

The Endogeneity of the Exchange Rate as a Determinant of FDI: A Model of Entry and Multinational Firms

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Abstract

This paper argues that when the exchange rate and projected sales in the host country are jointly determined by underlying macroeconomic variables, standard regressions of FDI flows on both exchange rate levels and volatility are subject to bias. The results hinge on the interaction of macroeconomic uncertainty, a sunk cost, and heterogeneous productivity across firms. They indicate that a multinational firm's response to increases in exchange rate volatility will differ depending on whether the volatility arises from shocks in the firm's native or host country. It is the first study to depart from the representative-firm framework in an analysis of direct investment behavior with money.

JEL Classifications: F1, F2, F4

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1 Introduction

Foreign direct investment (FDI) has become an increasingly important channel for resource flows across national borders. In 1990, production by overseas branches of multinational firms reached 16 percent of the world's total manufacturing output (Lipsey 1998). The share of FDI in total net resource flows to developing countries more than doubled during the 1990s, reaching 82 percent in 2001 (IMF 2002). In industrialized countries, the ratio of foreign direct investment (FDI) to total gross fixed capital formation ranges from zero to almost fifty percent, depending on the country and the year.¹ Thus, the questions of how and why FDI responds to exchange rate fluctuations, first raised in the 1980s, have become acutely relevant to open-economy macroeconomic analysis. This paper addresses the issue by expanding the general equilibrium analysis characteristic of recent optimum currency area theory—where the exchange rate depends on “fundamental” variables that may also impact local demand—to encompass the entry behavior of the multinational firm.

Although empirical and partial-equilibrium analyses suggest that exchange rate uncertainty may be important in a firm's decision to engage in production activity overseas, the literature incorporating the multinational enterprise (MNE) into models of the global economy has remained separate from studies of exchange rate behavior and policy. Previous studies of exchange rate variability and multinational firms have often treated the exchange rate as exogenous.² The model presented here combines the concept of an exchange rate driven by macroeconomic variables with a sunk cost, which motivates firms' sensitivity to uncertainty in the trade and industrial organization literature. The results indicate that while macroeconomic volatility in the MNE's native country and the country hosting its direct investment venture *both* increase exchange rate volatility, they can have quite different effects on flows of FDI.

In addition, the model here is unique in that it departs from the representative-firm framework to look at the effect of volatility on entry. The introduction of heterogeneous productivity levels across firms, based on Melitz (2003), explains why smaller, less productive firms might be deterred from investing overseas by uncertain macroeconomic conditions, while larger, more productive firms are not. A comparison of the zero-cutoff profit conditions for domestic and foreign entrants also allows the decomposition of the impact of monetary volatility into factors affecting all firms and those affecting only entering foreign firms through their exposure to exchange rate fluctuations. The analysis presented in Section 4 illustrates the two effects and provides a theoretical explanation for the observation by Hausmann and Fernandez-Arias (2000) that FDI appears to be cushioned from some types of macroeconomic risk that have been shown to curtail other types of private investment. It also reconciles the conflicting estimates of the direction of the effect of exchange

¹In Japan, for example, the ratio of FDI inflows to gross fixed capital formation has remained close to zero for over fifteen years, whereas in 2000, the figure peaked at 16 percent in the United States and 49 percent in the U.K. (IMF 2003 and 2004).

²Notable exceptions are Goldberg and Kolstad (1995), who conjecture that the effect of exchange rate volatility on FDI should depend on the correlation of exchange rate shocks with demand shocks, and Aizenman (1992) and Devereux and Engel (2001), which are the first models incorporating the MNE into a general equilibrium framework with money, endogenous exchange rates, and a representative firm.

rate volatility on flows of FDI evident in empirical studies based on partial equilibrium models.

The argument presented in this paper rests on two premises. First, it assumes that there is a repeated sunk cost involved in production at home and overseas—some fixed overhead cost such as legal retainers, rental and maintenance contracts, or a property tax that is paid, negotiated, or legislated in advance. In this respect, the model draws on the option value literature sparked by Pindyck (1988), Dixit and Pindyck (1994), and Campa (1993), as well as on trade models incorporating multinational firms with plant-level fixed costs (Horstmann and Markusen 1992 and Brainard 1997) and sunk costs, which are defined here as fixed costs that must be paid before the realization of a random shock (Grossman and Razin 1985 and Helpman, Melitz, and Yeaple 2003). Because the sunk cost is paid or negotiated in one period under a given exchange rate, but revenues are earned and repatriated at a later date, firms care about fluctuations in the value of the host-country currency.

Second, it is assumed that there are common macroeconomic forces that influence both the exchange rate and the volume of sales by overseas branches. These forces could involve productivity growth or any number of unobservable variables governing the international asset market. However, this study focuses on fluctuations in the growth rate of the money supply as the mechanism influencing both realizations of the exchange rate and, due to sticky prices, the demand for consumption goods in the host country. The exchange rate is a function of the ratio of the home (host-country) and foreign (native-country) money supply. It covaries negatively with the host country's demand for goods, as a positive shock to the home money supply weakens the value of the home currency but simultaneously increases real income—and therefore sales by both domestically owned firms and multinationals operating in the home market. Conversely, a contractionary monetary shock in the host country may generate a more favorable exchange rate at which to convert profits, but will depress local sales. In comparison, a contractionary monetary shock arising in the MNE's native country can adversely affect the value of the home currency with no mitigating influence (or, in the case of complete markets, an exacerbating influence) on sales overseas. Firm owners—consumers—can hedge against these macroeconomic shocks using nominal bonds, but the sunk cost and segmented goods markets introduce an asymmetry which reduces the effectiveness of the hedging when monetary volatility is not equal across countries, so that uncertainty impacts firm entry decisions nonetheless.

Thus, there are two rigidities driving the results of the model. The real rigidity, a sunk cost, motivates firms' sensitivity to uncertainty. The nominal rigidity, sticky prices, causes the uncertainty generated by monetary shocks to influence both the exchange rate and consumer demand in the host market. Because the exchange rate and the demand for goods are impacted differently by monetary shocks, depending on their origin, the analysis below indicates that studies regressing FDI flows on measures of exchange rate volatility may be subject to bias. Considering FDI and exchange rates as variables jointly determined by underlying macroeconomic factors provides an explanation of why the empirical literature analyzing whether exchange rate variability encourages or deters investment by multinational firms remains inconclusive.

The remainder of the text is organized as follows: First, insights from existing theoretical and empirical studies which guided the construction of this model are discussed in Sections 1.1 and 1.2. Sections 2.1 and 2.2 describe the consumer’s optimization problem and relevant first-order conditions. Section 2.3 defines an expression for expected discounted profits and describes the firm’s pricing behavior under uncertainty. Section 2.4 explains the calculation of aggregate productivity and the price level and 2.5 explains how aggregate productivity is related to the “threshold” productivity level, or the labor productivity of the least productive entrant. Part 3 presents the key equilibrium conditions governing investment behavior. A special section, 3.1, is devoted to discussing issues of geographic preference and asset-market structure. An analysis of the results is presented in Part 4, where the decision criteria of foreign investors considering a direct investment venture is decomposed into factors affecting all firms operating in the host market and factors rooted in exchange-rate risk, which affect only entrants from overseas. It evaluates the net impact of monetary policy variables on entry by domestic and foreign-owned firms, as well as implications for aggregate prices and consumption in the host market. Part 5 concludes the paper with a discussion of the results and possibilities for future research.

1.1 The Theoretical Debate

Existing partial equilibrium models provide important insight into the mechanics of the MNE’s decision-making behavior, but treat exchange rate fluctuations as exogenous, isolating them from macroeconomic shocks that simultaneously affect demand. Consequently, theoretical arguments based on these models are divided as to whether exchange rate uncertainty will increase or decrease FDI. Authors proposing that exchange rate variations could promote investment abroad assert the long-standing result in trade theory that cross-border investment is a substitute for trade when tariffs or other barriers prevent the free flow of goods (Goldberg and Kolstad 1995, Cushman 1985 and 1988). Mundell (1957) provides the first mathematical proof of this result. Numerous studies provide evidence that exchange rate uncertainty may function as a *de facto* trade barrier, implying by default that it should increase FDI.³ A related position espouses the “production flexibility” approach— that volatility increases the value of having a plant in both countries, enabling an MNE to decide at any time either to export from home or to produce in its foreign facility, depending on where conditions are most favorable (Sung and Lapan 2000). Assuming that exchange rate fluctuations are exogenous, multinational firms can take advantage of them by shifting production to the countries where the value of the local currency makes input costs look cheapest, *ceteris paribus*.⁴ In earlier work, Itagaki (1981) develops a financial flexibility argument. He posits that

³Examples of such studies include Cushman (1983) and Dell’Ariccia (1999). Cote (1994) and Barkoulas, Baum, and Caglayan (2002) discuss the conflict that exists within the sizeable literature investigating exchange-rate volatility and trade.

⁴An emerging literature focuses on the implications of skewness in the distribution of exchange rate shocks, demonstrating that investors will hurry to invest in a country whose currency appears to have suddenly depreciated to a level below its expected value. Such an extreme depreciation reduces the burden of fixed entry costs paid in local currency, resulting in a temporary surge in “firesale” FDI (Chakrabarti and Scholnick 2002, Krugman 1998). This research is inspired by studies such as Froot and Stein (1991) and Blonigen (1997) (and challenged by Stevens (1998)),

an increase in exchange rate risk may incite a firm to invest abroad as a way of hedging against a short position in its balance sheet. A depreciation of the firm's home currency might reduce the value of domestic assets relative to foreign liabilities, but would simultaneously increase the value of assets and revenue streams for its affiliates in foreign countries.

However, theoretical models also exist predicting that exchange rate uncertainty will instead suppress FDI. These arguments assert that unpredictable fluctuations in the exchange rate introduce added uncertainty into both the production costs and future revenues of overseas operations, deterring potential investors. Several studies (Rivoli and Salorio 1996 and Campa 1993), rooted in the work of Pindyck (1988) and Dixit and Pindyck (1994), declare that currency volatility deters the entry of multinational firms by increasing the "option value" associated with waiting before incurring the sunk costs necessary to produce overseas. They consider that a firm effectively holds an option to invest overseas in any given period. A fixed cost paid in advance (sunk) acts as an exercise price. The return from exercising the option is the expected present discounted value of profits earned from production in the foreign country. Exchange rate risk introduces uncertainty about the size of the return, increasing the value of holding on to the option to wait and motivating the firm to postpone investing until a future period. A salient feature of this literature is that the results hold even for risk-neutral firms, as the key engine is the sunk cost. Without it, there would be no cost to producing when the prevailing exchange rate allows positive returns and exiting when it does not, eliminating any value attached to waiting.

Several models which predict that exchange-rate volatility may discourage FDI are driven instead by risk-aversion in firm management. Cushman (1985 and 1988) and Goldberg and Kolstad (1995) specify conditions under which exchange-rate volatility may reduce the certainty-equivalence value of expected profits from overseas operations— a deterrent to risk-averse prospective investors. The authors show that a deterrent effect arises as long as demand and the elasticity of technical substitution are of a form such that they completely offset the effect of currency fluctuations on profit remittances. More concretely, exchange rate volatility will deter FDI as long as a depreciation of the host-country currency, which would undercut the value of repatriated profits in terms of the firm's native-country currency, is not met by an offsetting increase in host-country demand or reduction in host-country input costs. Like the literature contending that uncertainty promotes FDI, these studies either omit any simultaneous effects of underlying macroeconomic variables on demand and the exchange rate, or consider a correlation between the two but do not explicitly characterize its relationship to the underlying variables.

