Topic 2: International Comovement

Part1: International Business cycle Facts: Quantities

Issue: We now expand our study beyond consumption and the current account, to study a wider range of macroeconomic variables. We will learn about the literature studying business cycles in an international context.

Questions:

- How much do national business cycles move together?
- Is this due more to similar shocks, or due to spillovers?
- Through what markets are shocks transmitted?

a) <u>How to measure business cycles</u>:

To characterize business cycle facts we decompose a time series, y_t, into a

- cyclical component, y_t^c, and a
- secular (or trend) component, y_t^s

$$y_t = y_t^c + y_t^s$$

(a) The Hodrick and Prescott (1997) Filter

Given a time series y_t , for $t = 1, 2, \ldots T$, pick y_t^c and y_t^s to

$$\min_{\{y_t^c, y_t^s\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t^c)^2 + \lambda \sum_{t=2}^{T-1} \left[(y_{t+1}^s - y_t^s) - (y_t^s - y_{t-1}^s) \right]^2 \right\}$$

$$\text{subject to} \quad y_t^s + y_t^c = y_t$$

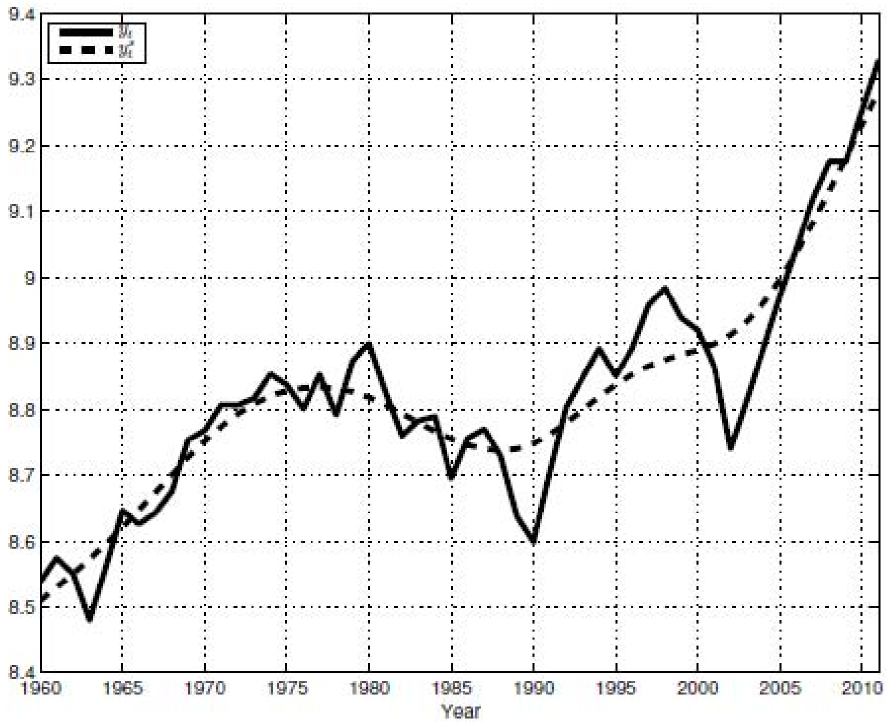
where λ is a parameter.

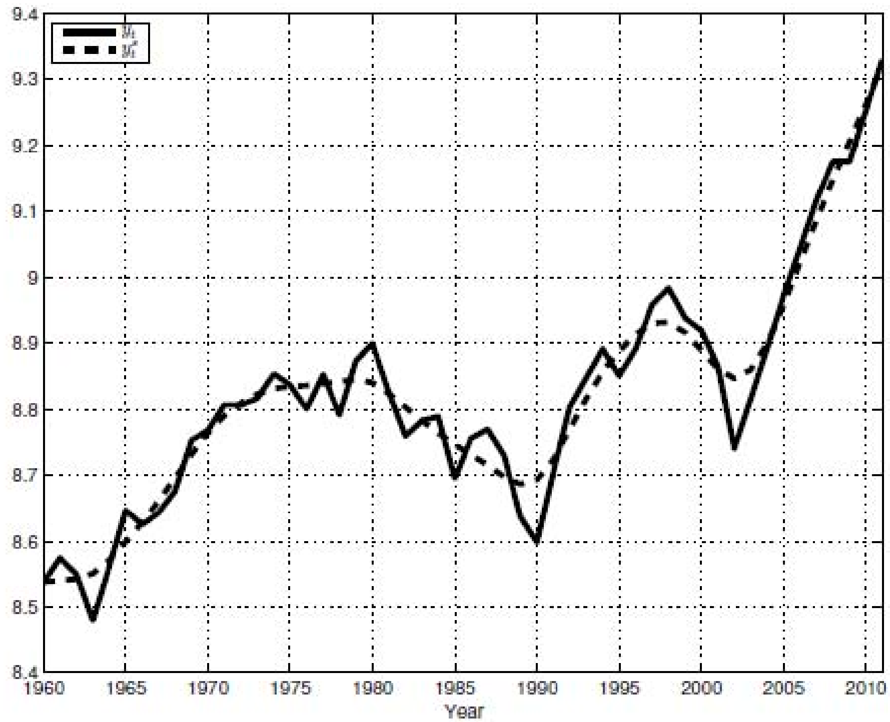
When $\lambda \to \infty$, changes in the growth rate of y_t^s become infinitely costly, and the HP trend component converges to a log-linear trend.

When $\lambda \to 0$, the cycle disappears ($y^c = 0$), and the secular trend is the time series itself ($y_t^s = y_t$).

HP Filtered Trend of Argentine Output

$(\lambda = 100)$ $(\lambda = 6.25)$





HP-6.25 attributes bulk of the 1989 crisis and of the 2001 crisis to trend. But both were cyclical rather than secular for both were followed by rapid recovery. Thus, we will use $\lambda = 100$ for remainder of section.

b) List of business cycle facts:

- Tables 11.1 and 11.2 from Backus et al.

- Data: 10 industrial countries and an aggregate of Europe, quarterly (1970:1 – 1990:2), and Hodrick-Prescott (HP) filtered to focus on business-cycle frequencies in data.

- Collect observations on the following:
  - Volatility: standard deviation
  - Persistence: autocorrelation
  - Comovement: correlations

Table 11.1
Properties of Business Cycles in OECD Economies

Country	Standard Deviation (%)		Ratio of Standard Deviation to That of y					Autocorr.
	y	nx	c	I	g	L	A	y
Australia	1.45	1.23	0.66	2.78	1.28	0.34	1.00	.60
Austria	1.28	1.15	1.14	2.92	0.36	1.23	0.84	.57
Canada	1.50	0.78	0.85	2.80	0.77	0.86	0.74	.79
France	0.90	0.82	0.99	2.96	0.71	0.55	0.76	.78
Germany	1.51	0.79	0.90	2.93	0.81	0.61	0.83	.65
italy	1.69	1.33	0.78	1.95	0.42	0.44	0.92	.85
japan	1.35	0.93	1.09	2.41	0.79	0.36	0.88	.80
Switzerland	1.92	1.32	0.74	2.30	0.53	0.71	0.67	.90
United Kingdom	1.61	1.19	1.15	2.29	0.69	0.68	0.88	.63
United States	1.92	0.52	0.75	3.27	0.75	0.61	0.68	.86
Europe	1.01	0.50	0.83	2.09	0.47	0.85	0.98	.75

Country	Correlations with output					
	C	I	G	NX	L	A
Australia	.46	.67	.15	-.01	.12	.98
Austria	.65	.75	-.24	-.46	.58	.65
Canada	.83	.52	-.23	.-26	.69	.84
France	.61	.79	.25	-.30	.77	.96
Germany	.66	.84	.26	-.11	.59	.93
Italy	.82	.86	.01	-.68	.42	.96
Japan	.80	.90	-.02	-.22	.60	.98
Switzerland	.81	.82	.27	-.68	.84	.93
UK	.74	.59	.05	-.19	.47	.90
US	.82	.94	.12	-.37	.88	.96
Europe	.81	.89	.10	-.25	.32	.85

Table 11.2
International Comovements in OECD Economies

Country	*Correlation of Each Country's Variable with Same U.S. Variable*					
	y	*c*	I	*g*	L	A
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
France	.41	.39	.22	−.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	−.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Table 1 Real GDP moments

	standard deviation	correlation with US
US	0.94	--
Canada	1.03	0.79
France	0.82	0.58
Germany	1.53	0.33
Italy	1.16	0.53
Japan	1.27	0.50
UK	1.07	0.71
China	1.03	0.79
Mexico	1.66	0.57
Korea	1.10	0.18

Source: International Financial Statistics, IMF. Quarterly real GDP, 1980:1-2016:3. Seasonally adjusted, logged and HP filtered.

Table 11-1

Domestic Volatilities:

- Consumption is less volatile than output, reflecting consumption-smoothing.
- Investment is more volatile than output: 2-3 times.
- Countries differ much: output is more volatile in the US (sdev = 1.92); least in France (0.9).

Employment is procyclical

- The Solow residual is strongly procyclical, but less volatile than output.
- Technology shocks help explain fluctuations in output, but they need endogenous fluctuations in labor supply to amplify their effects on output.

Net exports:

- The trade balance is countercyclical in all 10 countries
  This is due to the volatile cyclical movement of investment.

