Solutions for Homework 2, Bergin, 2017

Substitute production function, definition of investment and law of motion for capital into resource constraint.

$$C_{H,s}+C_{F,s}+\frac{1}{4}\sum_{j=1}^{4}\left(K_{H,s-j+1}-(1-\delta)K_{H,s-j}\right)+\frac{1}{4}\sum_{j=1}^{4}\left(K_{F,s-j+1}-(1-\delta)K_{F,s-j}\right)=A_{H,s}K_{H,s-4}^{\theta}L_{H,s}^{1-\theta}+A_{F,s}K_{F,s-4}^{\theta}L_{F,s}^{1-\theta}$$

Write the Lagrangian:

$$E_t\sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{1}{2}\frac{\sigma}{\sigma-1}\left[C_{H,s}^{\mu}\left(1-L_{H,s}\right)^{1-\mu}\right]^{\frac{\sigma-1}{\sigma}}+\frac{1}{2}\frac{\sigma}{\sigma-1}\left[C_{F,s}^{\mu}\left(1-L_{F,s}\right)^{1-\mu}\right]^{\frac{\sigma-1}{\sigma}}\right.$$

$$\left.+\lambda_s\left(C_{H,s}+C_{F,s}+\frac{1}{4}\sum_{j=1}^{4}\left(K_{H,s-j+1}-(1-\delta)K_{H,s-j}\right)+\frac{1}{4}\sum_{j=1}^{4}\left(K_{F,s-j+1}-(1-\delta)K_{F,s-j}\right)-A_{H,s}K_{H,s-4}^{\theta}L_{H,s}^{1-\theta}-A_{F,s}K_{F,s-4}^{\theta}L_{F,s}^{1-\theta}\right)\right]$$

1) First order conditions:

C: $0.5\mu\frac{\sigma-1}{\sigma}U_{i,s}\Big/C_{i,s}=\lambda_s$ for $i=H,F$

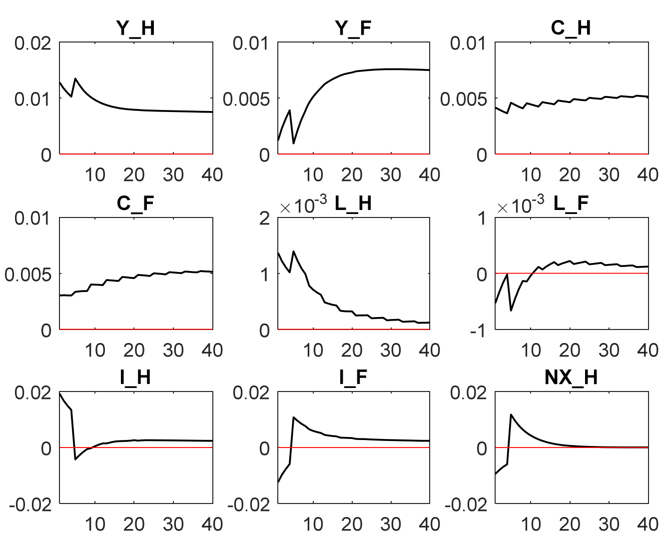
Combining home and foreign versions: $U_{H,s}\big/C_{H,s}=U_{F,s}\big/C_{F,s}$, which is the international risk sharing condition.

L: $$\frac{0.5(1-\mu)\frac{\sigma-1}{\sigma}U_{i,s}\Big/L_{i,s}}{\lambda_s}=(1-\theta)Y_{i,s}/L_{i,s}$$

K: derivative with respect to K implies the FOC:

$$E_t\left[\frac{1}{4}\sum_{j=1}^{4}\left(\beta^{j-1}\lambda_{s+j-1}-\beta^{j}(1-\delta)\lambda_{,s+j}\right)-\beta^4\lambda_{s+4}\theta Y_{H,s+4}/K_{H,s}\right]=0$$

3) Impulse responses for sigma = 0.5



Home output rises with the productivity shock, and rises further four periods later, when new capital comes online. Foreign output rises less, despite the transmission of productivity, due to the fall in foreign labor effort and capital. However, home and foreign consumption move similarly to each other, as it is optimal to smooth consumption across time and across countries. Consumption rises a bit more later since resources are needed early for investment. Investment rises because the marginal product of capital rises with the productivity shock. Foreign investment falls since it is efficient to concentrate production in the more productive home country. Home labor effort rises because of the rise in home marginal product of labor; it fall in foreign because of the fall in capital stock there. Net exports fall initially as resources are shifter from foreign to home to invest in capital in home to take advantage of the higher productivity.