There are two studies which incorporate multinational firms within a general equilibrium approach, where the exchange rate is endogenous, but they also generate conflicting results. The first study, Aizenman (1992), juxtaposes the production flexibility approach onto the Dixit-Pindyck-type conceptualization of the option value.⁵ Increases in volatility increase the value of diversification,

which analyze FDI and exchange rate levels.

⁵Aizenman explicitly discusses the value of having the option to produce in either of two plants located in two different countries (production flexibility). The sunk investment cost in his model also creates a financial option value like that discussed above, deterring investment in the face of uncertainty.

which pushes firms to shift production to the country where it is cheapest, but also discourages investment by increasing the uncertainty surrounding the return on exercising the Dixit-Pindyck option to invest abroad. Aizenman demonstrates that a floating exchange rate will transmit the effects of country-specific shocks across national borders, which erodes the ability of firms to diversify risk by shifting production across borders. In this sense, the exchange rate volatility associated with a flexible regime can be construed as deterring FDI. In the second study, Devereux and Engel (2001) find that when firms price in the currency of the local market they are serving (pricing-to-market), production by all firms, including the affiliates of multinationals, is higher under a flexible exchange rate than a fixed exchange rate. Hence, exchange-rate volatility associated with a flexible regime is loosely linked here with increased production by multinationals. These models are extremely important contributions in that they are the first to incorporate the multinational firm into an open-economy framework with money. Notwithstanding, it is difficult to interpret them in light of the question, “Does exchange rate volatility deter FDI?” because they incorporate a representative firm, implying that either all firms invest abroad or none do.

1.2 Empirical Evidence

Existing empirical tests are based on the partial equilibrium models described above or on gravity models and offer two principal conclusions: (1) it is not clear what relationship exists between exchange-rate uncertainty and FDI and (2) local fixed costs make it more likely that FDI will be discouraged by exchange-rate volatility. With regard to the first issue, both Cushman (1985 and 1988) and Goldberg and Kolstad (1995) find that volatility increases the willingness of U.S. MNEs to locate facilities abroad, in accordance with the early trade theory explaining FDI as a substitute for exports. Zhang (2001) supports their results, finding a positive and significant relationship between exchange rate volatility and FDI flowing into the European Union (EU) from both inside and outside the EU. Nevertheless, there is ample evidence to the contrary. Whereas Sekkat and Galgau (2004) also find a positive link for flows between EU nations, they find that increases in the variance of bilateral exchange rates deter inflows originating outside of the EU. Amuedo-Dorantes and Pozo (2001) report that results may not be robust to the way volatility is measured: there is a positive coefficient associated with volatility of the exchange rate measured as a the standard deviation within a rolling window but a negative coefficient emerges when a GARCH construction is used. Both coefficients are significant. Chakrabarti and Scholnick (2002) find a negative relationship between exchange rate volatility and FDI flows from the U.S. to 20 OECD countries. Using micro-level data, Campa (1993) shows that volatility deters entry by foreign firms contemplating investment in the U.S.

There is less conflict over the influence that fixed costs exert over direct investment behavior. Two papers present evidence that the fixed costs prominent in trade and industrial-organization literature examining multinational firms are important in understanding the response of MNEs to exchange rate uncertainty.⁶ Campa’s (1993) study shows that the negative impact of exchange

⁶See Russ (2004) for a survey of FDI and fixed costs in trade and IO theory.

rate uncertainty on entry by MNEs is more probable and more profound when sunk costs are large. He explains this phenomenon using the options theory reasoning: in the *absence* of sunk costs, a firm would simply produce overseas in any period when conditions were favorable, “and volatility would have no effect on the entry decision (p.619).” Although Tomlin (2000) indicates that some of Campa’s econometric results may be sensitive to model specification, she clearly demonstrates that local fixed costs, such as advertising expenses, are alone sufficient to quell entry by foreign firms deciding whether to produce in the U.S.

Finally, Goldberg and Kolstad (1995) present empirical evidence explicitly calling into question the practice of considering exchange rate volatility and shocks to demand in the host-country market as separate entities. They emphasize that for the majority of the sample used in their analysis, a depreciation of host-country currency is associated with a simultaneous increase in the host-country’s demand for goods (Goldberg and Kolstad 1995, p.866). They further show that the share of capacity firms choose to locate abroad may be affected by covariance between host country demand and exchange rate movements, but provide mixed evidence as to the direction of the relationship and its robustness across countries. The result invites the construction of a general equilibrium framework tying demand and the bilateral exchange rate to common underlying fundamental variables, depicting the multinational firm’s response to the net effect that macroeconomic shocks exert on both the exchange rate and sales abroad. This paper is the first to introduce FDI into a new open-economy macroeconomic model— where exchange rates and local demand are jointly determined— while preserving the fixed-cost component emphasized in analyses of direct investment within the trade and industrial-organization literature and introducing the heterogeneity which governs entry dynamics in recent trade models.⁷

2 The Model

The model below capitalizes on the insights of partial equilibrium theoretical examinations of MNE behavior and existing empirical evidence, while exploiting the capacity of a general equilibrium framework to connect both demand and the exchange rate to fluctuations in a common underlying variable— money. It builds on the conceptualization of MNEs put forth by Devereux and Engel (2001), the first study that has incorporated multinational firms into an Obstfeld-Rogoff-type general equilibrium model with money. In their model with two countries (Home and Foreign), there is no trade in goods— all goods are produced in the country where they are consumed. Only the profits of multinational firms cross national borders, as in Figure 1. The model here incorporates a local (i.e. plant-level) sunk cost to motivate the sensitivity of direct investment activity to exchange-rate volatility observed in the data. It also adds heterogeneity across firms to explain why exchange rate uncertainty combines with the sunk cost to deter entry by some firms— but not all— into the overseas market.

To this end, this paper superimposes Devereux and Engel’s MNE onto a framework originally

⁷For a survey and empirical testing of the implications of heterogeneity in trade theory, see Bernard, Jensen, and Schott (2003).

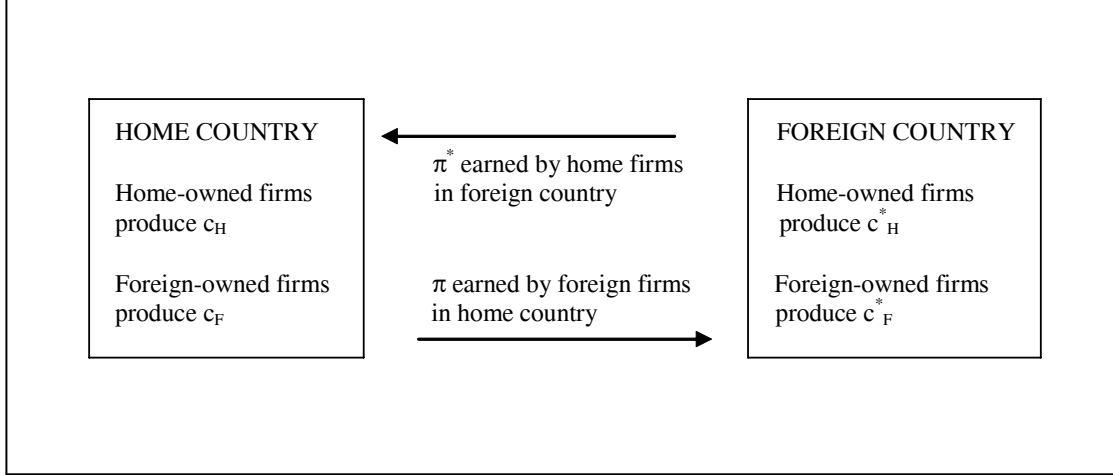


Figure 1: Global Map

developed by Melitz (2003) to explain the decision of firms to export. Melitz's analysis does not include the nominal rigidities (sticky prices) used here, but introduces heterogeneity among firms through a random productivity parameter. It accounts for the entry and exit of firms within the industry, while allowing positive profits for the most productive firms. This approach has been used by Helpman, Melitz, and Yeaple (2003) to explain why some firms export and others invest abroad, also in an economy without nominal rigidities. The two-country model here involves modifying the optimizing behavior of consumers and producers in Melitz's study by introducing risk aversion, as does Ghironi and Melitz (2005).⁸

2.1 The Consumer's Problem

In the open-economy model, the representative consumer in the Home country maximizes lifetime utility subject to an intertemporal budget constraint in a setting of complete international asset markets:

$$\max_{\{C_t, B_{t+1}, L_t, M_t\}} E_t \left[\sum_{t=0}^{\infty} \beta^t U_t(C_t, \frac{M_t}{P_t}, L_t) \right]$$

$$s.t. \quad P_t C_t + M_t + \sum_{z^{t+1}|z^t} q(z^{t+1}|z^t) B(z^{t+1}) = W_t L_t + \pi_t + M_{t-1} + B_t$$

where $0 < \beta < 1$ and $q(z^{t+1}|z^t)$ is the price at time t of the bond $B(z^{t+1})$, which is denominated in Home currency and has a payoff of one unit of home currency given that one of a set of possible states (z) of the macroeconomy is realized at time $t+1$.⁹ Utility is a function

⁸The study here does not assume *ex ante* that wages are equal across countries. However, it is confined to a two-country world, rather than Melitz's more generalized n -country economy.

⁹ $\sum_{z^t|z^{t-1}} B(z^t)$ represents a complete set of state-contingent bonds denominated in the Home currency. The probability of particular states are not explicitly represented here to simplify the exposition. They can be seen in Chari, Kehoe, and McGrattan (2002).

of aggregate consumption, C , and labor, L ,

$$U_t = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln \left(\frac{M_t}{P_t} \right) - \kappa L_t.$$

Aggregate consumption is an index reflecting preferences with constant elasticity of substitution (CES) across the set of all goods produced by both Home- and Foreign-owned firms ($n_H(t), n_F(t) \leq 1$),¹⁰

$$C_t = \left[\int_0^{n_H(t)} c_H(i, t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F(t)} c_F(i, t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}},$$

The aggregate price index is

$$P_t = \left[\int_0^1 p_H(i, t)^{1-\mu} di + \int_1^2 p_F(i, t)^{1-\mu} di \right]^{\frac{1}{1-\mu}}. \quad (1)$$

The demand curves for individual Home and Foreign goods are given by

$$c_H(i, t) = \left(\frac{p_H(i, t)}{P_t} \right)^{-\mu} C_t \quad c_F(i, t) = \left(\frac{p_F(i, t)}{P_t} \right)^{-\mu} C_t. \quad (2)$$

Multiplying $c_j(i, t)$ by $p_j(i, t)$ for $j = H, F$, an expression for the total expenditure on a particular good, $r_j(i, t)$, can be derived

$$r_j(i, t) = p_j(i, t) c_j(i, t) = \left(\frac{p_j(i, t)}{P_t} \right)^{1-\mu} R_t$$

where $R_t = P_t C_t$, the total consumer expenditure in the Home country. Since expenditures are the flip side of revenues in a general equilibrium framework, total expenditures on a particular good also equal total revenues for the firm which produces it. Analogous expressions apply in the Foreign country.¹¹

¹⁰ H stands for variables corresponding to Home-owned firms and F for variables corresponding to Foreign-owned firms.