- This is contrary to the simple PV model of the CA we studied where investment was exogenous. (Temporary rise in output should lead to a CA surplus.)

- Can be explained by allowing investment to rise in response to output by large amount, as in last lecture.

Persistence: Output quite persistent, autocorr from 0.5 - 0.9.

Table 11-2: International correlations: (with U.S.)

cor(Y,Y*) > cor (C,C*) for all cases

This is referred to as the Consumption Correlation Puzzle:

Reasons this fact is puzzling:

- If asset markets pool consumption risk, then consumption should move similarly across countries.
- True even in a simple PV model of the CA with non-contingent bonds if think in a two-country context:
- A fall in home country endowment leads to a smaller fall in consumption because borrow from abroad.
- Foreign lenders cut their consumption in response to rise in real interest rate.

Updated data from *SGU* (2017) text: average of sample of 120 counties, 1960-2011; basic facts still hold

Global Ranking of Volatilities

Business-Cycle Statistic	World Average
$\frac{\sigma_m}{\sigma_y}$	3.23
$\frac{\sigma_i}{\sigma_y}$	3.14
$\frac{\sigma_x}{\sigma_y}$	3.07
$\frac{\sigma_g}{\sigma_y}$	2.26
$\frac{\sigma_c}{\sigma_y}$	1.05

Fact 3: The ranking of cross-country average standard deviations from top to bottom is imports, investment, exports, government spending, consumption, and output.

Cyclicality

Business-Cycle Statistic	World Average
corr(c, y)	0.69
corr(i, y)	0.66
corr(x, y)	0.19
corr(m, y)	0.24
corr(tb, y)	-0.18
corr(ca, y)	-0.28
corr$(g/y, y)$	-0.02

Fact 4:

Consumption, investment, exports, and imports are procyclical.

Fact 5:

The trade balance and the current account are countercyclical.

Fact 6:

The share of government consumption in output is acyclical.

Persistence

Business-Cycle Statistic	World Average
corr(y_t, y_{t-1})	0.71
corr(c_t, c_{t-1})	0.66
corr(g_t, g_{t-1})	0.76
corr(i_t, i_{t-1})	0.56
corr(x_t, x_{t-1})	0.68
corr(m_t, m_{t-1})	0.61

Fact 7: All components of demand (c, g, i, x) and supply (y, m) are positively serially correlated.

Excess Volatility of Poor and Emerging Countries

Business-Cycle Statistic	Poor	Emerging	Rich
σ_y	6.1%	8.7%	3.3%

Fact 8: Business Cycles in poor and emerging countries are about **twice** as volatile as business cycles in rich countries.

Less Consumption Smoothing in Poor and Emerging than in Rich Countries

Business-Cycle Statistic	Poor	Emerging	Rich
σ_c/σ_y	1.12	0.98	0.87

Fact 9: The relative consumption volatility is higher in poor and emerging countries than in rich countries.

updated data on international comovement from Bergin (2017)

1) Output correlations remaining high, especially among OECD countries

2) But output correlations fluctuate over time. Five-year window shows near perfect correlation in recent “Global Recession”, but was temporary, and recently returned to lower correlations.

Table 1 Real GDP moments

	standard deviation	correlation with US
US	0.94	--
Canada	1.03	0.79
France	0.82	0.58
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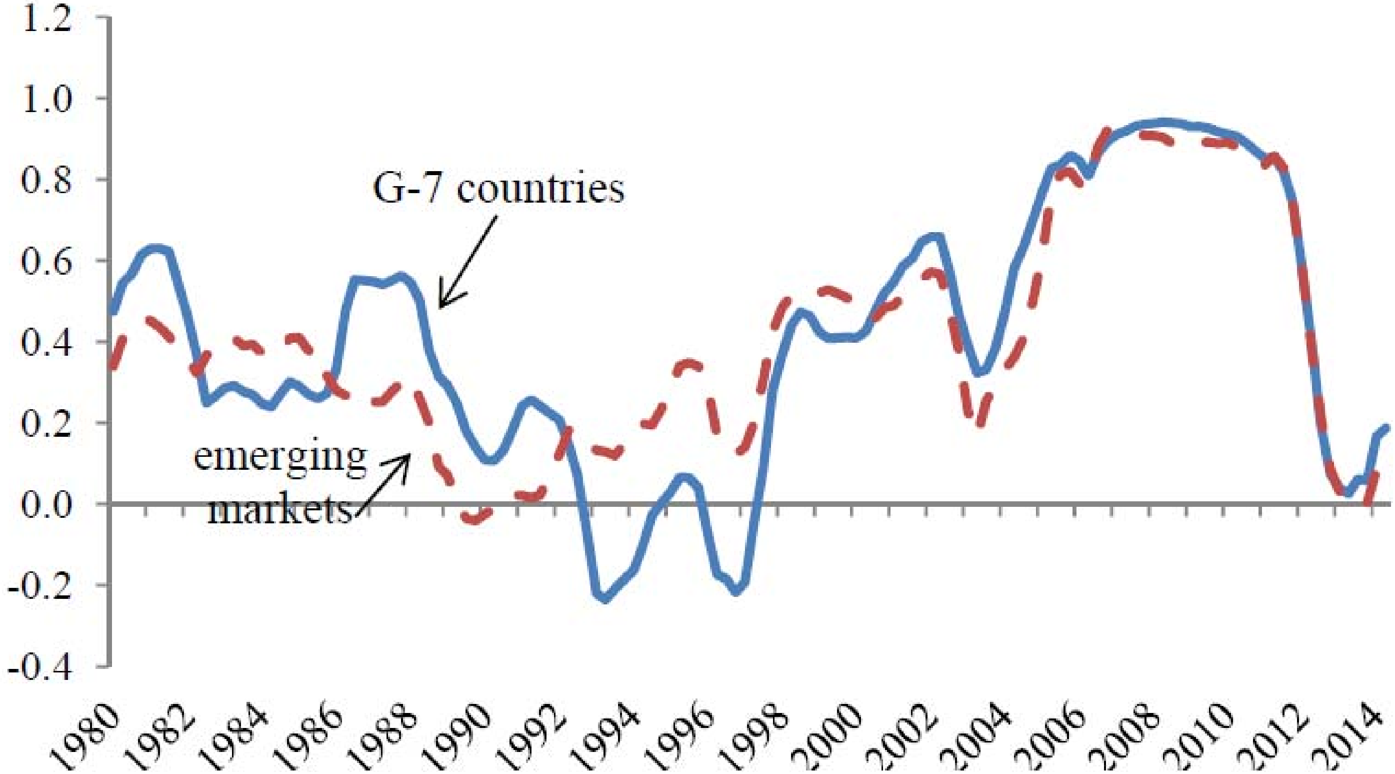
Source: International Financial Statistics, IMF. Quarterly real GDP, 1980:1-2016:3. Seasonally adjusted, logged and HP filtered.

Fig. 1. Correlation of GDP with US in rolling 5 year window



Solid line represents average among G7 countries (excluding US), which includes Canada, France, Germany, Italy, Japan, United Kingdom. Dashed line represents average among emerging markets countries in our sample, which includes China, Korea, Mexico. Source: International Financial Statistics, IMF. Quarterly real GDP, 1980:1-2016:3. Seasonally adjusted, logged and HP filtered.

Part 2: A simple two country business cycle model

- We here will use the approach of Real Business Cycle (RBC) models, pioneered by Backus, et al (1992 JPE) which set the agenda for the resulting literature.

- This differs from models we've used in previous lectures, in that output no longer is an exogenous endowment, but now is produced using capital and labor inputs.

- The majority of papers in this literature use two-country models.

  This is different from the models considered so far in class, which were small-open economy models.
  We present here a streamlined version of Backus, et al (1992 JPE), eliminate inventory accumulation.

Description:

- Two countries: home (h) and foreign (f).
- One world consumption good (for now)
- Production of output using capital and labor

Preferences:

Utility of representative household: cares about both consumption (C) and leisure $(1\text{-}L)$, where L is labor.

$$U_{it} = \left(C_{it}^{\ \mu} (1 - L_{it})^{1-\mu} \right)^{1-\frac{1}{\sigma}} \quad i = h, f$$

Agent allocates one unit of time between work and leisure.

Production

Production a function of labor (L) and capital (K) and productivity term A:

$$Y_{it} = F\left(K_{it}, L_{it}\right) = A_{it} K_{it}{}^{\theta} L^{1-\theta}{}_{it} \quad i = h, f$$

Since both countries produce the same good, the resource constraint is:

$$Y_{ht} + Y_{ft} = C_{ht} + C_{ft} + I_{ht} + I_{ft} + G_{ht} + G_{ft}$$

Capital formation uses time-to-build structure.
Additions to the stock of fixed capital require inputs of the produced good for 4 periods:

$$K_{t+1} = (1-\delta)K_t + s^1{}_t$$

(For the home country; analogous for foreign. Skipped i subscripts on everything to avoid confusion)

Where $s^j{}_t$ is the number of investment projects at date t that are j periods from completion.

$$s^j{}_{t+1} = s^{j+1}{}_t$$

It takes 4 periods for a capital good to be built and increase the capital stock. So put in 1/4 of value added each period:

If add up all the investment expenditure made in a period on the projects at various stages of completion, it equals:

$$I_t = \sum_{j=1}^{4} \frac{1}{4} s_t^j$$

Shocks

Separate technology shock in each country, but can be correlated.

$$\begin{bmatrix} A_{ht+1} \\ A_{ft+1} \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} A_{ht} \\ A_{ft} \end{bmatrix} + \begin{bmatrix} \varepsilon_{ht+1} \\ \varepsilon_{ft+1} \end{bmatrix}$$

Where epsilons have covariance matrix:

$$E_t \left[\begin{pmatrix} \varepsilon_{ht} \\ \varepsilon_{ft} \end{pmatrix} \begin{pmatrix} \varepsilon_{ht} & \varepsilon_{ft} \end{pmatrix} \right] = V$$

Correlations in technology are captured by off-diagonal elements of rho matrix and V matrices.

