1) Two-period open economy model

Consider the two-period endowment economy discussed in class, but augment it to allow for a government budget deficit. Let T represent lump sum taxes, and let B^G represent government issues of bonds. (Private holdings of bonds (B) can include government issued bonds, B^G.)

Write the household problem as a function of taxes:

$$\max_{C_1,C_2} \ U(C_1) + \beta U(C_2)$$

$s.t. \ Y_1 - I_1 - T_1 - C_1 = B \quad$ period 1 budget constraint

$Y_2 + (1+r)B - I_2 - T_2 - C_2 = 0 \quad$ period 2 budget constraint

where $U(C_t) \equiv \dfrac{1}{1-\sigma} C_t^{1-\sigma}, \beta = \dfrac{1}{1+r}$

Where the government faces constraints:

$$B^G = G_1 - T_1 \quad \text{period 1}$$
$$(1+r)B^G = T_2 - G_2 \quad \text{period 2}$$

Define private and government saving:

$$s_t^P = Y_t - T_t - C_t$$
$$s_t^G = T_t - G_t$$

For the purpose of this exercise, use the following values: $Y_1 = Y_2 = 100, I_1 = I_2 = 20, r = 0.10$

a) Suppose $G_1 = G_2 = 20, T_1 = 10$, so that there is a government budget deficit of 10 in period 1. Compute and discuss the implications for private saving and the current account in period 1. Interpret in light of the twin deficits hypothesis.

b) Now suppose instead that the government budget deficit of 10 is due to $G_1 = 30, T_1 = 20, G_2 = 20$. Now compute and discuss the implications for private saving and the current account in period 1.

---

2) Intertemporal Current Account Model

This problem set asks you to work through the omitted details of the intertemporal Current Account problem discussed in class. Consider a representative agent problem for a small open economy. The country receives an exogenous endowment of the single type of good, and the country can save only in the form of a real riskless bond paying a fixed return $r = \frac{1}{\beta} - 1$.

$$Max \quad E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s)$$

$s.t.$

$$B_{s+1} - B_s = Y_s + rB_s - C_s - I_s - G_s \equiv CA_s$$

$$where \quad U(C_t) = C_t - \frac{1}{2}C^2{}_t$$

$$C_t < 1$$

a) Set up a dynamic programming problem and derive the first-order conditions.

b) Find the consumption function ($C_t = f\left(B_t, E_t\left[\{NO_s\}_{s=t..t+\infty}\right]\right)$) by recursively substituting the intertemporal Euler equation into the intertemporal budget constraint. Substitute this back into the single-period budget constraint to find an equation that describes the current account ($CA_t = f\left(E_t\left[\{NO_s\}_{s=t..t+\infty}\right]\right)$). (Define net output as $NO = Y - G - I$.) Interpret the result.

c) Discuss what would happen to the current account in the present period and in the long run in this model under the following scenarios (give magnitudes as well as the direction of changes in the current account):
   i) a purely temporary rise in output endowment by amount X this period.
   ii) a permanent rise in government purchases by X.
   iii) a permanent rise in output by X that is anticipated to begin **next** period.

d) The Balance of Payments accounts (BOP) define the current account as the sum of the trade balance (exports minus imports), net factor income from abroad, and unilateral transfers. So in the present model (which does not have unilateral transfers, and where net factor income consists only of interest on bonds) the trade balance differs from the current account only in that it excludes net interest income, rB. Consider again the purely temporary rise in output in part (c-i) above. Discuss how the effect on the trade balance differs from the effect on the current account, both in the present period and in the long run. (You may assume that the country starts off with a zero net foreign asset position in the initial period, if this makes it easier to analyze.)