¹¹That is,

$$\begin{aligned} c_H^*(i, t) &= \left(\frac{p_H^*(i, t)}{P_t^*} \right)^{-\mu} C_t^* & c_F^*(i, t) &= \left(\frac{p_F^*(i, t)}{P_t^*} \right)^{-\mu} C_t^* \\ r_H^*(i, t) &= \left(\frac{p_H^*(i, t)}{P_t^*} \right)^{1-\mu} R_t^* & r_F^*(i, t) &= \left(\frac{p_F^*(i, t)}{P_t^*} \right)^{1-\mu} R_t^*. \end{aligned}$$

2.2 First-Order Conditions and the Exchange Rate

First-order conditions, as in Devereux and Engel (2001),¹² yield the wage relation,

$$W_t = \kappa P_t C_t^\rho; \quad (3)$$

a money-demand equation,

$$\frac{M_t}{P_t} = \frac{\chi}{1 - E_t[d_{t+1}]} C_t^\rho; \quad (4)$$

where $d_{t+1} = \beta \frac{P_t C_t^\rho}{P_{t+1} C_{t+1}^\rho}$, the consumption-based nominal interest rate; and a bond-pricing equation,

$$q(z^{t+1}|z^t) = \beta \frac{P_t C_t^\rho}{P_{t+1} C_{t+1}^\rho}. \quad (5)$$

It is assumed that the growth rate of the money supply is a lognormally distributed random variable defined by

$$\frac{M_t}{M_{t-1}} = (1 + \psi) e^{\nu_t},$$

where ψ is a constant and ν_t is an i.i.d. random variable with a normal distribution of mean $-\frac{1}{2}\sigma_m^2$ and constant variance σ_m^2 .¹³ It is assumed that the variance is bounded such that the level of the money supply is always greater than zero and finite. Obstfeld and Rogoff (1998, p.39) provide an exact solution to (4) in the special case of logarithmic preferences for real balances used here. The solution demonstrates that consumption in a given period is a function only of real money balances and underlying parameters, or

$$C_t^\rho = \frac{M_t}{P_t} \left(\frac{1 - \beta\theta}{\chi} \right), \quad (6)$$

where $\theta = E_t \left[\frac{M_{t-1}}{M_t} \right] = \frac{e^{\sigma_m^2}}{1 + \psi}$, a constant restricted in this analysis so that $\theta < \frac{1}{\beta}$. Although no goods are traded in the model, the transfer of currency to a country experiencing a negative shock in the form of payoffs on bondholdings boosts real money balances (and thus consumption) due to the sticky-price mechanism depicted in the firm's maximization problem below.

Let S_t represent the nominal exchange rate, expressed as units of Home currency per unit of Foreign currency. Then, the bond-pricing equations for the Home- and Foreign-country consumers can be set equal to each other and iterated to show that the real exchange rate in this model is equal to the ratio of the marginal utility of consumption in each country, and, using the Home and Foreign versions of (6), the solution for S_t is:¹⁴

$$S_t = \frac{P_t C_t^\rho}{P_t^* C_t^{*\rho}} = \frac{M_t(1 - \beta\theta)}{M_t^*(1 - \beta\theta^*)}. \quad (7)$$

¹²First-order conditions are derived explicitly for this paper in Russ (2004b).

¹³The results are qualitatively identical whether or not the money-supply growth process is characterized by a mean preserving spread (i.e., whether ν_t is distributed $N(-\frac{1}{2}\sigma_m^2, \sigma_m^2)$ or $N(0, \sigma_m^2)$). It is assumed that the variance is bounded such that the level of the money supply is always greater than zero and finite.

¹⁴See full technical appendix for a detailed derivation, as outlined in Chari, Kehoe, and McGrattan (2002).

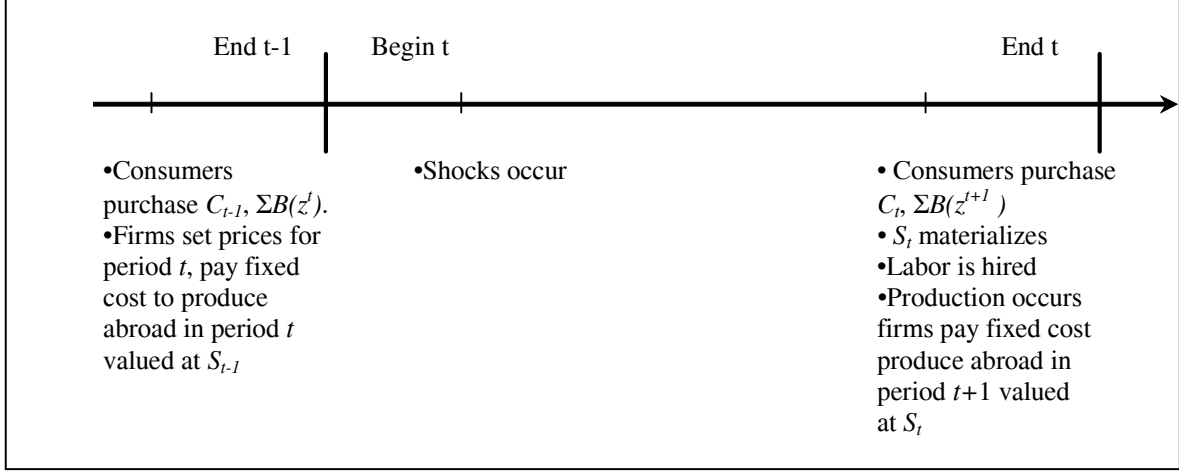


Figure 2: Timeline

Uncertainty regarding the exchange rate in this model stems directly from the randomly distributed disturbance in the the growth rate of the money supply in each country. Increased volatility in the disturbances implies greater uncertainty with regard to future levels of the exchange rate.

2.3 Firms

It is useful at this point to illustrate the timeline of events. Prospective market entrants are endowed with a permanent idiosyncratic productivity index, φ . At the end of period $t - 1$, a particular firm i owned by an entrepreneur in country j must decide whether it will produce in the following period and set the price for its unique good, which will apply to any production it undertakes in period t , as in Figure 2. Production is linear in labor and is characterized by the technology

$$c_H(i, t) = \varphi_H(i) l_H(i, t), \quad (8)$$

where $\varphi_H(i)$ is the productivity level of a Home firm i . The parameter is known at the time the firm decides whether to produce and what price to charge. The quantity $l_H(i, t)$ is the amount of labor used by Home firm i in its domestic plant in period t . Variables for consumption and production activity in the Foreign country are denoted by an asterisk, so that the identical technology for production by a Home firm abroad is represented by

$$c_H^*(i, t) = \varphi_H(i) l_H^*(i, t).$$

The firm takes the pricing and entry decisions of other firms as given and seeks to maximize the expected market value of total nominal profits from domestic and overseas plants. Producers anticipate potential fluctuations in demand and wages in the host country as a result of volatility in the host-country money supply. In addition, they consider potential fluctuations in the exchange

rate when deciding whether to enter the overseas market, which could occur due to monetary shocks both in the host-country and in its native country (that is, due to shocks to both M and M^*). They therefore place a subjective value on each potential state of the economy using a stochastic discount factor, $d_t = \beta \frac{P_{t-1}C_{t-1}^\rho}{P_t C_t^\rho}$ for Home firms, and $d_t^* = \beta \frac{P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}}$ for Foreign firms.¹⁵ The results are qualitatively similar whether the stochastic discount factor or a constant, \bar{d} , is used. This means that, as in the Dixit-Pindyck framework, the central results do not depend on whether firm managers are risk-averse or risk-neutral.

The firm's problem is not only to decide whether and how much to produce in its own country, but also whether to undertake production abroad:¹⁶

$$\max E_{t-1}[d_t (\pi_H(i, t) + \pi_H^*(i, t))],$$

where

$$\pi_H(i, t) = p_H(i, t)c_H(i, t) - W_t l_H(i, t) - P_t f ,$$

and

$$\pi_H^*(i, t) = S_t p_H^*(i, t)c_H^*(i, t) - S_t W_t^* l_H^*(i, t) - S_{t-1} P_t^* f_{MNE}^*.$$

Given its knowledge of $\varphi_H(i)$ and its expectations of economic conditions in the next period, each Home-owned firm decides whether to pay a fixed overhead cost, $P_t f$, in the Home country and $S_{t-1} P_t^* f_{MNE}^*$ in the Foreign country, if it chooses to invest overseas, becoming a multinational enterprise. Therefore, to calculate profits from operations abroad, a Home firm takes into account the exchange rate, S_t , at which it will have to pay wages for Foreign workers and repatriate revenues earned overseas, as well as the fixed overhead costs, which it agrees to pay before shocks materialize at the exchange rate S_{t-1} . The cost can be considered “sunk” because it is paid before the firm knows what its revenues will be from sales in the following period. However, it differs from the sunk cost conceptualized in Melitz (2003), where it is a one-time upfront payment required to draw the productivity parameter. To make the model tractible in the open-economy macroeconomic framework, it is assumed here that firms costlessly draw φ and that the sunk cost is a repeated overhead cost. As a result, this model foregoes important observed properties of firm dynamics, such as the endogenous persistence of entry decisions, which are emphasized in recent models of heterogeneous firms in general equilibrium, notably Ghironi and Melitz (2005), where the productivity threshold discussed below is a state variable because firms draw their permanent productivity parameter after they pay a one-time entry fee.

It is useful to emphasize here that this is a model of *horizontal* direct investment, where a firm

¹⁵The stochastic discount factor is the expected ratio of marginal utility in the present and immediate future, which serves as a measure of how much a shock in period t will impact the well-being of the consumers who own the firm (Cochrane 2001, Chapter 1). As mentioned in Section 2.2, the discount factor, d_t , is also a consumption-based nominal interest rate. In this sense, it represents the opportunity cost of investing in productive activities rather than in bonds at the end of period $t - 1$.

¹⁶Define the expression $E_{t-1}[d_{t-1}\pi_H^T(i, t)]$ as $E[d_{t-1}\pi_H^T(i, t)|\Omega_{t-1}]$, where Ω_{t-1} is the set of all variables observed through the end of period $t - 1$ (which includes $\varphi(i)$).

produces a unique good in multiple countries, but for local consumption in each country, not for trade or assembly somewhere else. The model abstracts from cross-border flows of physical capital. Cross-border capital flows, where imported capital is used in domestic production using domestic technology, are distinct from FDI, where a firm uses a single technology to produce identical goods in multiple countries. (See Russ (2004a) for a more detailed discussion and a literature review of studies modeling cross-border capital flows.) FDI in the case here is the payment of some fixed cost ($S_{t-1}P_t^*f_{MNE}^*$) to gain entry into the local market for a particular period. Examples of such recurring fixed costs include annual property taxes, retainer fees for local accounting and legal firms, or fees involved in maintaining local marketing and distribution networks.