Equilibrium:

- We will assume that financial markets are complete. People in either country have access to a full set of conditional assets they can buy to insure against shocks.
- We could try to model explicitly all the assets and find the solution for the competitive equilibrium: see notes further below.
- Under complete markets, solution will be Pareto optimum.
- So we can also solve for the equilibrium as a single optimization problem of a social planner that maximizes the weighted sum of utilities of the two countries.
- So solve following subject to the constraints above:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[\psi U\left(C_{ht}, \left(1 - L_{ht}\right)\right) + \left(1 - \psi\right) U\left(C_{ft}, \left(1 - L_{ft}\right)\right)\right]$$

- Combining FOC for consumption with that for labor).

$$\frac{-U'_{L_h,t}}{U'_{C_h,t}} = F'_{L_h,t}$$

equating marginal utility of lost leisure to marginal utility of extra consumption if provide additional labor.

- Combining FOC for home and foreign consumptions):

$$U'_{C_h,t} = \left(\frac{1-\psi}{\psi}\right) U'_{C_f,t}$$

International Risk sharing condition, equating changes in marginal utility across countries.

Solution:

- Combine these optimality conditions with the resource constraints.

- Solve for a deterministic steady state (dropping uncertainty)

- Take a log-Linear approximation around the steady state.

- Solve the linear system of equations, such as by method of Blanchard and Kahn (1980): find unstable roots of system by eigen values; imposing the associated eigenvectors.

Derivation of a first-order accurate approximation

The next 12 slides are optional material for those interested in knowing more of the theory behind solution of a DSGE model.

The model developed above gives rise to equilibrium conditions of the form

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{1}$$

where

$x_t = n_x \times 1$ vector of predetermined (or state) variables

$y_t = n_y \times 1$ vector of nonpredetermined (or control) variables

x_0 is an $n_x \times 1$ vector of initial conditions

Terminal condition: $\lim_{j\to\infty} E_t \begin{bmatrix} x_{t+j} & y_{t+j} \end{bmatrix}' \to \begin{bmatrix} \bar{x} & \bar{y} \end{bmatrix}'$

Let

$$n = n_x + n_y$$

A large class of dynamic stochastic general equilibrium models can be written in the form given in (1). And most studies in real and monetary business cycle analysis use models belonging to this class. Of course, there are also many types of models that do not fit into that class. For example, models with occasionally binding constraints.

Partition state vector x_t

$$x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix}$$

$x_t^1 =$ vector of endogenous predetermined state variables
$x_t^2 =$ vector of exogenous state variables

We assume that the exogenous state evolves as:

$$x_{t+1}^2 = \tilde{h}(x_t^2, \sigma) + \sigma\tilde{\eta}\epsilon_{t+1}, \tag{2}$$

$\sigma =$ parameter scaling the amount of uncertainty. ($\sigma = 0$ is perfect foresight.)

Solution to models that are described by (1) and (2) can then be expressed as:

$$y_t = \widehat{g}(x_t) \tag{3}$$

$$x_{t+1} = \widehat{h}(x_t) + \sigma \eta \epsilon_{t+1} \tag{4}$$

where

$$\eta = \begin{bmatrix} \emptyset \\ \tilde{\eta} \end{bmatrix}.$$

The shape of the functions $\widehat{h}$ and $\widehat{g}$ will in general depend on the amount of uncertainty in the economy.

Key idea of perturbation: parameterize the amount of uncertainty as follows

$$\widehat{g}(x_t) = g(x_t, \sigma)$$

$$\widehat{h}(x_t) = h(x_t, \sigma)$$

Then we can write the solution to the model described by (1) and (2) as

$$y_t = g(x_t, \sigma) \tag{5}$$

$$x_{t+1} = h(x_t, \sigma) + \sigma \eta \epsilon_{t+1} \tag{6}$$

Perturbation methods perform a *local* approximation of $g(x, \sigma)$ and $h(x, \sigma)$ around a particular point $(\bar{x}, \bar{\sigma})$

First-order Taylor series expansion of g and h around $(x,\sigma) = (\bar{x},\bar{\sigma})$

$$\begin{aligned} g(x,\sigma) &= g(\bar{x},\bar{\sigma}) + g_x(\bar{x},\bar{\sigma})(x-\bar{x}) + g_\sigma(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma}) + h.o.t. \\ h(x,\sigma) &= h(\bar{x},\bar{\sigma}) + h_x(\bar{x},\bar{\sigma})(x-\bar{x}) + h_\sigma(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma}) + h.o.t. \end{aligned}$$

$h.o.t.$ = higher order terms
Unknowns: $g(\bar{x},\bar{\sigma}), g_x(\bar{x},\bar{\sigma}), g_\sigma(\bar{x},\bar{\sigma}), h(\bar{x},\bar{\sigma}), h_x(\bar{x},\bar{\sigma}), h_\sigma(\bar{x},\bar{\sigma})$
To identify these terms, substitute the proposed solution given by equations (5) and (6) into equation (1), and define

$$\begin{aligned} F(x,\sigma) &\equiv E_t f(g(h(x,\sigma)+\eta\sigma\epsilon',\sigma), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) \quad (7) \\ &= 0. \end{aligned}$$

Here we are dropping time subscripts. We use a prime to indicate variables dated in period $t+1$.

Because $F(x,\sigma)$ must be equal to zero for any possible values of x and σ, it must be the case that the derivatives of any order of F must also be equal to zero. Formally,

$$F_{x^k\sigma^j}(x,\sigma) = 0 \quad \forall x,\sigma,j,k, \tag{8}$$

where $F_{x^k\sigma^j}(x,\sigma)$ denotes the derivative of F with respect to x taken k times and with respect to σ taken j times.

What point to approximate around?

We need to evaluate the derivatives of $F(x,\sigma)$, $F_{x^k\sigma^j}(x,\sigma)$, at the point we are approximating the equilibrium around. In general this is difficult if not impossible. But there are some points for which evaluation of those derivatives is possible.

One such point is the non-stochastic steady state, $(x,\sigma) = (\bar{x}, 0)$, where $\bar{x}$ denotes the non-stochastic steady state value of x_t. For this point we know: $y_t = \bar{y}$, $y_{t+1} = \bar{y}$, and $x_{t+1} = \bar{x}$, where $\bar{y}$ denotes the non-stochastic steady state of y_t.

Another point one can evaluate the derivatives of $F(x,\sigma)$ at is $x_t \neq \bar{x}$ and $\sigma = 0$. This works in cases in which one can find the exact deterministic solution of a model. In that case one can find y_t, y_{t+1} and x_{t+1} for $(x_t,\sigma) = (x_t, 0)$ but needs to resort to approximation techniques to characterize the solution to the stochastic version of the economy.

For the remainder of this chapter we will focus on approximation around the non-stochastic steady state $(x,\sigma) = (\bar{x}, 0)$.

Let's write again the first-order Taylor series expansion of g and h but this time around the non-stochastic steady state, $(x, \sigma) = (\bar{x}, 0)$

$$g(x, \sigma) = g(\bar{x}, 0) + g_x(\bar{x}, 0)(x - \bar{x}) + g_\sigma(\bar{x}, 0)(\sigma - 0)$$

$$h(x, \sigma) = h(\bar{x}, 0) + h_x(\bar{x}, 0)(x - \bar{x}) + h_\sigma(\bar{x}, 0)(\sigma - 0)$$

We wish to find:

$g(\bar{x}, 0)$

$g_x(\bar{x}, 0)$

$g_\sigma(\bar{x}, 0)$

$h(\bar{x}, 0)$

$h_x(\bar{x}, 0)$

$h_\sigma(\bar{x}, 0)$

Find $g(\bar{x}, 0)$ and $h(\bar{x}, 0)$

From (5)

$$g(\bar{x}, 0) = \bar{y}$$

From (6)

$$h(\bar{x}, 0) = \bar{x}$$

Find h_σ and g_σ

Recall (7)

$$\begin{aligned} 0 &= F(x,\sigma) \\ &= E_t f(g(h(x,\sigma)+\eta\sigma\epsilon',\sigma), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) \end{aligned}$$

The first derivative of $F(x,\sigma)$ with respect to σ evaluated at $(x,\sigma) = (\bar{x},0)$

$$\begin{aligned} 0 &= F_\sigma(\bar{x},0) \\ &= f_{y'}(\bar{y},\bar{y},\bar{x},\bar{x})\left[g_x(\bar{x},0)h_\sigma(\bar{x},0)+g_\sigma(\bar{x},0)\right] \\ &\quad +f_y(\bar{y},\bar{y},\bar{x},\bar{x})g_\sigma(\bar{x},0) \\ &\quad +f_{x'}(\bar{y},\bar{y},\bar{x},\bar{x})h_\sigma(\bar{x},0) \end{aligned}$$