4)

VARIABLE	MEAN	STD. DEV.	VARIANCE
Y_H	1.1241	0.0179	0.0003
Y_F	1.1241	0.0179	0.0003
C_H	0.8402	0.0058	0.0000
C_F	0.8402	0.0058	0.0000
L_H	0.3061	0.0024	0.0000
L_F	0.3061	0.0024	0.0000
I_H	0.2839	0.0448	0.0020
I_F	0.2839	0.0448	0.0020
NX_H	0.0000	0.0381	0.0015

It roughly matches data in that consumption is less volatile than output. By the way, it matches data in that investment more volatile than output, and it t fails in that net exports are too volatile.

Yes, consumption correlation is high, and output correlation is low.

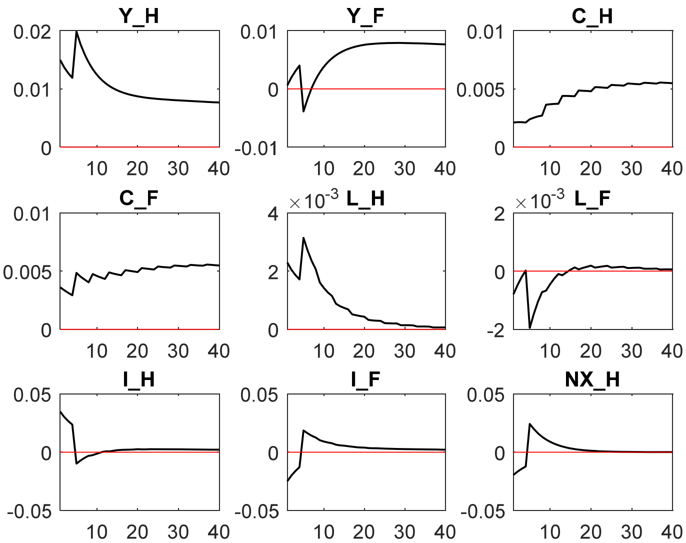
Variables	Y_H	Y_F	C_H	C_F
Y_H	1.0000	-0.1887	0.7907	0.4297
Y_F	-0.1887	1.0000	0.4297	0.7907
C_H	0.7907	0.4297	1.0000	0.8904
C_F	0.4297	0.7907	0.8904	1.0000

5) A higher intertemporal elasticity makes output more variable (higher standard deviation), and lowers the international consumption correlation. But it also lowers the international output correlation. This probably is due to the fact that consumers are willing to put more resources into investment and sacrifice consumption in the short run to raise consumption more later on. A higher intertemporal elasticity also means households are willing to raise labor effort more in short run, substituting utility from leisure in the future in exchange for lower leisure today.

With sigma = 2:
MATRIX OF CORRELATIONS (HP filter, lambda = 1600)

Variables	Y_H	Y_F	C_H	C_F
Y_H	1.0000	-0.5156	0.0007	0.8028
Y_F	-0.5156	1.0000	0.8028	0.0007
C_H	0.0007	0.8028	1.0000	0.5755
C_F	0.8028	0.0007	0.5755	1.0000

Impulse responses for sigma = 2



6) First order conditions:

C: $U_C^{'} = 0.5\mu\left[\mu C_{i,s}^{\frac{\gamma-1}{\gamma}} + (1-\mu)(1-L_{i,s})^{\frac{\gamma-1}{\gamma}}\right]^{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)-1} C_{i,s}^{\frac{-1}{\gamma}} = \lambda_s$ for $i = H, F$