Substituting $\frac{c_H(i,t)}{\varphi_H(i)}$ for $l_H(i,t)$ and taking the derivative of $E_{t-1}[d_t\pi_H(i)]$ with respect to $p_H(i,t)$, one can calculate the price a firm will set when it decides whether to enter the market:

$$\begin{aligned} \frac{\partial E_{t-1}[d_t\pi_H(i,t)]}{\partial p_H(i,t)} : E_{t-1} \left[d_t \left(c_H(i,t) + p_H(i,t) \left(\frac{\partial c_H(i,t)}{\partial p_H(i,t)} \right) - \frac{W_t}{\varphi_H(i)} \left(\frac{\partial c_H(i,t)}{\partial p_H(i,t)} \right) \right) \right] &= 0 \\ (\mu - 1)p_H(i,t)E_{t-1} [d_t c_H(i,t)] &= \mu E_{t-1} \left[\frac{W_t c_H(i,t)}{\varphi_H(i)} \right] \\ p_H(i,t) &= \frac{E_{t-1} [d_t W_t C_t]}{\alpha \varphi_H(i) E_{t-1} [d_t C_t]}, \end{aligned}$$

where α is the inverse of the markup ($\alpha = \frac{\mu-1}{\mu}$). A firm will set a price for its unique good equal to a fixed markup over the expected discounted marginal cost. Substituting the wage relation and the formula for consumption, equations (3) and (6), the pricing rule is

$$p_H(i,t) = \frac{\kappa(1 - \beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}{\alpha\chi\varphi_H(i)E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \quad (9)$$

For Foreign-owned firms deciding whether to operate in the Home market, the pricing rule reduces to a similar expression,¹⁷

$$\begin{aligned} p_F(i,t) &= \frac{E_{t-1} \left[d_t^* \frac{W_t C_t}{S_t} \right]}{\alpha\varphi_F(i)E_{t-1} \left[d_t^* \frac{C_t}{S_t} \right]} \\ &= \frac{\kappa(1 - \beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}{\alpha\chi\varphi_F(i)E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}. \end{aligned} \quad (10)$$

¹⁷Since the markup, the expected discounted wage, and the aggregate price level are the same for all firms, the ratio of output for any two firms i and j operating in the Home economy in period t is a function only of the ratio of $\varphi(i)$ to $\varphi(j)$:

$$\frac{c(i,t)}{c(j,t)} = \frac{p(i,t)^{-\mu} P_t^\mu C_t}{p(j,t)^{-\mu} P_t^\mu C_t} = \left(\frac{\varphi(i)}{\varphi(j)} \right)^\mu.$$

This key characteristic of the model is derived by substituting the pricing rules into the demand equations (2). Similarly, the revenues of any two firms are also a function of the ratio of their productivity levels.

2.4 Aggregation

There is a continuum of prospective entrants owned by the Home country over the $[0,1)$ interval and of prospective entrants owned by Foreign agents over the interval $[1,2]$. All prospective entrants costlessly draw a productivity parameter. However, because the fixed costs act as barriers to entry, only a certain fraction of prospective entrants will actually enter the Home market and produce goods for consumption. The variable $n_H(t)$ represents the proportion of Home-owned firms and $n_F(t)$ is the proportion of Foreign-owned firms which enter the Home economy. Then, there will be a continuum of goods produced by Home firms over $[0, n_H(t))$ and by Foreign firms over $[1, 1+n_F(t)]$ in the Home market. If all firms draw their productivity parameter from an identical underlying distribution, then in equilibrium, there will also be a distribution characterizing the productivity levels of all firms that decide to produce— $\eta_H(\varphi, t)$ for entering Home firms and $\eta_F(\varphi, t)$ for entering Foreign firms, both with positive support over a subset of $(0, \infty)$. The distribution $\eta_j(\varphi, t)$ (for $j = H, F$) reflects the probability that a firm has drawn a particular productivity level, given that the firm is sufficiently productive to enter the market with nonnegative expected profits in period t . Let $\bar{\varphi}_j(t)^{\mu-1}$ denotes the average output-weighted productivity level among country j -owned firms. Then, for any firm i owned by country j that enters the Home market, $\bar{\varphi}_j(t)^{\mu-1}$ is given by,¹⁸

$$\bar{\varphi}_j(t)^{\mu-1} \equiv \int_0^\infty \varphi_j(i)^{\mu-1} \eta_j(\varphi, t) d\varphi = \int_0^\infty \varphi^{\mu-1} \eta_j(\varphi, t) d\varphi$$

Define $\bar{p}_H(t)^{1-\mu}$ as the expected contribution to the overall price level of a Home good chosen at random from among Home firms in the economy. Then, because prices differ only according to each firm's level of productivity, $\bar{p}_H(t)^{1-\mu}$ is computed by using the equilibrium distribution of entrants' productivity levels to find the average contribution that the price of each available good

¹⁸More precisely, Melitz (2003) points out that $\bar{\varphi}_H(t)$ and $\bar{\varphi}_F(t)$ are expressions of the output-weighted harmonic mean of productivity levels for Home and Foreign firms operating in the Home economy during period t . To see this, note that (for $j = H, F$) the ratio of output for any two firms a and b will be

$$\frac{c_j(\varphi_j^a(t))}{c_j(\varphi_j^b(t))} = \left(\frac{\varphi_j^a(t)}{\varphi_j^b(t)} \right)^\mu.$$

Then,

$$\begin{aligned} \bar{\varphi}_j(t)^{\mu-1} &= \int_0^\infty \varphi_j(t)^{\mu-1} \eta_j(\varphi(t)) d\varphi \\ \bar{\varphi}_j(t)^{-1} &= \int_0^\infty \varphi_j(t)^{-1} \left(\frac{\varphi_j(t)}{\bar{\varphi}_j(t)} \right)^\mu \eta_j(\varphi(t)) d\varphi \\ &= \int_0^\infty \varphi_j(t)^{-1} \left(\frac{c_j(\varphi(t))}{c_j(\bar{\varphi}_j(t))} \right)^\mu \eta_j(\varphi(t)) d\varphi. \end{aligned}$$

makes to the aggregate price level:

$$\begin{aligned}\bar{p}_H(t)^{1-\mu} &= \int_0^\infty p_H(i, t)^{1-\mu} \eta_H(\varphi, t) d\varphi \\ &= \left(\frac{\kappa(1-\beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}{\alpha\chi E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{1-\mu} \bar{\varphi}_H(t)^{\mu-1}.\end{aligned}$$

Since the equilibrium distribution is the same for all Home firms, $\bar{p}_H(t)^{1-\mu}$ is also the same across all Home firms. Similarly, the expected price of a good chosen at random among Foreign firms in the economy is

$$\bar{p}_F(t)^{1-\mu} = \left(\frac{\kappa(1-\beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}{\alpha\chi E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{1-\mu} \bar{\varphi}_F(t)^{\mu-1}$$

The aggregate price level can be computed as though all Home-owned firms charged $\bar{p}_H(t)^{1-\mu}$ and all Foreign-owned firms charged $\bar{p}_F(t)^{1-\mu}$, as in Melitz (2002, p.7):

$$\begin{aligned}P_t &= \left[\int_0^{n_H(t)} \bar{p}_H(t)^{1-\mu} di + \int_1^{1+n_F(t)} \bar{p}_F(t)^{1-\mu} di \right]^{\frac{1}{1-\mu}} \\ &= [n_H(t)\bar{p}_H(t)^{1-\mu} + n_F(t)\bar{p}_F(t)^{1-\mu}]^{\frac{1}{1-\mu}} \\ &= \frac{\kappa(1-\beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}{\alpha\chi E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} [n_H(t)\bar{\varphi}_H(t)^{\mu-1} + n_F(t)\bar{\varphi}_F(t)^{\mu-1}]^{\frac{1}{1-\mu}}.\end{aligned}\tag{11}$$

Let $\bar{\varphi}_t$ denote the aggregate productivity level for the entire Home economy, given by

$$\bar{\varphi}_t = \left[\frac{n_H(t)}{N_t} \bar{\varphi}_H(t)^{\mu-1} + \frac{n_F(t)}{N_t} \bar{\varphi}_F(t)^{\mu-1} \right]^{\frac{1}{\mu-1}},\tag{12}$$

where N_t equals $n_H(t) + n_F(t)$, the composite continuum (a measure of the total variety) of Home and Foreign goods actually produced in the Home country. Using (11) and (12), the aggregate price level can now be expressed as

$$P_t = \frac{\kappa(1-\beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right] N_t^{\frac{1}{1-\mu}}}{\alpha\chi E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right] \bar{\varphi}_t}.\tag{13}$$

The aggregate price level is thus a function of the aggregate productivity level, $\bar{\varphi}_t$.

2.5 Investment Behavior and Productivity

Only firms sufficiently productive to cover their fixed costs will enter and produce in the market, implying that there is some threshold productivity level, $\hat{\varphi}$, below which a firm will not be able to enter the market with the expectation of positive profits. The distribution of successful firms' productivity levels, $\eta_j(\varphi, t)$, is therefore the probability of drawing a particular φ , given that $\varphi \geq \hat{\varphi}_j$. Let all firms, regardless of ownership, draw from the same (stationary) distribution of productivity levels, with density $g(\varphi)$ and cumulative distribution $G(\varphi)$. Then the probability that a Home-owned firm will draw a particular level of productivity, given that it enters and produces in the Home market, is

$$\eta_H(\varphi, t) = \eta_H(\varphi, \hat{\varphi}_H(t)) = \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi}_H(t))} & \text{if } \varphi \geq \hat{\varphi}_H(t) \\ 0 & \text{if } \varphi < \hat{\varphi}_H(t). \end{cases} \quad (14)$$

This definition of the equilibrium distribution acknowledges that any Home firm which draws $\varphi < \hat{\varphi}_H(t)$ exits the market before initiating production. The equilibrium distribution for Foreign firms operating in the Home market is characterized in the same way, but using a potentially different cutoff level, $\hat{\varphi}_F(t)$.

The definition of the equilibrium distributions imply that the expression for the average productivity levels for Home- and Foreign-owned firms operating in the Home country, $\bar{\varphi}_H(t)$ and $\bar{\varphi}_F(t)$, can be rewritten as a function of the threshold levels, $\hat{\varphi}_H(t)$ and $\hat{\varphi}_F(t)$:

$$\begin{aligned} \bar{\varphi}_H(t) &= \bar{\varphi}(\hat{\varphi}_H(t)) = \left[\frac{1}{1-G(\hat{\varphi}_H(t))} \int_{\hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{\frac{1}{\mu-1}} \\ \bar{\varphi}_F(t) &= \bar{\varphi}(\hat{\varphi}_F(t)) = \left[\frac{1}{1-G(\hat{\varphi}_F(t))} \int_{\hat{\varphi}_F(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right]^{\frac{1}{\mu-1}}. \end{aligned}$$

Both the Home- and Foreign-owned firms draw from an identical distribution, $g(\varphi)$. The distribution of productivity levels in the Home economy is truncated, as depicted in Figure 3,¹⁹ at the point $\hat{\varphi}_H(t)$ for Home-owned firms and $\hat{\varphi}_F(t)$ for Foreign-owned firms. The points of truncation will be the same or different depending on how easy it is for native firms to enter relative to foreign firms, an issue which is discussed in detail in Section 3. The aggregate productivity level in the Home country, equation (12), is thus a function of the sectoral cutoff productivity levels.