Let $f_i \equiv f_i(\bar{y},\bar{y},\bar{x},\bar{x})$ for $i = y',' y, x', x$

Note that we can evaluate f_i because we know the function f and we know the steady state $(\bar{y},\bar{x})$

Rearrange to obtain

$$\left[\begin{array}{cc} f_{y'}g_x + f_{x'} & f_{y'} + f_y \end{array}\right]\left[\begin{array}{c} h_\sigma \\ g_\sigma \end{array}\right] = 0$$

This is a linear homogenous equation in n unknowns. For it to have a unique solution it must be that

$$\left[\begin{array}{c} h_\sigma \\ g_\sigma \end{array}\right] = \left[\begin{array}{c} \emptyset \\ \emptyset \end{array}\right] \tag{9}$$

This is an important result. It says that up to first-order accuracy one need not correct the constant term or the slope term of the approximation for the presence of uncertainty. The policy function is the same as under perfect foresight but for the additive stochastic error term. (the solution displays the certainty equivalence principle)

Up to first order accuracy the solution is:

$$\begin{aligned} y_t &= \bar{y} + g_x(\bar{x}, 0)(x - \bar{x}) \\ x_{t+1} &= \bar{x} + h_x(\bar{x}, 0)(x - \bar{x}) + \sigma\eta\epsilon_{t+1} \end{aligned}$$

Consider the unconditional expectations of x_t of the first-order accurate approximation:

$$\begin{aligned} E(x_t) &= E\left\{\bar{x} + h_x(\bar{x}, 0)(x_t - \bar{x}) + h_\sigma(\bar{x}, 0)(\sigma - 0)\right\} \\ &= \bar{x} + h_x(\bar{x}, 0)(E(x_t) - \bar{x}) + 0 \end{aligned}$$

It follow that up to first order accuracy:

$$Ex_t = \bar{x} \quad \text{and} \quad Ey_t = \bar{y}$$

or in words the unconditional expectation is the same as the mean. Hence first-order accurate approximations will not be helpful to approximate average risk premia (they would all be zero) or the average welfare associated with different monetary or fiscal policy that all give rise to the same nonstochastic steady state (all policies give the same welfare in the steady state).

Find $h_x(\bar{x}, 0)$ and $g_x(\bar{x}, 0)$

Start again from (7)

$$\begin{aligned} 0 &= F(x, \sigma) \\ &= E_t f(g(h(x, \sigma) + \eta\sigma\epsilon', \sigma), g(x, \sigma), h(x, \sigma) + \eta\sigma\epsilon', x) \end{aligned}$$

The first derivative of $F(x, \sigma)$ with respect to x evaluated at $(x, \sigma) = (\bar{x}, 0)$

$$\begin{aligned} 0 &= F_x(\bar{x}, 0) \\ &= f_{y'} g_x h_x + f_y g_x + f_{x'} h_x + f_x \end{aligned}$$

Rearrange to

$$\begin{bmatrix} f_{x'} & f_{y'} \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix} h_x = - \begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix}$$

To solve this expression for h_x and g_x use a Schur decomposition.

- Calibration:

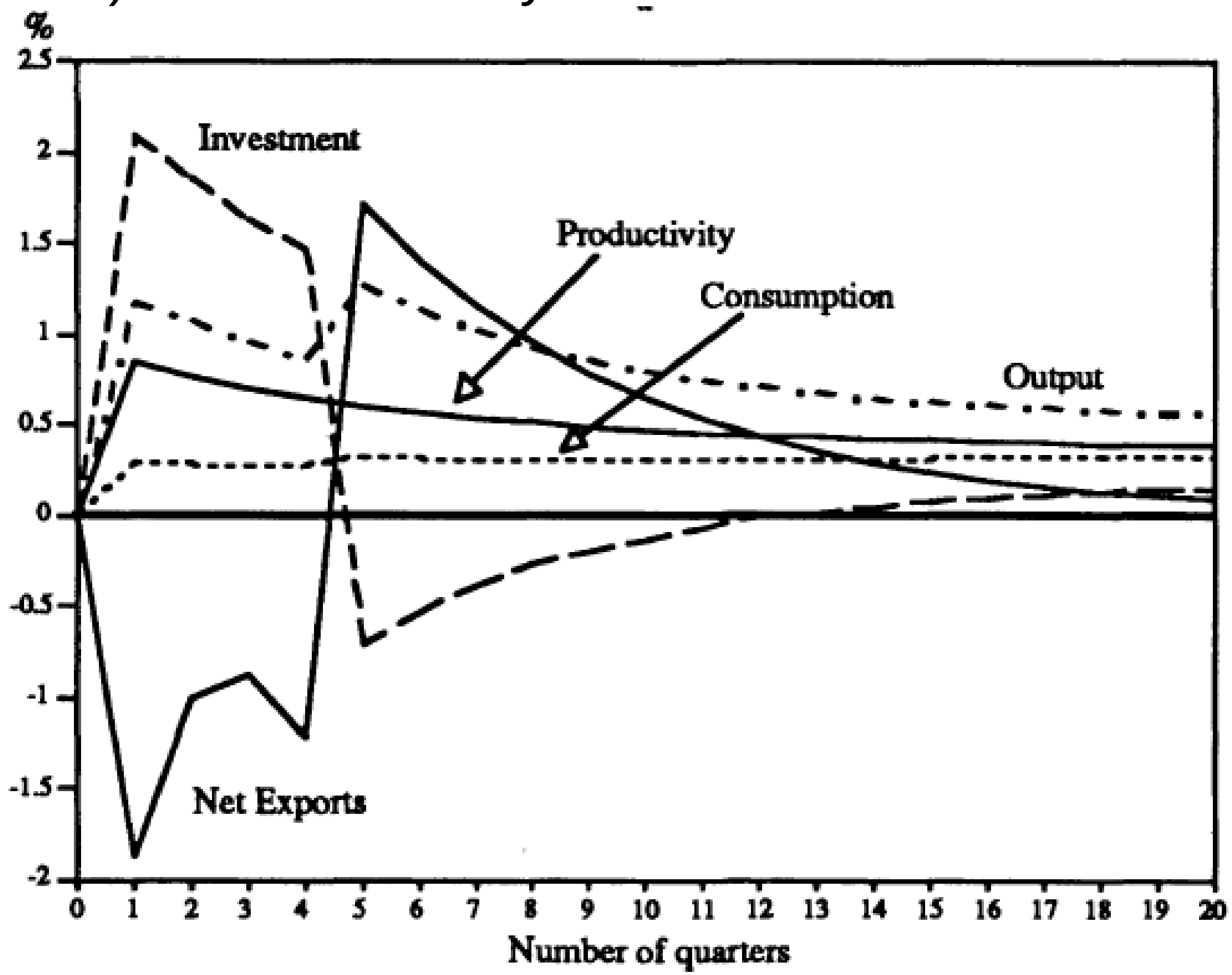
  - o discount factor = 0.99 (assume quarterly period).
  - o Intertemporal elasticity equals 0.5.
  - o Technology shocks have persistence 0.9, and cross persistence of 0.09. Correlation of epsilons are 0.258.

- Simulate: 20 runs of 100 periods each.

- Hodrick-Prescott (HP) filter and compute same statistics as for actual data from the real economy.

- Compare the moments from simulated data to those from actual data.

d) Results: Consider a 1 % rise in A (positive epsilon for one period) in home country.



First do impulse responses:

Home:

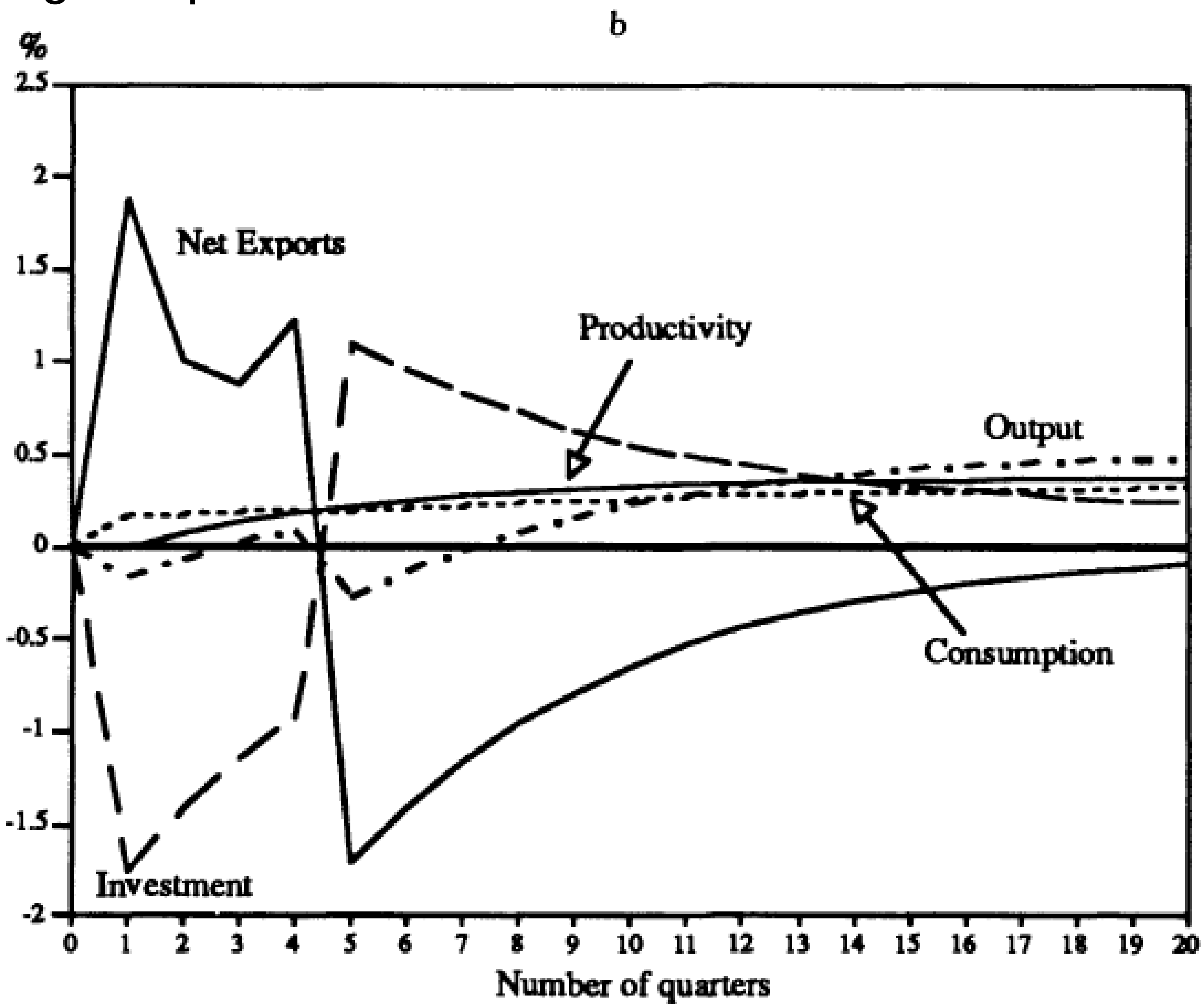
- Rise in productivity raises output.

- Also raises investment because of marginal productivity of capital. Investment is very volatile in an open economy since it easy to borrow from abroad to finance investment.

- This makes net exports go negative (not shown explicitly, but is apparent).

- Also raises consumption as smoothed.

Foreign response:



Foreign:

- Foreign investment moves the opposite way because want to shift resources to where are the most productive. As a result output moves opposite as well. Falls at first.

- But consumption moves very similarly. Even though output falls, consumption rises like in home country.

- This is due to social planner / risk sharing.

Moments:

	A. Business Cycle Properties						
	Standard Deviation (%)		*Ratio of Standard Deviation to That of y*				*Autocorr.*
Economy	*y*	*nx*	*c*	I	L	A	*y*
U.S. data	1.92	0.52	.75	3.27	.61	.68	.86
Benchmark	1.50	3.77	.42	10.99	.50	.67	.62
Transport cost	1.35	0.37	.47	2.91	.47	.75	.61
Autarky	1.26		.54	2.65	.91	.99	.62

B. International Comovements

Economy	Correlation of Foreign and Domestic Variables				
	y	*c*	I	L	A
U.S. data	.66	.51	.53	.33	.56
Benchmark	−.21	.88	−.94	−.94	.25
Transport cost	−.05	.89	−.48	−.48	.25
Autarky	.08	.56	−.31	−.31	.25

Correlations:

- Also see in correlations: output correlation is less than in data (-0.21 versus 0.66), but consumption correlation is higher than in data (.88 versus .51).

- Otherwise match things pretty well. (Investment a bit too volatile)

- Main problem is **consumption correlation puzzle**: consumption is less correlated in data than is output. Model says the opposite.

One Possible solution: Transport costs:

- Try a version of model with costs of trading goods.

- In a world budget constraint, impose a cost that is a quadratic function of net exports. So if try to import goods to raise investment, becomes expensive.

$$Y_{ht} + Y_{ft} = C_{ht} + C_{ft} + I_{ht} + I_{ft} + G_{ht} + G_{ft} + \tau\, nx_t^2$$

$$where \quad nx_t \equiv Y_{ht} - C_{ht} - I_{ht} - G_{ht}$$

- Mechanically, just add this term on to budget constraint before do first order conditions. Calibrate, so on average cost is only about 1%.

- Result: lower response of net exports, and thereby investment response to technology shock. But not affect consumption or output correlation much. Output cor rise from -.21 to -.05. Consumption cor rise from 0.88 to 0.89.

Part 3: Business Cycle Facts: Relative Prices

Stylized facts

- Define: Terms of Trade (TOT) = price of exports / price of imports.

- The relative price data reported here is the inverse of the usual definition of the TOT given above.

- Regularities: Look at table 11.5

- The ‘terms of trade’ is highly variable: Standard deviations are usually 2-3 times that of output.

- It is also highly persistent, with an autocorrelation near 0.8.

Table 11.5
Properties of the Terms of Trade in OECD Economies

Country	SD $p(\%)$	Autocorr. p	Correlation of: (p, nx)	(p, y)
Australia	5.78	.82	−.10	−.27
Austria	1.73	.46	−.24	.04
Canada	2.99	.85	.05	−.05
France	3.52	.75	−.50	−.13
Germany	2.66	.85	−.08	−.11
Italy	3.50	.78	−.66	.38
Japan	7.24	.86	−.56	−.22
Switzerland	2.85	.88	−.61	.41
United Kingdom	3.14	.80	−.58	.09
Europe	3.68	.83	.30	−.20

Model

- To describe relative prices, we need two types of goods.

- Assume each country produces a distinctive good. Home produces good 1; foreign good 2. Households in each country consume both goods.

- Changes in model: Use star to indicate foreign variable, H and F to indicate good.

- Two goods market clearing conditions:

$$Y_{ht} = C_{ht} + C*_{ht} + I_{ht} + I*_{ht} + G_{ht} + G*_{ht}$$

$$Y*_{ft} = C_{ft} + C*_{ft} + I_{ft} + I*_{ft} + G_{ft} + G*_{ft}$$

- Budget constraint is (using home goods as a numeraire)

$$Y_{ht} + p_t Y_{ft}^* = C_{ht} + C_{ht}^* + p_t \left(C_{ft} + C_{ft}^* \right) + \ldots$$

where p_t is the relative price of foreign goods in terms of home goods (p_f/p_h), or from home perspective, the relative price of imported goods in terms of exported goods

So p_t is the inverse of the terms of trade as conventionally defined above.

- Model home consumption as an aggregation (a function "g") over home and foreign good:

$$C_t = g(C_{ht}, C_{ft}) = \left[C_{ht}^{\varpi} C_{ft}^{1-\varpi} \right]$$

Where start off using Cobb-Douglas for the aggregation function. (Can use same aggregation function for investment and government demands.)

- Put this in utility function, and derive optimal choice between the two goods based on relative price. Intratemporal substitution again:

$$\frac{U'_{cf}}{U'_{ch}} = p_t$$

- Using chain rule over the utility function, can express p_t (the inverse TOT) as the ratio of derivatives of the aggregation function over the two types of goods.

$$p_t = \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ft}} / \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ht}} \quad = \frac{1-\varpi}{\varpi}\left(\frac{C_{ht}}{C_{ft}}\right)$$

- Can compute net exports (in units of home goods):

$$nx_t = C_{ht}* - p_t C_{ft}$$

Results

Calibrate:

- Share of imports in GNP = 0.15

Simulation results for TOT:

- Persistence: 0.83, similar to data. Inherit persistence from technology shock.

- Correlation of TOT with NX is negative, similar to data.

- Volatility: Sdev of TOT is much less in model than in data (data is 7X larger).

Table 11.6

Properties of the Terms of Trade in Theoretical Economics

Country	*SD* $p(\%)$	*Autocorr.* p	*Correlation of:* (p, nx)	(p, y)
U.S. data	3.68	.83	.30	−.20
Benchmark	0.48	.83	−.41	.49
Two shocks (technology and government spending)	0.57	.67	−.05	.39
Large import share	0.66	.83	−.41	.55
Small elasticity	0.76	.77	−.80	.51

Puzzle:

- So have a "**relative price puzzle**."
  It is clear we can't resolve this puzzle in this model just by varying parameter values.

- Discuss Ideas of how to resolve?

- Recall that the intratemporal optimality condition shows that the relative price is directly related to the ratio of imports to consumption of domestic goods.

$$p_t = \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ft}} / \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ht}}$$

$$= \frac{1-\varpi}{\varpi}\left(\frac{C_{ht}}{C_{ft}}\right)$$

in percent changes

$$\tilde{p}_t = \tilde{C}_{ht} - \tilde{C}_{ft}$$

- Model matches volatility of the import ratio. But is no way to increase the volatility of the terms of trade, due to tight connection between quantities and relative prices here.

- If there is a negative technology shock abroad that raises the relative price of imported goods in the home country, there is a fall in the quantity of imported goods.

- The tight link between price and quantities implies that technology shocks that lead to moderate swings in quantities cannot generate big swings in prices.

Alternative

- Consider using a different aggregator, with an intratemporal elasticity different from unity:

$$C_t = g\left(C_{ht}, C_{ft}\right) = \left[\varpi C_{ht}^{1-\frac{1}{\psi}} + \left(1-\varpi\right) C_{ft}^{1-\frac{1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

where ψ is the elasticity of intratemporal substitution.