Combining home and foreign versions to get the international risk sharing condition:

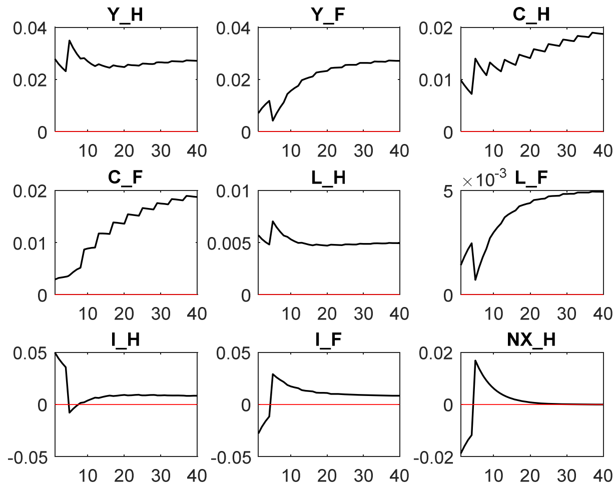
$$\left[\mu C_{H,s}^{\frac{\gamma-1}{\gamma}} + (1-\mu)(1-L_{H,s})^{\frac{\gamma-1}{\gamma}}\right]^{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)-1} C_{H,s}^{\frac{-1}{\gamma}} = \left[\mu C_{F,s}^{\frac{\gamma-1}{\gamma}} + (1-\mu)(1-L_{F,s})^{\frac{\gamma-1}{\gamma}}\right]^{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)-1} C_{F,s}^{\frac{-1}{\gamma}}$$

L: $0.5(1-\mu)\left[\mu C_{i,s}^{\frac{\gamma-1}{\gamma}} + (1-\mu)(1-L_{i,s})^{\frac{\gamma-1}{\gamma}}\right]^{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)-1} (1-L_{i,s})^{\frac{-1}{\gamma}} / \lambda_s = (1-\theta)Y_{i,s}/L_{i,s}$

Combine with derivative wrt C:

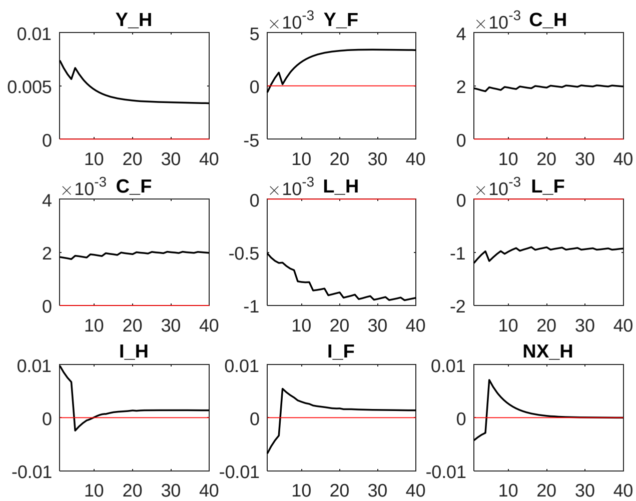
$$\frac{(1-\mu)}{\mu}\left(\frac{1-L_{i,s}}{C_{i,s}}\right)^{\frac{-1}{\gamma}} = (1-\theta)Y_{i,s}/L_{i,s}$$

Case with gamma = 4:



Gamma = 4: corr(C_H,C_F) = 0.1300 (high intratemporal elasticity with low intertemporal elasticity). The social planner is trying to smooth home marginal utility over time. So when home leisure falls due to the rise in productivity, the social planner tries to raise home consumption more to compensate. So home consumption rises more than foreign consumption in response to a positive home productivity shock.

Case with gamma = 0.2:



Gamma = 0.2: corr(C_H,C_F) = 0.997. Now the high intertemporal elasticity implies that the home household is willing to accept the loss of leisure now in exchange for higher leisure in the future. And the low intratemporal elasticity implies limited scope for a rise in consumption to compensate for low leisure today. So it is optimal to allow a big rise in home labor effort and a small rise in home consumption.