¹⁹Figure 3 is an illustration supposing that $g(\varphi)$ is some distribution resembling a normal distribution. This does not have to be the case. It is merely required that the distribution have a finite $\mu - 1$ -degree moment (that the “ $\mu - 1$ th” moment be finite— see Melitz (2002)).

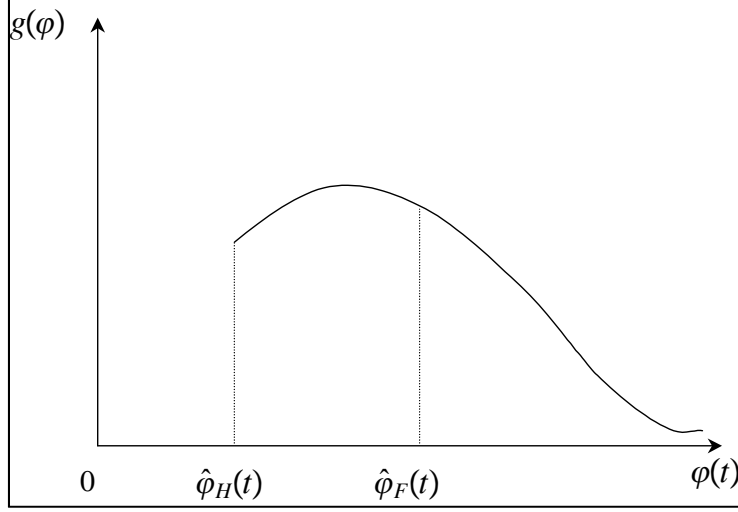


Figure 3: Probability Density of Labor Productivity in the Home Economy

3 The Zero-Cutoff Profit Condition

Two constraints governing firm entry allow one to solve for the threshold productivity levels, $\hat{\varphi}_H(t)$ and $\hat{\varphi}_F(t)$. Threshold productivity levels are found at the point where a firm is just productive enough that its expected discounted profits equal zero. If expected discounted profits are lower, agents considering engaging in production activities will use funds they might have sunk into the fixed cost for next-period production to invest instead in bonds or to increase their present consumption. Let $\pi_F(\varphi_F(i), t)$ represent profit earned by a Foreign-owned firm denominated in Foreign currency. To determine the equilibrium solution, characterized by $\{P_t, \hat{\varphi}_H(t), \hat{\varphi}_F(t)\}$, one can use the zero-profit conditions (ZCPs) governing investment in the Home economy,

$$E_{t-1}[d_t \pi_H(\hat{\varphi}_H(t), t)] = 0$$

for Home-owned firms and

$$E_{t-1}[d_t^* \pi_F(\hat{\varphi}_F(t), t)] = 0$$

for Foreign-owned firms.

The equations for expected discounted profits are very similar for Home- and Foreign-owned firms. To see this, one can note from (7) that

$$d_t^* = \left(\frac{S_t}{S_{t-1}} \right) d_t,$$

allowing expected profits for Foreign-owned firms to be rewritten as

$$E_{t-1} \left[d_t \left(\frac{1}{S_{t-1}} \right) (p_F(\hat{\varphi}_F(t), t) c_F(\hat{\varphi}_F(t), t) - W_t l_H(\hat{\varphi}_F(t), t)) \right] = E_{t-1} \left[d_t \left(\frac{S_t}{S_{t-1}} \right) P_t f_{MNE} \right],$$

which demonstrates more explicitly that it is the sunk cost making Foreign-owned firms in the Home market worry about the prospect of changes in the exchange rate and therefore respond differently than Home-owned firms to monetary uncertainty. If the respective fixed costs were equal and either the exchange rate were fixed (and normalized to 1) or conditions in both countries were governed by a common monetary innovation or monetary authority, then a Home and Foreign firm's expected discounted profits from sales in the Home-country market would be distinguishable only by their unique productivity levels. Further, the threshold productivity levels would be identical.

Substituting the equations for prices and aggregate consumption derived above into the ZCP conditions yields expressions for the threshold productivity levels,²⁰

$$\hat{\varphi}_H(t) = \left(\frac{f E_{t-1} \left[\frac{M_{t-1}}{M_t} \right]}{a_1 M_{t-1} (1 - \alpha) E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}} \quad (15)$$

$$\hat{\varphi}_F(t) = \left(\frac{f_{MNE} E_{t-1} \left[\frac{M_{t-1}^*}{M_t^*} \right]}{a_1 M_{t-1} (1 - \alpha) E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}, \quad (16)$$

where $a_1 = N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}} \left(\frac{\alpha E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{\kappa E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right)^{\frac{1}{\rho}}$. Dividing $\hat{\varphi}_H(t)$ by $\hat{\varphi}_F(t)$ provides a measure of Foreign investors' relative willingness to invest in the Home economy:

$$\frac{\hat{\varphi}_F(t)}{\hat{\varphi}_H(t)} = \left(\frac{f_{MNE} E_{t-1} \left[\frac{M_{t-1}^*}{M_t^*} \right]}{f E_{t-1} \left[\frac{M_{t-1}}{M_t} \right]} \right)^{\frac{1}{\mu-1}}. \quad (17)$$

Equation (17) relates the threshold productivity level of Foreign firms operating in the Home economy to that of the Home firms in terms of the fundamental variables, M and M^* ; the fixed costs, f and f_{MNE} ; and the elasticity of substitution, μ .

In addition to permitting equation (15) to be solved for $\hat{\varphi}_H$,²¹ equation (17) provides an expression for the ratio of the productivity levels of the least productive Home and Foreign firm, which allows an investigation into the effect of changes in underlying parameters on the relative difficulty Foreign agents face when investing in the Home country. Let the ratio be designated γ . Since the money-supply growth process is assumed to be stationary, γ is a constant given by

$$\gamma = \frac{\hat{\varphi}_F(t)}{\hat{\varphi}_H(t)} = \left[\left(\frac{1 + \psi}{1 + \psi^*} \right) \left(\frac{f_{MNE}}{f} \right) \right]^{\left(\frac{1}{\mu-1} \right)} e^{\left(\frac{1}{\mu-1} \right) (\sigma_{m^*}^2 - \sigma_m^2)}. \quad (18)$$

²⁰See Appendix A.3 for detailed derivation.

²¹See Appendix A.4 for proof.

This expression is the same whether firm managers are risk-averse or risk-neutral. If $\gamma = 1$, then Home and Foreign firms have equal access to the Home market. As γ increases, more Home firms relative to Foreign firms expect entry to be profitable, meaning that it is harder for Foreign firms than native firms to enter the Home market. A casual look at (18) indicates that raising the fixed cost incurred by Foreign entrants relative to that paid by domestically owned entrants increases γ , reflecting the increased difficulty Foreign firms would face relative to Home firms when entering the Home market.

The effects of the differences in the mean and variance of the money-supply growth rates, as well as the intuition behind them, are discussed in Part 4. However, it is interesting to note at this point that the symmetric case, where $f = f_{MNE}$ and monetary growth processes are identical, is the mirror of the result that Bacchetta and van Wincoop (2000) derive for trade when preferences are separable in consumption and labor/leisure. Under these conditions, they find that exchange rate uncertainty does not affect the level of trade flows, which is the same under a fixed or floating exchange rate regime. Similarly, in the symmetric case here $\gamma = 1$, implying that expectations of exchange rate volatility bear no influence on FDI flows at all. Foreign-owned firms in the Home market respond to Home monetary volatility in exactly the same way as Home-owned firms (and not at all to Foreign monetary volatility) when making their entry decision.

3.1 A Note on Complete Markets, Factor-Price Equalization, and Geographic Preference

The availability of a complete set of state-contingent bonds allows consumers to insure against country-specific risk to the extent that nominal wages are equal in terms of the Home currency. As Devereux and Engel (2001) point out, this risk sharing results in factor-price equalization—wages will be equal in the Home and Foreign countries when expressed in the same currency. To see that this is true here also, one can examine the wage relation for each country's representative consumer (equation (5) and its Foreign-country counterpart, $W_t^* = P_t^* C_t^{*\rho}$), along with the formula for the real exchange rate (10):

$$W_t = \kappa P_t C_t^\rho = \frac{\kappa P_t C_t^\rho}{\kappa P_t^* C_t^{*\rho}} \kappa P_t^* C_t^{*\rho} = S_t W_t^*.$$

Because the wage is equal across countries, the attractiveness of investing in one's native market as compared with overseas is determined solely by the relative costs of entry. As long as $\hat{\varphi}_F(t)$ is at least as large as $\hat{\varphi}_F^*(t)$, which is assumed in the following analysis, there will be Home firms which do not invest in the Foreign market and Foreign firms which do not invest in the Home market. Foreign firms will always choose to invest in the Foreign country, with some fraction of the entrants also choosing to invest in the Home country. It is noted here that if the stochastic processes governing the growth rate of the money supply in each country are identical—that is, $\psi = \psi^*$ and $\sigma_m^2 = \sigma_{m^*}^2$ —then this is true as long as $f_{MNE} \geq f^*$. For cases where $\sigma_m^2 \neq \sigma_{m^*}^2$ in the analysis below, it is assumed that f^* is sufficiently small in comparison to f_{MNE} as to maintain this property. Intuitively speaking, firms will always invest in their native country before investing

abroad as long as the fixed entry cost overseas does not look too “cheap” relative to the cost of entering their own native market. This assumption adheres to the spirit of Dunning’s argument that there is an additional “cost of doing business abroad” associated with overseas operations (Dunning 1973). Relaxing this assumption in a setting with incomplete markets and/or real wage rigidity would lead to a model of geographic preference, which is ground for further research.

4 The Effect of Exchange-Rate Uncertainty on Entry by Foreign Firms

As mentioned above, the relationship between the threshold productivity levels of Home and Foreign firms operating in the Home market reveals the effect of the fundamental variables and the structural parameters governing demand on the relative willingness of Foreign investors to engage in ventures overseas. Another way to look at γ is as a parameter embodying the effect of exchange-rate risk on entry by Foreign firms. Rearranging (18),

$$\begin{aligned}\hat{\varphi}_F(t) &= \gamma \hat{\varphi}_H(t) \\ &= \left[\left(\frac{1+\psi}{1+\psi^*} \right) \left(\frac{f_{MNE}}{f} \right) \right]^{\left(\frac{1}{\mu-1} \right)} e^{\left(\frac{1}{\mu-1} \right) (\sigma_m^2 - \sigma_m^{*2})} \hat{\varphi}_H(t),\end{aligned}$$

it is evident that the minimum productivity level for Foreign entrants into the Home economy is composed of two effects. The first effect, embodied in $\hat{\varphi}_H(t)$, reflects the influence of any factor that would create a more or less welcoming environment for any investor contemplating a startup in the Home country—affecting all firms, both Home- and Foreign-owned, in the same manner. The second, contained in γ , represents the influence of variables that impact Foreign-owned firms differently than domestically owned firms due to the exchange-rate risk incurred when investors pay the fixed overhead cost in period $t-1$ required to start production in period t . The term γ reflects the size of the sunk cost for Foreign firms relative to that of domestic firms, $\frac{f_{MNE}}{f}$. However, if this consideration is neutralized by letting $f_{MNE} = f$, then γ represents only the net effect of expected changes in monetary variables across the two countries, $\left(\frac{1+\psi}{1+\psi^*} \right)^{\left(\frac{1}{\mu-1} \right)} e^{\left(\frac{1}{\mu-1} \right) (\sigma_m^2 - \sigma_m^{*2})}$, which Foreign producers in the Home country care about because they pay the fixed cost valued at rate $\frac{1}{S_{t-1}}$, but repatriate profits at the rate $\frac{1}{S_t}$.