- This alters the intratemporal condition (in log deviations):

$$\tilde{p}_t = \frac{1}{\psi}\left(\tilde{C}_{ht} - \tilde{C}_{ft}\right)$$

- Idea: If make intratemporal elasticity (ψ) small, then the change in p will be big for a given change in import share.

- If goods are not very substitutable, a fall in supply of importable good will require a very big rise in the price of importables to make everyone willingly consume less of them.

- But empirical estimates imply a range of 0.5 to 5 for the elasticity; even a small value of 0.5 is not small enough to generate the observed price volatility.

Conclusion: How to break the tight link between relative prices and quantities is a topic pursued in subsequent literature, and we will discuss this further in later lectures.

Backus-Smith puzzle

- A related puzzle involves the comovement of international relative prices and relative consumption levels.

- The real exchange rate, q, is defined as the ratio of national consumer price indexes in each country, foreign to home.

- Note: The national price indexes and real exchange rate are functions of the terms of traded used above (see below).

- Backus and Smith (1993) documented that correlations between international relative consumption ratios and real exchange rates are negative or zero.

- Confirmed with more recent data, as in table below taken from Corsetti et al (REStud 2008).

Aside: Derive national price index:

Define the consumption index as above: $C_t = C_{h,t}^{\varpi} C_{f,t}^{1-\varpi}$

Define the price index, P, as the minimum expenditure required to purchase one unit of the consumption index

$$P_t \equiv \min C_{h,t} + p_t C_{f,t} \qquad \text{s.t. } C_t = C_{h,t}^{\varpi} C_{f,t}^{1-\varpi} = 1$$

Implies demands:

$$C_{h,t} = \left(\frac{\varpi}{1-\varpi}\right)^{1-\varpi} p_t^{1-\varpi} \quad \text{and} \quad C_{f,t} = \left(\frac{\varpi}{1-\varpi}\right)^{-\varpi} p_t^{-\varpi}$$

Plug into definition: $P_t = C_{h,t} + p_t C_{f,t}$

$$P_t = \left(\frac{\varpi}{1-\varpi}\right)^{1-\varpi} p_t^{1-\varpi} + p_t \left(\frac{\varpi}{1-\varpi}\right)^{-\varpi} p_t^{-\varpi} = \left[\left(\frac{\varpi}{1-\varpi}\right)^{1-\varpi} + \left(\frac{\varpi}{1-\varpi}\right)^{-\varpi}\right] p_t^{1-\varpi}$$

Aside cont.: Find the real exchange rate:

Assume symmetry: foreign consumption index has weight ϖ on foreign good consumption. Note:

$$P^*_t = \left[\left(\frac{1-\varpi}{\varpi} \right)^{\varpi} + \left(\frac{1-\varpi}{\varpi} \right)^{\varpi-1} \right] p_t^{\varpi}$$

Note: Implies consumption home bias if ϖ >½.

Plug into definition of real exchange rate:

$$q_t \equiv \frac{P^*_t}{P_t} = p_t^{2\varpi-1}$$

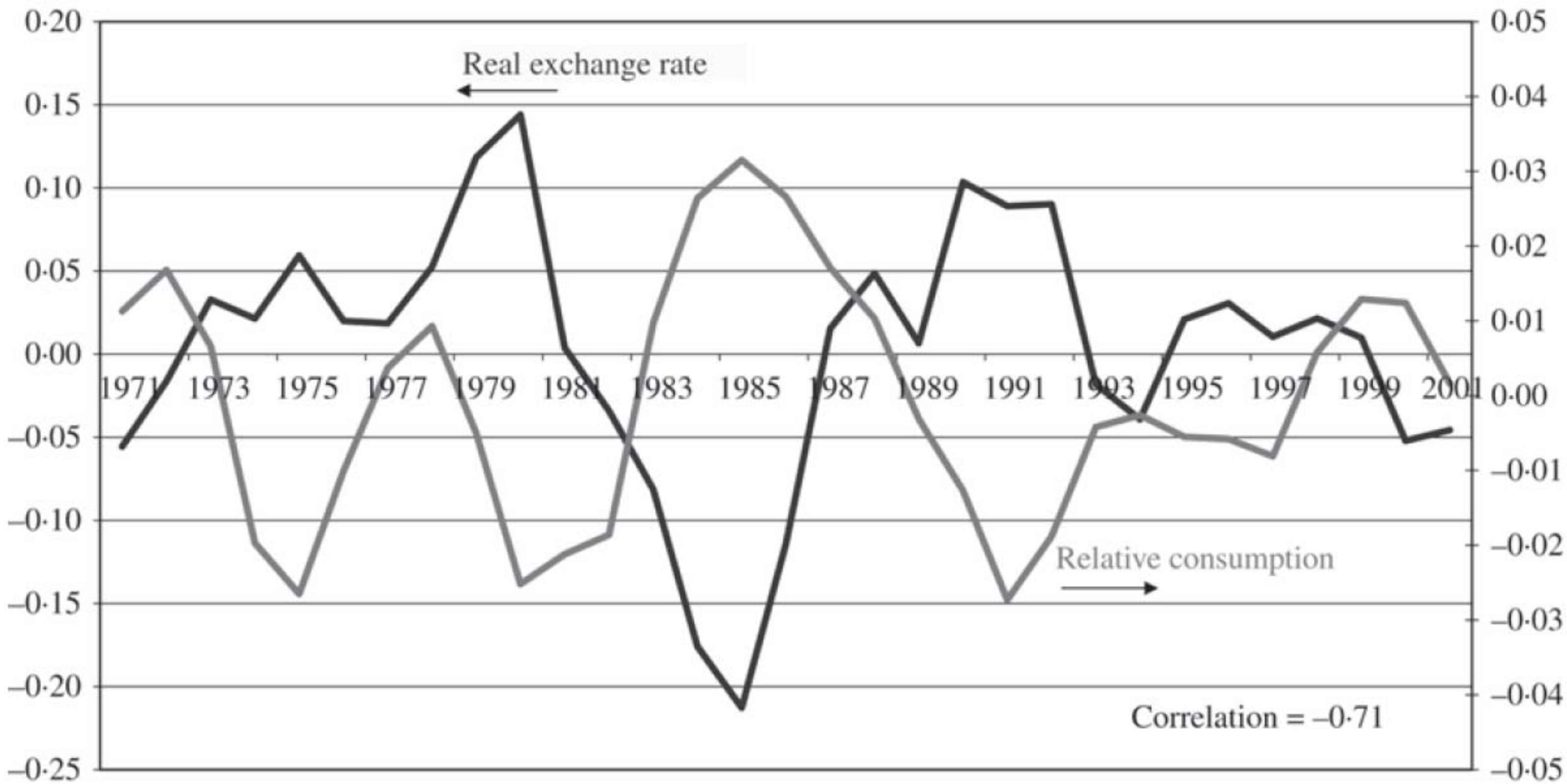
So the real exchange rate, q_t, is a direct function of the (inverse) terms of trade used in the model above, p_t.

Note: if there is no home bias (ϖ =1/2), so that preferences identical across countries, then q is constant at unity.

Correlations between real exchange rates and relative consumptions†

	Correlation			
	HP-filtered		First-difference	
Country	U.S.	ROW	U.S.	ROW
Austria	−0·11	0·05	−0·11	−0·01
Belgium/Luxembourg	−0·16	0·50	−0·10	0·43
Canada	−0·52	−0·31	−0·33	−0·18
Denmark	−0·14	−0·10	−0·20	−0·14
Finland	−0·30	−0·49	−0·38	−0·46
France	−0·20	0·43	−0·20	0·02
Germany	−0·51	−0·27	−0·37	−0·06
Greece	−0·45	−0·35	−0·23	0·03
Ireland	−0·39	0·72	0·03	0·56
Italy	−0·28	−0·52	−0·21	−0·27
Japan	0·05	0·25	0·00	0·14
Netherlands	−0·45	−0·20	−0·26	−0·13
Portugal	−0·61	−0·77	−0·46	−0·57
Spain	−0·63	−0·64	−0·40	−0·31
Sweden	−0·56	−0·40	−0·32	−0·27
U.K.	−0·51	−0·21	−0·39	−0·12
U.S.	N/A	−0·71	N/A	−0·54
Median	−0·42	−0·27	−0·26	−0·13

Annual data 1970-2001, from OECD.



The real exchange rate and relative consumption are constructed using trade weights as described in the data appendix. Both series are logged and HP-filtered.

FIGURE 1

U.S. real exchange rate and relative consumption

- This empirical finding is contrary to economic theory, which predicts that with full risk sharing relative consumption is perfectly positively correlated with the real exchange rate.

- Intuition: Countries with relative low prices should receive a transfer to take advantage of cheap consumption.

- This is true either for a central planner problem studied above or complete asset market. Consider again a social planner problem from above:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[\mu U\left(C_{ht},(1-L_{ht})\right) + (1-\mu) U\left(C_{ft},(1-L_{ft})\right)\right]$$

where the budget constraint is written in terms of the aggregate consumption bundle, C and price index, P:

Budget constraint:

$$Y_{ht} + p_t Y_{ft} = P_t C_t + P_t^* C_t^* + ...$$

First order conditions:

$$\mu U_{ct}^{'} = \lambda_t P_t \qquad \text{and} \qquad (1-\mu) U_{ct}^{*'} = \lambda_t P_t^*$$

So:

$$\left(\frac{1-\mu}{\mu}\right)\frac{U_{ct}^{*'}}{U_{ct}^{'}} = \frac{P_t^*}{P_t}$$

The international ratio of marginal utilities of consumption is directly tied to the real exchange rate.

