lowers the current account in period 1. This is for two reasons: the need to finance investment expenditure, and because the extra output in the future leads to a fall in saving as households smooth consumption.

f) Assuming the case where G_1=G_2=0, equation (1) above shows that the net effect of a rise in r is to raise the CA. There are three types of effects at work, as seen in the long run budget constraint combined with the consumption Euler:

$$CA_1 = Y_1 - I_1 - G_1 - \frac{1}{\beta(1+r)}\left(Y_2 + (1+r_1)B_1 - G_2\right),$$

where r affects I_1, Y_2, and the discounting of future income by 1/(1+r):

1) Consumption tilting toward future, raises CA
2) lowers investment expenditure, which raises CA
3) lower capital means lower output in future. Consumption smoothing requires a rise in saving in period 1, which raises the CA.

Bonus: Note: it will never be optimal to have non-positive K and just go long on bonds, because for a K arbitrarily near zero, the MKP will be arbitrarily high, and this will always be higher than the interest rate on bonds.

Question 2: Interest Rate Parity Under Capital Controls

a) First order conditions:
(1) Consumption: $\boldsymbol{U'_{C,t} = P_t \lambda_t}$

(2) Home bonds: $\beta E_t[\lambda_{t+1}](1+i_t)=\lambda_t$

(3) Foreign bonds: $\beta E_t\left[e_{t+1}(1-\tau_{t+1})\lambda_{t+1}\right]\left(1+i^*_t\right)=e_t\lambda_t$

b) Interest rate parity: $$e_t\left(\frac{1+i_t}{1+i_t^*}\right)=E_t\left[e_{t+1}(1-\tau_{t+1})\frac{U'_{c,t+1}}{P_{t+1}}\right]\Big/E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

rewrite:

$$e_t\left(\frac{1+i_t}{1+i_t^*}\right)=\left(E_t\left[e_{t+1}(1-\tau_{t+1})\right]E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]+\text{cov}_t\left[e_{t+1}(1-\tau_{t+1}),\frac{U'_{c,t+1}}{P_{t+1}}\right]\right)\Big/E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

$$=E_t\left[e_{t+1}(1-\tau_{t+1})\right]+\text{cov}_t\left[e_{t+1}(1-\tau_{t+1}),\frac{U'_{c,t+1}}{P_{t+1}}\right]\Big/E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

The equation indicates that foreign assets would have to offer a higher interest rate when the risk premium term above is negative. This would be true when the exchange rate is negatively correlated with marginal utility: foreign currency assets pay off poorly (low e) when marginal utility is high (consumption low). Foreign assets are a bad hedge in this case against consumption risk. Note that the reverse would be true if the covariance term were positive.

c) The Fama regressions of the change in e on the forward rate or on the interest differential is:

$$(e_{t+1}-e_t)=a_0+a_1(i_t-i^*_t)+\varepsilon_t$$

Results show a_1 does not equal one, and usually is less than zero. This says that when the home interest rate is high, this predicts currency appreciation in the future. This indicates the omitted risk premium is time varying and highly volatile, acting as the dominant driver of the exchange rate.

d) In the above solution, if $e_{t+1}(1-\tau_{t+1})$ is guaranteed to be constant at unity each period and agents know this, then that pair of terms will drop out of the expectation operator, and the covariance term will be zero in the risk premium. So the capital tax should be set at $\tau_s=1-\frac{1}{e_s}$, for example. The capital tax is set each period to take back capital gains due to foreign currency appreciation.

Question 3: International Output Comovement

a) Backus, Kehoe and Kydland (1992) found international output correlation was high relative to international consumption correlations. Perri-Quadrini (2011) showed this comovement was particularly high during the financial crisis rising from 0.3 to 0.7.

b) In BKK (1992) financial integration was complete, which worked to reduce output comovement. This was due to more efficient reallocation of production resources away from one country towards another with higher productivity. But this effect can be reversed by adding various frictions to the financial market in the model. In this case financial integration means that credit

shocks or asset valuations in one country can be transmitted abroad, and affect output in both countries. Choose one case and elaborate: balance sheet effects and leverage constraint. (in in Devereux-Yetman 2010), International Banks (Kollmann et al, 2011) , credit constraints with transmission of shock tightening constraint to other countries (Perri and Quadrini 2011), or credit shocks with equalized spreads across countries (Dedola and Lombard, 2012).

c) Burstein, Kurz and Tesar (2008) showed that trade associated with production sharing raises GDP correlation, whereas trade of final goods is associated with lower GDP correlation. If goods from the two countries are substitutes, then shocks affect countries oppositely; if goods complements that productions levels tend to move together. (Could also cite papers dealing with the trade collapse.)

d) Evidence against trade: trade is still a small share of GDP for most countries, whereas asset trade has grown much.
Evidence in favor of financial argument: financial variables correlated across countries for crisis period. (Kollmann, Perri-Quadrini)
Against financial argument: tends to create high consumption correlation, higher than output, which is counterfactual.

Question 4: International Price Puzzle and Monetary Policy

a) The volatility of the real exchange rate shows that PPP does not hold at each point in time. Unit root tests on the real exchange rate indicate that PPP holds as a long-run condition, but the halflife is long, usually 3-5 years. The real exchange rate is highly volatile and persistent. It is also correlated with the real exchange rate.

b) Chari, Kehoe McGrattan (2003) showed how a sticky price model with prices in local currency can explain this. Volatility required a low intertemopral elasticity (to make the interest rate move a lot in response to shocks, which made the nominal exchange rate move more as in UIP). Persistence was helped by staggered contacts. Could also cite Johri and Lahiri (2008) which used the CKM model augmented with learning by doing and habits in leisure to get added persistence. (Could also cite Rodriguez. Note cannot cite Obstfeld and Rogoff 1995 for this, since PPP held in their model because they used price stickiness in the currency of the producer.

c) Engel (2012) showed in a model of LCP price stickiness that if there is exchange rate misalignment, there is a misallocation of resources between countries. This results because it is not efficient for consumes to pay different prices for the same good (in the absence of trade costs). Can also cite Devereux-Engel (2003), but this paper showed that in the case where PPP fails (LCP) there is not much benefit to using monetary policy to manipulate the nominal exchange rate, as it does not affect consumer decisions between goods.