What generates the wedge embodied in γ , despite the presence of complete markets? In the original Devereux and Engel (2001) model of multinational firms cited in this paper, there are no sunk costs paid in local currency and they point out, as discussed in Section 3.1, that it does not matter who owns the firms, since all firm owners are fully insured. In this model, when Home and Foreign money supply functions are identical, this observation still holds—the covariance of consumption and the net profits of Foreign-owned firms operating in the Home country is always equal to the covariance of consumption and the net profits of Home-owned firms (in absolute value). Then there is no difference in the entry behavior of Home- and Foreign-owned firms when fixed

ρ	μ	κ	χ	β	ψ	ψ^*	f, f_{MNE}
2	6	2	1	0.96	0.25	0.25	0.1

Table 1: Calibration

costs are equal. However, when the underlying monetary volatilities differ, the sunk cost paid in local currency causes these covariances to diverge in size. This is the relationship reflected in the parameter γ defined in equation (18)—essentially, it embodies the difference in the covariances as a function of the underlying monetary supply process parameters and any magnification of the impact of the divergence arising from differences in the sunk costs. Thus, even though owners of multinational firms can use nominal bonds to hedge against the impacts of monetary shocks, the asymmetry generated by the local sunk cost in the presence of segmented goods markets diminishes the effectiveness of hedging when monetary volatility in their own country is higher than that of the host country.

4.1 Exchange Rate Risk and the Host Country’s Money-Supply Growth Rate

The two separate effects at work in determining the cutoff level of productivity for Foreign firms in the Home market are quite distinct and can actually exert opposing influences on Foreign entry. An increase in Home money-supply volatility decreases the value of γ ,

$$\frac{\partial \gamma}{\partial \sigma_m^2} = -\frac{\gamma}{(\mu - 1)} < 0, \quad (19)$$

which means that the relative difficulty facing Foreign-owned firms face when entering the Home market declines, pushing down $\hat{\varphi}_F(t)$. The impact can be shown both analytically, as in Appendix C, and graphically by computing equilibrium levels of entry under different levels of Home monetary volatility. The graphs below are produced using a Pareto distribution for $g(\varphi)$ following Helpman, Melitz, and Yeaple (2003). The period $t - 1$ money supply is normalized to 1 and the remaining parameters in the cutoff equations are calibrated as follows:

The value for ρ , the coefficient of relative risk aversion, is taken from Devereux and Lane (2003) and is consistent with estimates reported in Deaton (1992). The coefficient on labor in the utility function, κ , is also the same as in Devereux and Lane (2003). The subjective discount rate in the utility function, β , is assigned the value used in Bergin (2003) and the value for the substitution elasticity between goods is taken from the range of estimates in that study, with the Pareto shape parameter equal to the elasticity plus 0.1 to allow for a dispersion equal to just over 1, as estimated by Helpman, Melitz, and Yeaple (2003),²² and scale parameter $b = 1$, a normalization which forces the cutoff productivity levels to be greater than or equal to 1. To highlight the effect of changes in monetary volatility on the value of γ , it is assumed that the mean money supply growth rates are equal across countries ($\psi = \psi^*$) and that there is no extra overhead cost involved in doing business

²²The results are robust to any choice of k up to μ plus 15, which is a very wide range of levels of dispersion in firm size, far outside the empirical estimates in Helpman, Melitz and Yeaple.

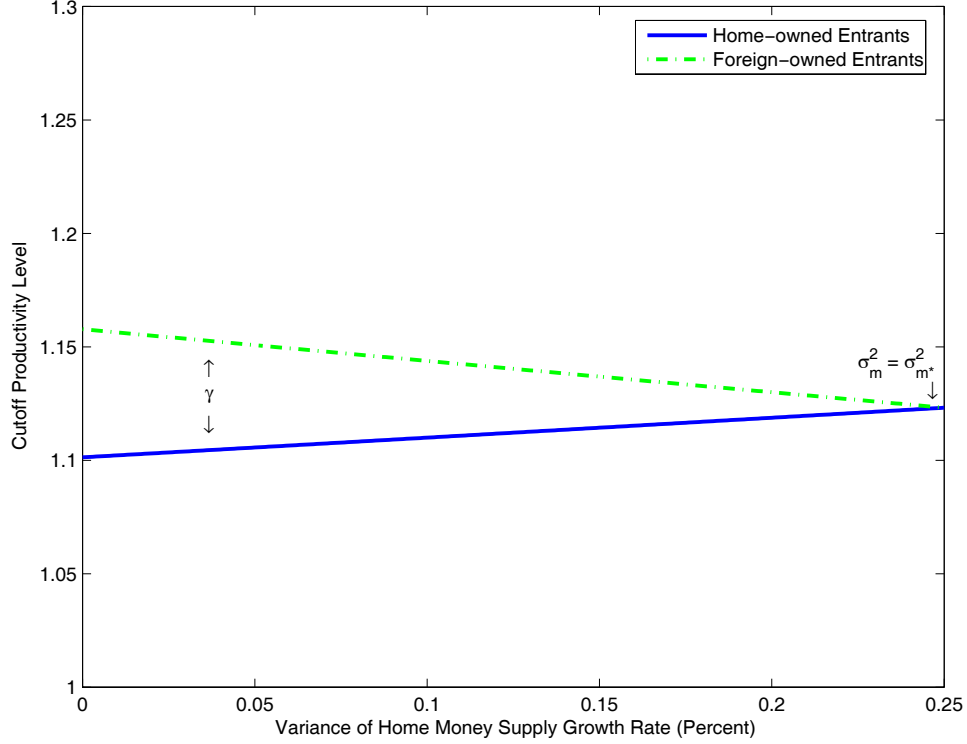


Figure 4: Volatility in the Growth Rate of the Home Money Supply and Productivity Thresholds

abroad ($f = f_{MNE}$). The values for ψ and ψ^* are set equal to focus on the role of shocks on entry behavior, as well as to satisfy the restriction that $\theta < \frac{1}{\beta}$ for $0 < \sigma_m^2 \leq 0.25$, the values of home monetary volatility considered in Figures 4 and 5. Foreign monetary volatility, $\sigma_{m^*}^2$, is set equal to 0.25, the maximum value of σ_m^2 , to illustrate that when $\sigma_m^2 = \sigma_{m^*}^2$, the lines in each graph meet and the economies become identical in every respect so that $\gamma = 1$ and exchange rate volatility does not impede or encourage entry by multinational firms relative to domestically owned firms.

Figure 4 shows that an increase in the variance of the growth rate of the Home money supply from zero to 0.25 percent generates overall less favorable conditions for prospective investors in the Home country, illustrated by the continuous rise in $\hat{\varphi}_H(t)$ and the fall in the proportion of all prospective Home investors that choose enter the domestic market (Figure 5). Yet the decreasing value of γ implies that Foreign firms operating in the Home market are protected from the perils of Home monetary uncertainty by virtue of their exchange-rate exposure. Even though an increase in Home monetary volatility makes investment in the Home country less inviting for domestic firms due to sunk costs and sticky prices, the negative effect is more than offset for Foreign firms because the threat of an unexpected fall in the growth rate of the money supply, which would depress sales in the Home market, always occurs alongside a simultaneous appreciation of the Home country's currency.

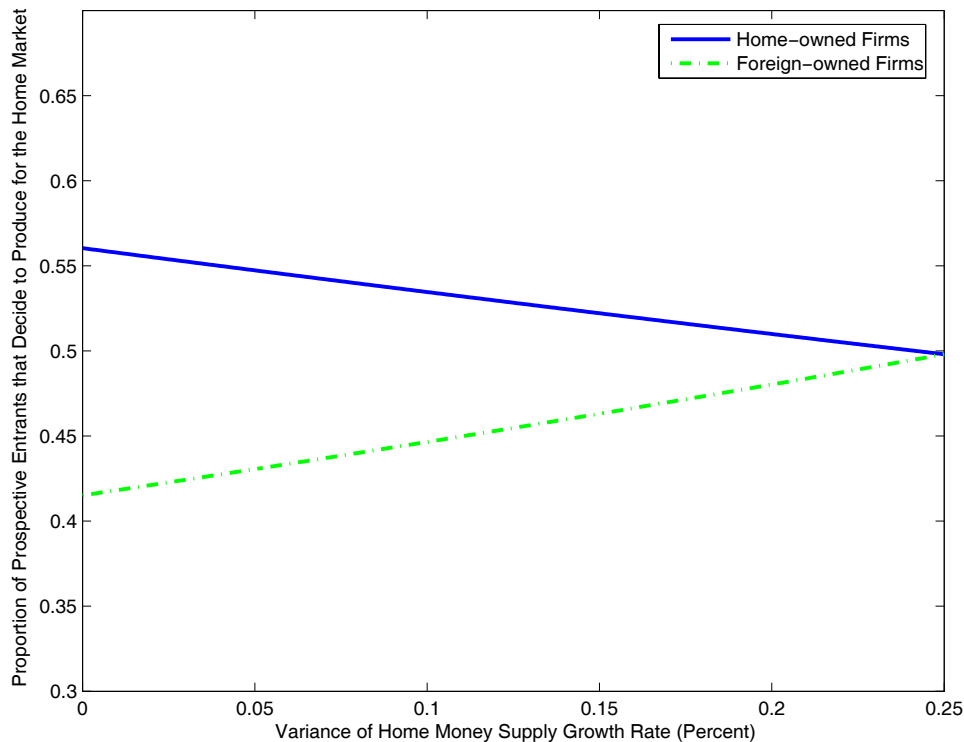


Figure 5: Volatility in the Growth Rate of the Home Money Supply and Entry

4.2 Exchange Rate Risk and the Native Country's Money-Supply Growth Rate

The two effects also work in opposite directions for changes in the volatility of Foreign money-supply growth. For increases in $\sigma_{m^*}^2$, the change in γ is positive,

$$\frac{\partial \gamma}{\partial \sigma_{m^*}^2} = \frac{\gamma}{(\mu - 1)} > 0. \quad (20)$$

Exchange rate fluctuations generated by shocks to the Foreign money-supply growth rate are not cushioned by offsetting fluctuations in Home-country sales. Indeed, an unexpected drop in the Foreign money supply not only results in an unexpected depreciation of the Home currency, it does so at a time of unexpectedly low real income in the Foreign country, a condition that may be spread to the Home-country market, as well, through risk sharing. Thus, exchange-rate risk introduced through volatility in the growth rate of the Foreign money supply is not offset and can potentially be exacerbated by fluctuations in sales by branches of Foreign firms operating in the Home economy.

