Solutions to Homework 1

---

1. Two Period Model

a) The household intertemporal budget constraint, in terms of taxes is:

$$C_1 + \frac{1}{1+r}C_2 = (Y_1 - I_1 - T_1) + \frac{1}{1+r}(Y_2 - I_2 - T_2)$$

The intertemporal constraint for the government is: $G_1 - T_1 = -\frac{1}{1+r}(G_2 - T_2)$

Combining these: $C_1 + \frac{1}{1+r}C_2 = (Y_1 - I_1 - G_1) + \frac{1}{1+r}(Y_2 - I_2 - G_2)$

This is the same constraint as the model studied in class.

Since the household optimization problem is the same as that considered in class, so the consumption smoothing Euler equation is the same: $C_1 = C_2$, and the consumption function for period 1 is the same: $C_1 = \left(\frac{1+r}{2+r}\right)NO_1 + \left(\frac{1}{2+r}\right)NO_2$.

If $G_1 = G_2 = 20, T_1 = 10$

$$C_1 = \left(\frac{1+r}{2+r}\right)(Y_1 - G_1 - I_1) + \left(\frac{1}{2+r}\right)(Y_2 - G_2 - I_2)$$

$$= \left(\frac{1+r}{2+r}\right)(100 - 20 - 20) + \left(\frac{1}{2+r}\right)(100 - 20 - 20) \quad = 60$$

Substitute this into the definition of private saving:

$$s_1^P = Y_1 - T_1 - C_1 = 100 - 10 - 60 = 30$$

and $CA_1 = Y_1 - G_1 - C_1 - I_1 = 100 - 20 - 60 - 20 = 0$

The current account is balanced because the high private saving compensates for the government budget deficit in the case of temporarily low taxes. The twin deficits hypothesis does not apply in this case.

b) If $G_1 = 30, G_2 = 20, T_1 = 20$

$$C_1 = \left(\frac{1+r}{2+r}\right)(Y_1 - G_1 - I_1) + \left(\frac{1}{2+r}\right)(Y_2 - G_2 - I_2)$$

$$= \left(\frac{1.1}{2.1}\right)(100 - 30 - 20) + \left(\frac{1}{2.1}\right)(100 - 20 - 20) = \left(\frac{1}{2.1}\right)(115) = 54.8$$

Substitute this into the definition of private saving:

$$s_1^P = Y_1 - T_1 - C_1 = 100 - 20 - 54.8 = 25.2$$

and $CA_1 = Y_1 - G_1 - C_1 - I_1 = 100 - 30 - 54.8 - 20 = -4.8$

There is a current account deficit because net output is temporarily low in period 1 due to the high government spending.

2) Intertemporal current account model

a) Bellman form:

$$V_t(B_t) = \max \left\{ \left(C_t - \frac{1}{2} C_t^{\,2} \right) + \beta E_t \left[V_{t+1}(B_{t+1}) \right] \right\} +$$
$$\lambda_t \left(Y_t + (1+r) B_t - C_t - I_t - G_t - B_{t+1} \right)$$

First order conditions:

$$C_t : (1 - C_t) = \lambda_t$$

$$B_{t+1} : \beta E_t \left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \right] = \lambda_t$$

Envelope theorem:

$$B_t : \quad \frac{\partial V_t}{\partial B_t} = \lambda_t (1+r) \quad \rightarrow \quad \frac{\partial V_{t+1}}{\partial B_{t+1}} = \lambda_{t+1} (1+r)$$

Euler equation:

$$(1 - C_t) = \beta (1+r) E_t \left[1 - C_{t+1} \right]$$

Since $r = {}^{1}\!/_{\beta} - 1$, this may be rewritten: $C_t = E_t \left[C_{t+1} \right]$

---

b) The intertemporal budget constraint is:

$$\sum_{s=t}^{\infty} \beta^{s-t} (C_s + I_s + G_s) = (1+r) B_t + \sum_{s=t}^{\infty} \beta^{s-t} (Y_s)$$

Regroup and impose expectations, since the constraint must hold ex-ante as well as ex-post:

$$\sum_{s=t}^{\infty} \beta^{s-t} E_t (C_s) = (1+r) B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t (Y_s - I_s - G_s)$$

substituting the Euler equation recursively for expected consumption, and rearrange:

$$\sum_{s=t}^{\infty} \beta^{s-t} C_t = (1+r) B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t (Y_s - I_s - G_s).$$

$$\frac{1}{1-\beta} C_t = \ldots$$

$$C_t = \frac{1-\beta}{\beta} B_t + (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t (Y_s - I_s - G_s)$$

$$= r B_t + (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t (NO_s)$$

Substitute back into the single-period budget constraint:

$$CA_t = Y_t + r B_t - I_t - G_t - C_t = NO_t + r B_t - C_t$$

$$= (NO_t) - (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[NO_s \right]$$

or $$CA_t = \beta(NO_t) - (1-\beta)\sum_{s=t+1}^{\infty} \beta^{s-t} E_t[NO_s]$$

This indicates that most of a temporary rise in net output will be saved: the country will run a positive current account. A permanent rise in net output, however, will lead to no increased saving, and no change in the current account.

---

c) (i) The equation above indicates that the current account equals beta times the shock, βX (assuming the current account was balanced prior to this). Households will smooth consumption by buying foreign assets. Consumption rises only by amount $(1-\beta)X$ in period t, and the current account rises by the remaining amount. The equation indicates that the current account returns to zero in periods thereafter.

(ii) Households will lower consumption by X each period because the permanent shock lowers lifetime wealth. The current account will be zero in period t and the subsequent periods.

(iii) Households will raise consumption today to smooth consumption in the face of increased intertemporal wealth, and this will cause a current account deficit in the current period. In particular, consumption rises by βX . The current account must fall by this amount in the current period, but there is a balanced current account in subsequent periods.

---

d) Use the fact that $TB_t = CA_t - rB_t$, and that $B_{t+1} = B_t + CA_t$. We found above that (assuming no net foreign assets in previous periods) the current account is βX in the initial period and zero afterward. So net foreign assets rise to βX in the period after the shock. This means that the trade balance is βX in the initial period and $-r\beta X = -(1-\beta)X$ in subsequent periods. Intuitively, the accumulation of assets in the initial period gives rise to a stream of interest income in future periods that allows households to finance a stream of net imports in all future periods.