In particular, assuming the marginal utility is a negative function of consumption, a rise in real exchange rate (P^*/P) requires a rise in relative home consumption (C/C^*).

Solution may lie in incompleteness of asset markets (Corsetti, Dedola, Leduc, RES 2008)

This paper shows that incomplete asset markets and low trade elasticities can provide an explanation for BS puzzle.

Model Assumptions:

- Two countries
- Endowment
- Each country endowed with one good; consumers consume both national goods.
- CES preferences specifying home bias and elasticity of substitution

Definitions:

- Consumption index

$$C = C_T = \left[a_H^{1-\rho} C_H^{\rho} + a_F^{1-\rho} C_F^{\rho}\right]^{1/\rho}, \qquad \rho < 1,$$

Where a_H governs home bias
and $\omega = (1-\rho)^{-1}$ is elasticity of sub between H and F goods.

- Define P_H as the price of home good and P_F foreign,
and define terms of trade: $\tau = \frac{P_F}{P_H}$

- Price index is:

$$P = P_T = \left[a_H P_H^{\rho/(\rho-1)} + (1-a_H) P_F^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho}$$

- and demands: $C_H = a_H \left(\frac{P_H}{P}\right)^{-\omega} C$

Logic of the result:

- Resource constraint under autarky: $PC/P_H = Y_H$.

  Rewrite demand: $$C_H = \frac{a_H}{a_H + (1-a_H)\tau^{1-\omega}} Y_H$$

  Take derivative with respect to terms of trade, decompose into substitution effect (SE) and income effect (IE).

$$\frac{\partial C_H}{\partial \tau} = \underbrace{\omega \frac{a_H(1-a_H)\tau^{-\omega}}{[a_H + (1-a_H)\tau^{1-\omega}]^2} Y_H}_{\text{SE}} \quad \underbrace{- \frac{a_H(1-a_H)\tau^{-\omega}}{[a_H + (1-a_H)\tau^{1-\omega}]^2} Y_H}_{\text{IE}} \quad > 0 \iff \omega > 1$$

- IE negative: worsening terms of trade makes home country poorer, lowering home demand for home good.
- SE positive: home good cheaper raises demand for it.
- IE can dominate if elasticity (ω) low.

- So a rise in τ can lower home consumption of home good.
- But will always raise foreign consumption of home good.
- So the sign of the correlation of terms of trade τ with relative cons. can switch depending on the elasticity ω.
- For a low ω, if there is a rise in home endowment, τ must fall (rise in price of home good) in order to raise home and hence world demand for the home good enough to accommodate the raise in supply (provided home bias).
- This lowers the foreign consumption.
- So get a negative correlation between τ and cons. ratio.

More formally, manipulate balanced trade condition to get:

$$\tau C_{\mathrm{F}} = C^*_{\mathrm{H}} \Longleftrightarrow \frac{C}{C^*} = \frac{a^*_{\mathrm{H}}}{1-a_{\mathrm{H}}}\tau^{\omega-1}\left[\frac{a^*_{\mathrm{H}}+(1-a^*_{\mathrm{H}})\tau^{1-\omega}}{a_{\mathrm{H}}+(1-a_{\mathrm{H}})\tau^{1-\omega}}\right]^{\omega/(1-\omega)}$$

Use this to solve log-linearized relationship:

$$\widehat{\mathrm{RER}} = \frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}\omega-1}(\widehat{C}-\widehat{C^*})$$

<u>Conclude:</u>

Can get negative correlation if $\omega < \frac{1}{2a_{\mathrm{H}}} < 1$
(if trade elasticity low and have home bias (a_H>1/2)).

Also shows that autarky is not automatically immune to the BS puzzle: get positive correlation if elasticity too high: ie. if ω=1 , RER = *(C=C*)*

Complementarity in two-good model can help with International Production correlation

Consider a model:

- two countries, two national goods.
- production of goods uses capital and labor
- CES utility, where national goods can be substitutes or complements.

Production: $Y_t = A_t K_{t-1}^{\theta} L_t^{1-\theta}$.

Goods market structure: $D = \{C, I, G\}$

$$D_t = \left(\nu^{\frac{1}{\phi}} \left(D_{H,t}\right)^{\frac{\phi-1}{\phi}} + \left(1-\nu\right)^{\frac{1}{\phi}} \left(D_{F,t}\right)^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}.$$

The corresponding price index is:

$$P_t = \left(\nu \left(P_{H,t} \right)^{1-\phi} + (1-\nu)\left(P_{F,t} \right)^{1-\phi} \right)^{\frac{1}{1-\phi}}$$

and demands

$$D_{H,t} / D_t = \nu \left(P_{H,t} / P_t \right)^{-\phi}$$

$$D_{F,t} / D_t = (1-\nu)\left(P_{F,t} / P_t \right)^{-\phi}.$$

two sets parameter values:

substitutes parameterization:

elasticity of intratemporal substitution $\phi = 1.5$

intertemporal elasticity $\sigma = 0.5$

complements parameteriziation:

elasticity of intratemporal substitution $\phi = 0.5$

intertemporal elasticity $\sigma = 1.5$

- Importantly, the case of complementarity between home and foreign goods indicates that agents are more willing to substitute across time than across goods within a period.

Fig. 6. Impulse response to a positive home productivity shock in benchmark model

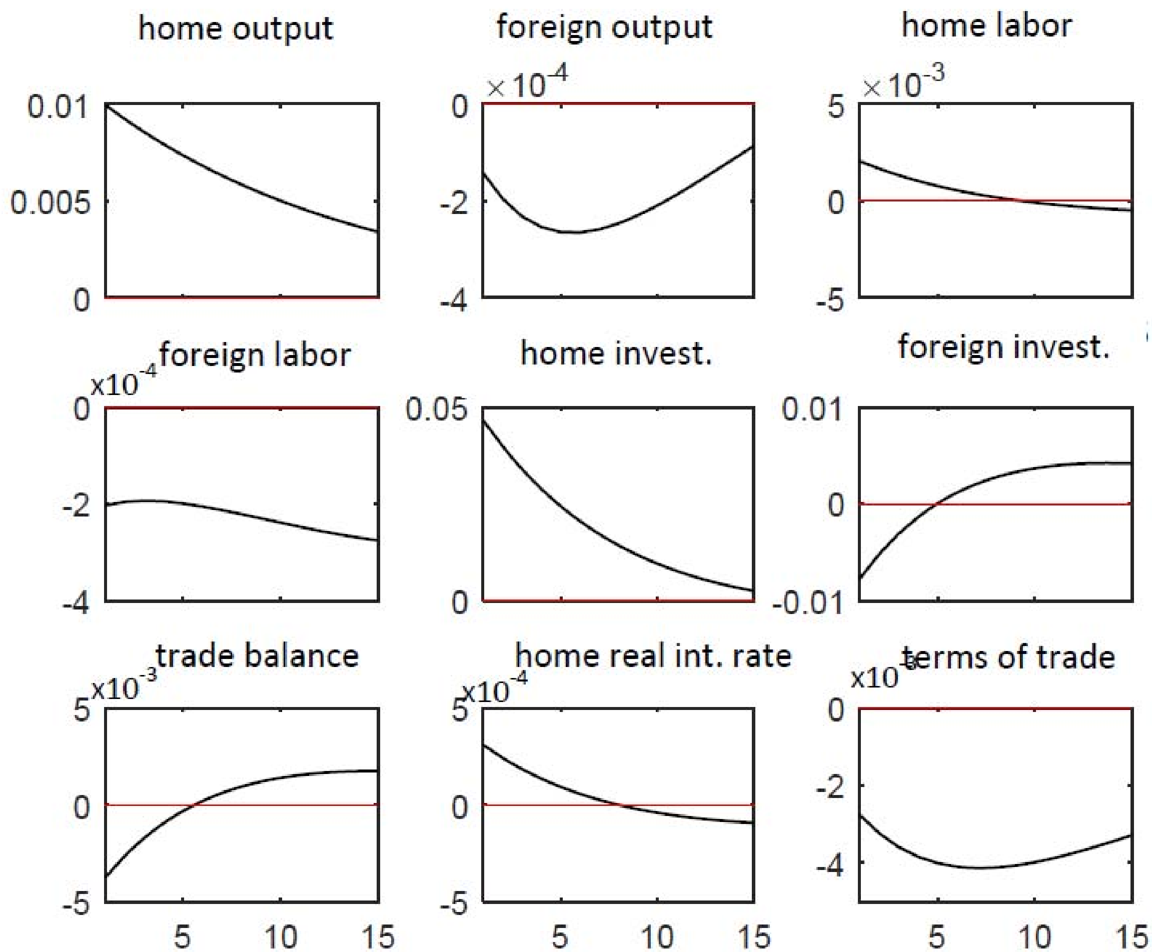
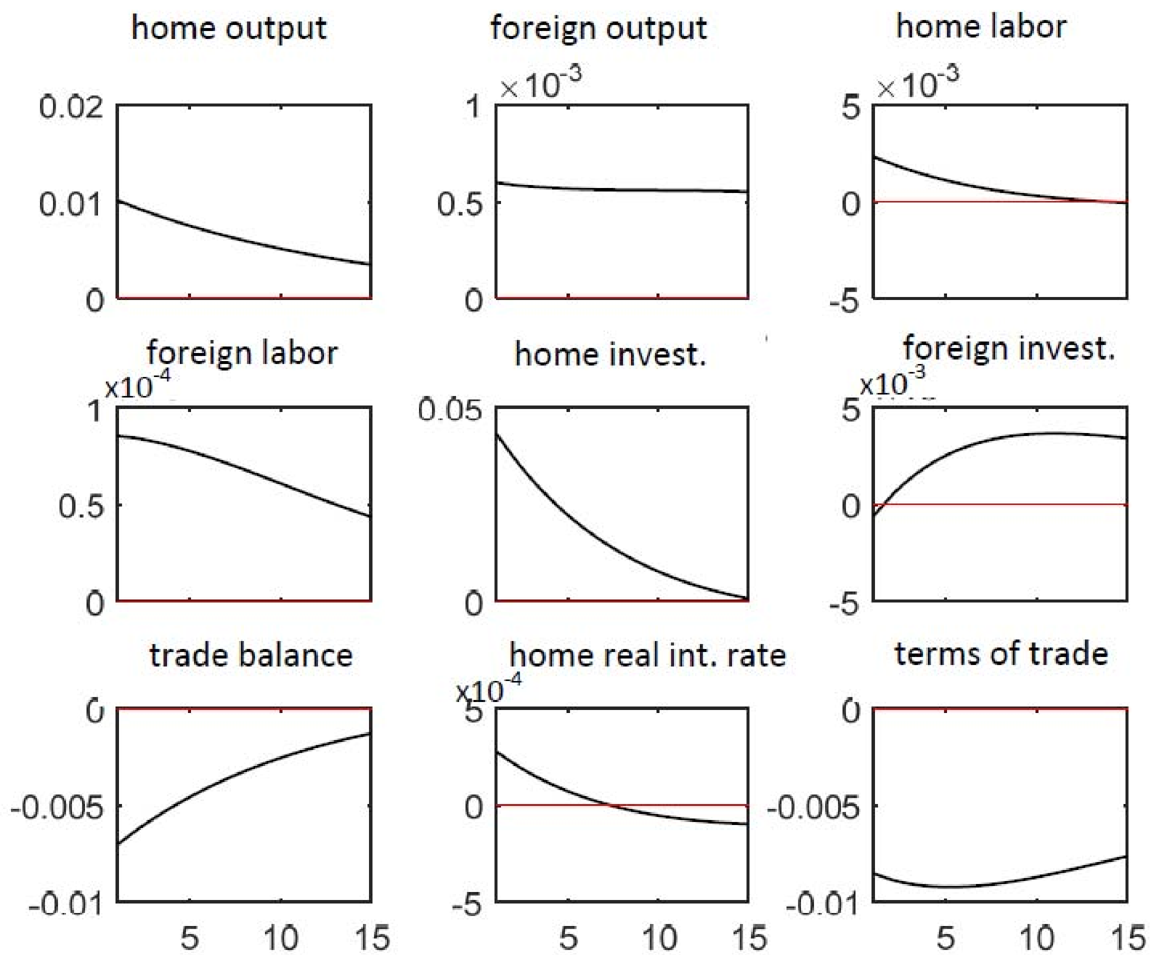