The cutoff productivity level of Foreign firms entering the Home market rises with increases in $\sigma_{m^*}^2$, both in a relative and an absolute sense. (See Figure 6, where now σ_m^2 is set equal to zero to emphasize the role of Foreign monetary volatility.) Fewer firms of larger average size will enter, complementing Campa's (1993) finding that exchange-rate volatility can deter entry by Foreign firms. Figure 7 illustrates that adverse exchange-rate risk arising from increases in the volatility

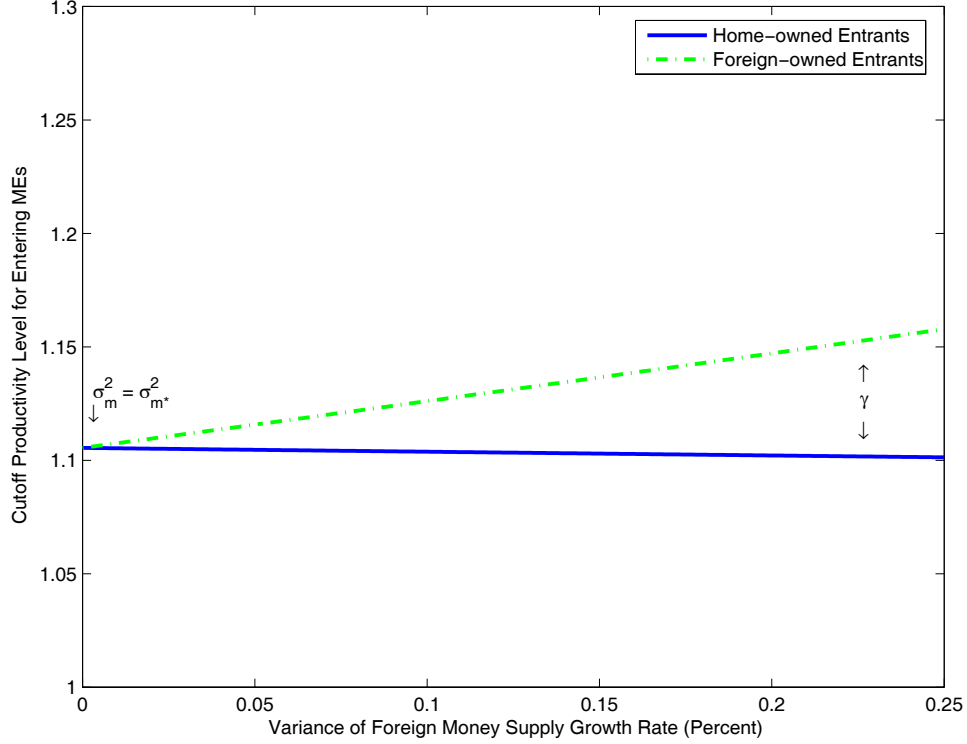


Figure 6: Volatility in the Growth Rate of the Foreign Money Supply and Productivity Thresholds

Foreign money-supply growth rate pushes the least productive Foreign-owned firms out of the Home market, even as their defection allows a very small number of less productive domestic investors to soak up their abandoned market share. The overall effects on entry is significant— a drop of more than ten percentage points, over twenty percent of existing Foreign-owned firms— in response to an increase in the variance of the money supply growth rate of only one quarter of one percent. Also important is that this deterrent effect increases with the size of local fixed costs,

$$\frac{\partial}{\partial f_{MNE}} \left[\frac{\partial \gamma}{\partial \sigma_{m*}^2} \right] = \frac{\gamma}{(\mu - 1)^2 f_{MNE}} > 0,$$

which coincides with the Campa's (1993) finding that the deterrent effect of exchange rate volatility grows with the size of sunk costs.

Thus, the results in (19) and (20) imply that exposure to unpredictable fluctuations in the exchange rate generated by volatility in underlying monetary variables can either encourage or discourage FDI, depending on whether the volatility comes from the host-country money supply or from a firm's native economy. This dual result offers a theoretical explanation for conflicting results in empirical tests of partial equilibrium models. The relative willingness of Foreign firms to invest in the Home economy grows as σ_m^2 increases (γ falls in Figure 4). However, increasing σ_{m*}^2 causes γ to rise, indicating that entry into the Home market is less attractive to Foreign firms. Remarkably, when monetary volatility is perfectly symmetric across the two countries ($\sigma_m^2 = \sigma_{m*}^2$),

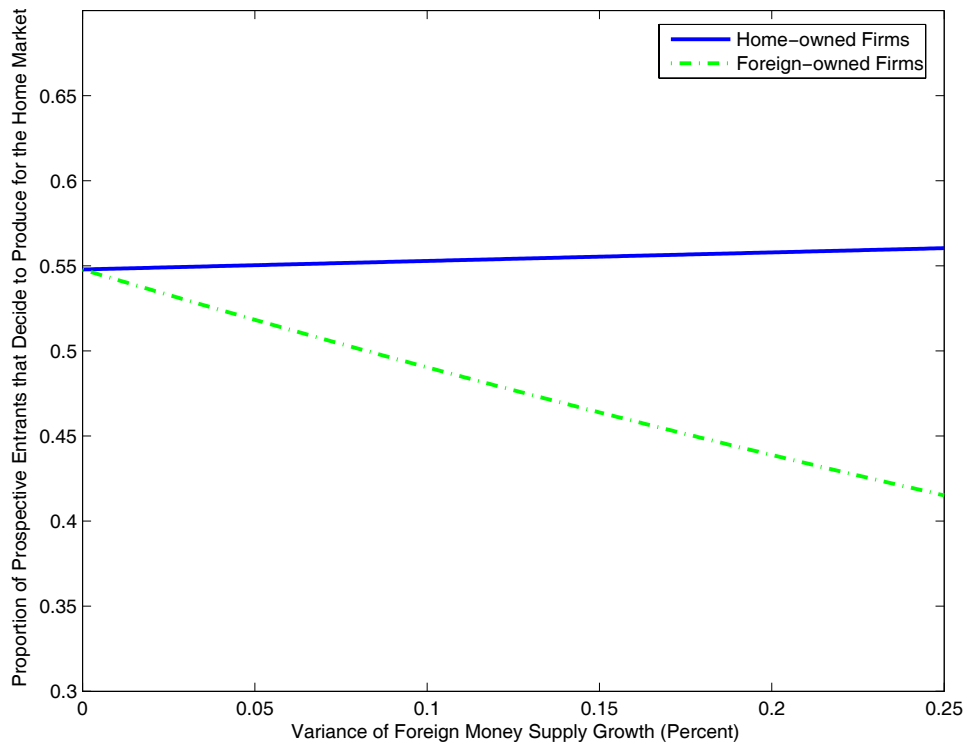


Figure 7: Volatility in the Growth Rate of the Foreign Money Supply and Entry

the ratio of Home and Foreign firms is not affected at all by monetary volatility— and therefore not by exchange rate uncertainty, either, in this stylized model. Further, whereas the volatility of the Home and Foreign money-supply growth rates have opposing effects on γ , they affect exchange rate volatility with the same sign,²³ bearing the implication that *there is no clear correlation between exchange rate volatility and FDI* unless one takes into account the origin of the volatility. Within the stylized framework of this study, a regression of FDI flows— either in absolute levels or as a fraction of total investment— on the variance of the exchange rate would yield a positive coefficient in samples where the variance of money growth rates in the host country were larger than that of the source country, a negative coefficient when the variance of foreign money growth rates were larger, a coefficient not significantly different from zero when the variances were equal, and a coefficient of questionable size, sign, and significance in samples where host and source monetary volatility “flip flopped” in relative size.

The factor-price equalization noted in Section 3.1 raises the question of whether the above results hold in a world with incomplete markets, where wages are not necessarily equal across countries. Under incomplete markets, multinational corporations factor into their decision-making the fact that the local wage rate will change due to shocks to the money supply. Nonetheless, the net effect of the uncertainty is similar. In fact, the full technical appendix²⁴ presents a model without bonds

²³In this model, $\text{var}(\log S_t) = \text{var}(m_t) + \text{var}(m_t^*)$ since M and M^* are independently distributed.

²⁴The full technical appendix is available at <http://www.econ.ucdavis.edu/faculty/knruss/> or upon request.

and simplified to one period with logarithmic utility in consumption and produces an equation identical to (18).

The sectoral responses to changes in monetary volatility have an impact on aggregate productivity, the price level, and consumption in the Home country. As Home-owned firms exit and more productive Foreign-owned firms enter in response to increases in σ_m^2 , aggregate productivity in the economy increases, generating downward pressure on the aggregate price level, resulting in a substantial increase in Home consumption, regardless of the parameterization of μ . In contrast, when increases in Foreign monetary volatility drive Foreign-owned firms out of the Home market, to be partially replaced by less productive Home-owned firms entering to capture a bit of the abandoned market share, the effect on prices and consumption is ambiguous, depending on the size of the elasticity of substitution between individual goods. When demand is relatively inelastic ($\mu = 2$, for instance), less productive Home-owned firms enter at a higher rate when Foreign-owned firms exit because a low elasticity allows a higher markup to compensate for their more costly production and smaller market shares. This aggressive entry by high-cost producers pushes up prices and exerts a mild negative effect on Home consumption. When demand is more elastic, as in the calibration used here, the higher average productivity among surviving Foreign-owned firms generates more competitive pressure on prices in the Home market, pushing the aggregate prices down slightly and yielding a minute positive effect on aggregate consumption.

5 Conclusions

The goal of this paper is to explain the conflicting findings of previous empirical work done in a partial equilibrium framework by showing that volatility in the exchange rate may or may not deter foreign direct investment, depending on which underlying variable is the source of the volatility. The result here provides a theoretical account of the link between FDI flows and the correlation between local demand and exchange-rate volatility investigated by Goldberg and Kolstad (1995). It bears the important implication that the variance of the exchange rate will impact the MNE's decision to enter a market, but whether it encourages or deters firms contemplating direct investment depends on whether the shocks originate in the company's own native country or overseas, in the host market. As described by Campa (1993), the extent to which MNEs worry about exchange rate volatility is closely related to the presence and magnitude of local sunk costs. Thus, in several ways, the model and its results are an extension of prior investigations of the determinants of direct investment in the trade and industrial-organization literature.

The findings presented here also contribute to the literature of open-economy macroeconomics on three fronts. First, they echo a key point in Melitz (2003) and Ghironi and Melitz (2005) that a country's aggregate level of labor productivity can change without any change in available technology, characterized here by $g(\varphi)$. These studies show that such changes can occur in response to changes in fixed costs, trade patterns, and productivity shocks, while this paper introduces the effect of second moments from the growth rate of the money supply, variables controlled by monetary

policymakers. Second, if one envisions production by multinationals for local consumption as a case of pricing-to-market, as Devereux and Engel (2001) propose, then the redistribution of production from the Foreign-owned to the Home-owned sector is a corollary to the conventional wisdom that pricing-to-market insulates an economy from shocks to the money supply arising in other countries of the world. Finally, the model suggests that both the behavior of overseas investors and the level and volatility of exchange rates could be jointly determined by common underlying macroeconomic variables. In this sense, the approach and findings in this paper are quite similar to those in the study of exchange rates and trade by Bacchetta and van Wincoop (2003). Taken together, the studies imply that regressions of FDI flows on both movements in exchange rate levels and on proxies for exchange rate uncertainty, such as its variance, are subject to the same types of endogeneity issues as studies of the impact of exchange rate uncertainty on trade flows.²⁵

The results point to several avenues for future research. It would be informative to introduce trade or vertical FDI into a version of the model with incomplete markets to look at the effect of exchange rate uncertainty and fluctuations in local costs of production on the concentration of productive capacity, in the spirit of nonmonetary models by Brainard (1997); Helpman, Melitz, and Yeaple (2003); Markusen and Venables (2002), and Aizenman and Marion (2005). It would also be extremely useful to incorporate a source of monetary variation that allows a more prominent (and realistic) role for monetary policy— in particular, an interest rate rule— to examine the effect of different policy responses when productivity shocks as the principal source of macroeconomic uncertainty. Aizenman (1992) and Aizenman and Marion (2004) have also emphasized in general equilibrium models of FDI with representative firms that the response of multinational firms to real and nominal shocks can be quite different. Further, adding a richer role for monetary policy is essential for analyzing the optimal use of policy in the presence of multinational firms. In addition, the model here abstracts from the more realistic dynamic behavior of entry achieved among exporting firms in Ghironi and Melitz (2005). Establishing state variables such as the number of firms or the stock of physical capital that generate endogenous persistence in the model due to one-time sunk costs or partial depreciation is of great importance to illuminate the impact of exchange-rate variability on the growth of a country's aggregate productivity and capital stock over time. Finally, a model exploring capital accumulation would need to address the fact that larger, more productive firms are likely to establish and maintain larger stocks of capital than the smaller, less productive firms near the entry threshold, a fact which is important in analyzing the size of the impact of macroeconomic uncertainty on flows of FDI.

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A Aggregation of Profits Earned by All Firms Operating in the Home Economy

The actual period- t profits of a firm owned by country j and earned in the Home country can be expressed as a function of revenues:

$$\begin{aligned}
\pi_j(i, t) &= p_j(i, t)c_j(i, t) - W_t \left(\frac{c_j(i, t)}{\varphi_j(i)} \right) - P_t f \\
&= p_j(i, t)c_j(i, t) - p_j(i, t)c_j(i, t) \left(\frac{W_t}{\varphi_j(i)p_j(i, t)} \right) - P_t f \\
&= \Gamma_0 r_j(i, t) - P_t f,
\end{aligned}$$

where, substituting the firm's pricing rule from Section 2.3, $\Gamma_0 = 1 - \frac{\alpha \chi_{E_{t-1}} \left[M_t^{\frac{1-\rho}{\rho}} \right] W_t}{\kappa(1-\beta\theta)E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]}$. The profit of a firm picked at random from the country j -owned sector of the Home economy is then computed using the equilibrium distribution of firm productivity levels:

$$\begin{aligned}
\bar{\pi}_j &= \int_0^\infty [\Gamma_0 r_j(i, t) - P_t f] \eta_j(\varphi, t) d\varphi \\
&= \Gamma_0 \int_0^\infty r_j(i, t) \eta_j(\varphi, t) d\varphi - P_t f \\
&= \Gamma_0 \int_0^\infty r_j(\bar{\varphi}_j(t), t) \frac{r_j(i, t)}{r_j(\bar{\varphi}_j(t))} \eta_j(\varphi, t) d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t), t) \int_0^\infty \left(\frac{\varphi}{\bar{\varphi}_j(t)} \right)^{\mu-1} \eta_j(\varphi, t) d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t), t) \left(\frac{1}{\bar{\varphi}_j(t)} \right)^{\mu-1} \int_0^\infty \varphi^{\mu-1} \eta_j(\varphi, t) d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_j(t), t) - P_t f.
\end{aligned}$$

Substituting back in the definition of Γ_0 and rearranging, it is clear that the average level of profit is also the level of profit earned by a firm with the average productivity level, or $\bar{\pi}_j = \pi_j(\bar{\varphi}_j(t), t)$. All firms in each market can now be treated as though they earned this average level of profit. Total profits earned by all Home- owned firms in the Home and Foreign markets in terms of the

Home currency are then computed as

$$\begin{aligned}\Pi_t &= \int_0^{n_H(t)} \pi_H(\bar{\varphi}_H(t), t) di + \int_0^{n_H^*(t)} \pi_H^*(\bar{\varphi}_H^*(t), t) di \\ &= n_H(t) \pi_H(\bar{\varphi}_H(t), t) + S_t n_H^*(t) \pi_H^*(\bar{\varphi}_F(t), t).\end{aligned}$$

B Reducing the Zero-Cutoff Profit Conditions

The Zero-Cutoff Profit Condition (ZCP) implies that

$$E_{t-1}[d_t \pi_H(\hat{\varphi}_H(t), t)] = E_{t-1} \left[d_t \left(1 - \frac{\alpha M_t E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right) r_H(\hat{\varphi}(t)) \right] - P_t f E_{t-1}[d_t] \equiv 0.$$

Noting that $p_H(\varphi_H(t)) c_H(\varphi_H(t)) = p_H(\varphi_H(t))^{1-\mu} P_t^\mu C_t = r_H(\varphi_H(t))$ and $d_t = E_{t-1} \left[\frac{\beta P_{t-1} C_{t-1}^\rho}{P_t C_t^\rho} \right]$, one can write

$$\begin{aligned}E_{t-1} \left[d_t \left(1 - \frac{\alpha M_t E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right) p_H(\hat{\varphi}_H(t), t)^{1-\mu} P_t^\mu C_t \right] &= P_t f E_t[d_{t-1}] \\ E_{t-1} \left[\frac{1}{P_t C_t^\rho} \left(1 - \frac{\alpha M_t E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right) p_H(\hat{\varphi}_H(t), t)^{1-\mu} P_t^{\mu-1} C_t \right] &= f E_{t-1} \left[\frac{1}{P_t C_t^\rho} \right].\end{aligned}$$

Using equation (6) to substitute for consumption, substituting the definitions for $p_H(\hat{\varphi}_H(t), t)$ and P_t given in equations (9) and (13), and simplifying, the expression becomes

$$\left\{ N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}} \left(\frac{\alpha}{\kappa} \right)^{\frac{1}{\rho}} \hat{\varphi}_H^{\mu-1}(t) \bar{\varphi}_t^{\frac{1-\rho(\mu-1)}{\rho}} \times E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \left(1 - \frac{\alpha M_t E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right) \right] \right\} = f E_{t-1} \left[\frac{1}{M_t} \right].$$

It is now possible to isolate $\hat{\varphi}_H(t)$ by rearranging terms to obtain equation (26) in the main text. The same process using the ZCP for Foreign firms operating in the Home country yields equation (27).

C The Existence of $\hat{\varphi}_H(t)$

Let Z be defined by

$$Z = \hat{\varphi}_H(t) - \left(\frac{f E_{t-1} \left[\frac{M_{t-1}}{M_t} \right]}{a_1 M_{t-1} (1 - \alpha) E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}}.$$

Using the definition of $\bar{\varphi}_j(t)$ given in Section 2.4 and recalling that $n_j = 1 - G(\hat{\varphi}_j(t))$, Z can be rewritten:

$$= \hat{\varphi}_H(t) - \Gamma_1 \left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi + \int_{\gamma \hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}},$$

where $\Gamma_1 = \left(\frac{f E_{t-1} \left[\frac{M_{t-1}}{M_t} \right]}{\left(\frac{\alpha E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]}{\kappa E_{t-1} \left[M_t^{\frac{1}{\rho}} \right]} \right)^{\frac{1}{\rho}} M_{t-1} (1 - \alpha) E_{t-1} \left[M_t^{\frac{1-\rho}{\rho}} \right]} \right)^{\frac{1}{\mu-1}} \cdot$ The partial derivative of Z with respect to $\hat{\varphi}_H(t)$ is

$$\begin{aligned} \frac{\partial Z}{\partial \hat{\varphi}_H(t)} &= 1 + \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2} \right) * \\ &\left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi + \int_{\gamma \hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} * [\gamma^{\mu-1} g(\gamma \hat{\varphi}_H(t)) + g(\hat{\varphi}_H(t))] \hat{\varphi}_H(t)^{\mu-1}. \end{aligned}$$

The derivative implies that Z is monotonically increasing as long as Γ_1 is a real number and has the same sign as $\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}$. This will be the case if both Γ_1 and $\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}$ are greater than zero, which occur when $\mu > \frac{1}{\rho} + 1$.

D The Impact of Volatility on Threshold Productivity

It is straightforward to ascertain the impact of Home monetary volatility on $\hat{\varphi}_H(t)$ by taking the its derivative with respect to σ_m^2 using the implicit function rule,

$$\frac{d\hat{\varphi}_H(t)}{d\sigma_m^2} = - \frac{\frac{\partial Z}{\partial \sigma_m^2}}{\frac{\partial Z}{\partial \hat{\varphi}_H(t)}}.$$

In Appendix C, it is established that for feasible parameter values, the denominator is positive. To assess the sign of the numerator, with the aim of simplifying notation it is useful first to define

$$\tilde{\varphi}_t = \int_{\hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi + \int_{\gamma \hat{\varphi}_H(t)}^{\infty} \varphi^{\mu-1} g(\varphi) d\varphi$$

and to note that, after substituting in the lognormal money supply process,

$$\Gamma_1 = \left[(1 + \psi) \left(\frac{\kappa}{\alpha} \right)^{\frac{1}{\rho}} \left(\frac{\theta f}{M_{t-1}} \right) e^{\left(\frac{\rho+1}{2\rho^2} \right) \sigma_m^2} \right]^{\frac{1}{\mu-1}}.$$

It is also useful to note that

$$\frac{\partial \tilde{\varphi}_t}{\partial \sigma_m^2} = \left(\frac{\partial \tilde{\varphi}_t}{\partial \gamma} \right) \left(\frac{\partial \gamma}{\partial \sigma_m^2} \right) = \frac{1}{\mu-1} (\gamma \hat{\varphi}_H(t))^{2-\mu} g(\gamma \hat{\varphi}_H(t)) > 0$$

and that

$$\frac{\partial \Gamma_1}{\partial \sigma_m^2} = \frac{1}{\mu-1} \Gamma_1^{\frac{-\mu}{\mu-1}} \left(\frac{\rho+1}{2\rho^2} \right) e^{\left(\frac{\rho+1}{2\rho^2} \right) \sigma_m^2} > 0.$$

Then, it is clear that the numerator, $\frac{\partial Z}{\partial \sigma_m^2}$, is negative

$$\frac{\partial Z}{\partial \sigma_m^2} = -\frac{\partial \Gamma_1}{\partial \sigma_m^2} \tilde{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}} - \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2} \right) \tilde{\varphi}_t^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} \frac{\partial \tilde{\varphi}_t}{\partial \sigma_m^2} < 0,$$

implying that $\frac{d\hat{\varphi}_H(t)}{d\sigma_m^2}$. One can similarly show that the impact of Foreign monetary volatility on $\hat{\varphi}_H(t)$ is negative by noting that

$$\begin{aligned} \frac{\partial \Gamma_1}{\partial \sigma_{m^*}^2} &= 0 \\ \frac{\partial \tilde{\varphi}_t}{\partial \sigma_{m^*}^2} &= \left(\frac{\partial \tilde{\varphi}_t}{\partial \gamma} \right) \left(\frac{\partial \gamma}{\partial \sigma_{m^*}^2} \right) = \frac{-1}{\mu-1} (\gamma \hat{\varphi}_H(t))^{2-\mu} g(\gamma \hat{\varphi}_H(t)) < 0. \end{aligned}$$