Fig. 7. Impulse response to a positive home productivity shock in model with alternative parameter values



	(1) productivity shock, benchmark parameters	(2) productivity shock, alternate parameters
<u>standard deviations:</u>		
output	0.0130	0.0132
employment	0.0028	0.0032
consumption	0.0027	0.0046
investment	0.0605	0.0557
net exports	0.0075	0.0129
Real interest rate	0.0004	0.0004
terms of trade	0.0070	0.0171
<u>correlations between home and foreign variables:</u>		
output	-0.0440	0.1107
employment	-0.1096	0.6181
consumption	0.4306	-0.0799
investment	-0.3139	-0.0282

- Logic for positive output comovement: As shock raises home output, home goods are more useful in combination with foreign goods, so foreign production must also rise.

- Also implies negative international consumption correlation: if the home country needs foreign as well as home goods for its rise in investment expenditure, it imports more foreign goods, driving down foreign consumption.

Part 4: Internatoinal Business Cycles and Trade Flows:

A. Engel and Wang (2011)

Some Questions:

1) What is main question addressed. What new stylized fact?

2) What would standard RBC model of BKK predict?

3) why can't explain even if add higher real exch. rate volatility?

4) What do they add to model, and how model it?

5) What is the main result?

6) What is the intuition for the result?

7) critiques/comments? Counterfactual implications, questionable calibrations, alternative explanations

8) Interesting implications or extensions come to mind?

New Stylized facts:

1) trade volatility: Standard deviation of real imports and exports are about 2-3 times that of GDP

2) Positive comovement: real imports and exports are procyclical and positively corelated with each other.

3) still true that net exports are countercyclical

Data: 25 OECD countries, quarterly, 1973Q1 – 2006Q3.

- Standard deviation relative to GDP, mean among countries:

Imports	3.25
Exports	2.73
net exports	0.78

- Correlations with GDP, mean among countries:

Imports	0.63
Exports	0.39
Net exports	-0.24

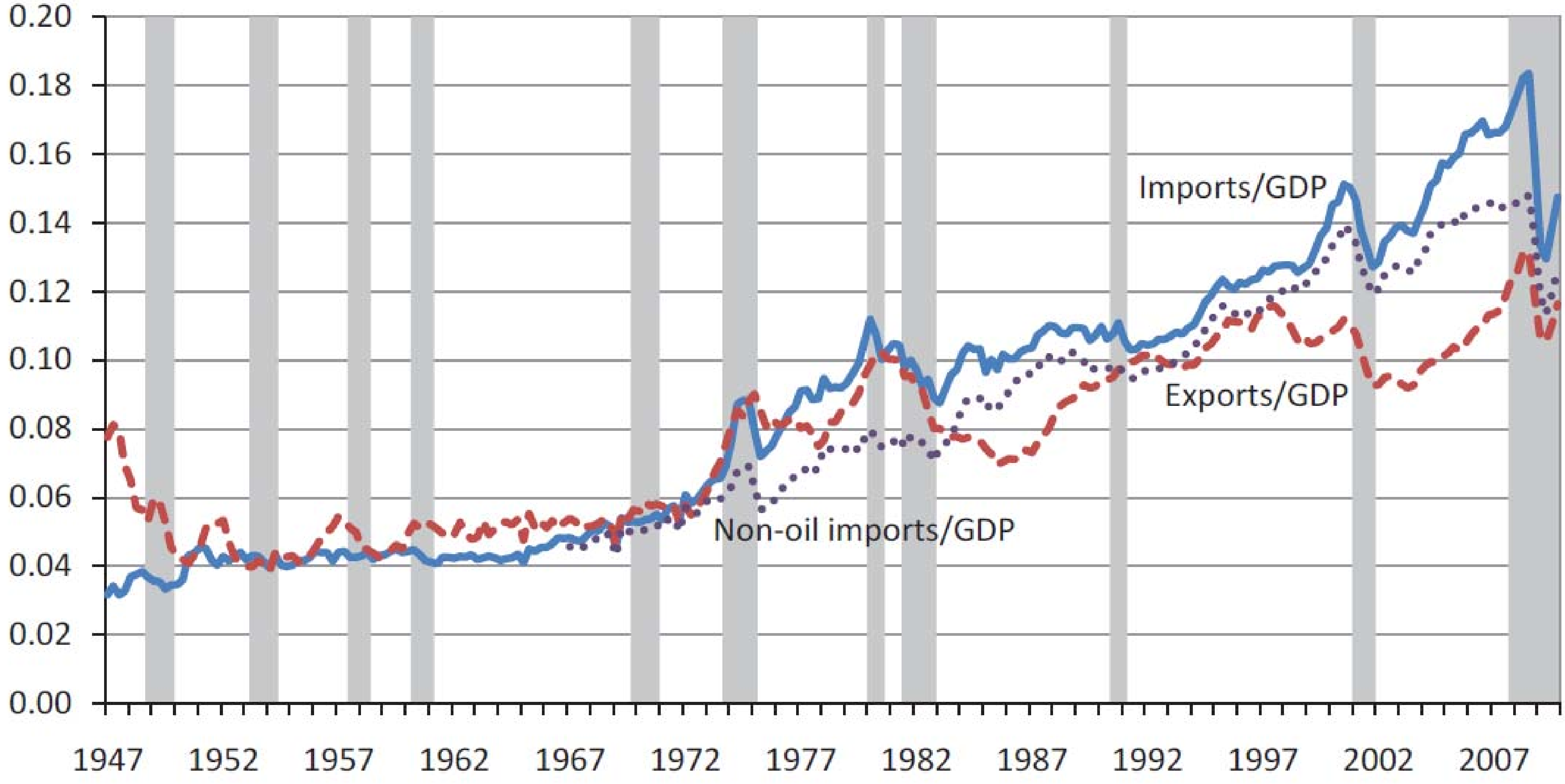
- Corr(Imports, Exports): 0.38

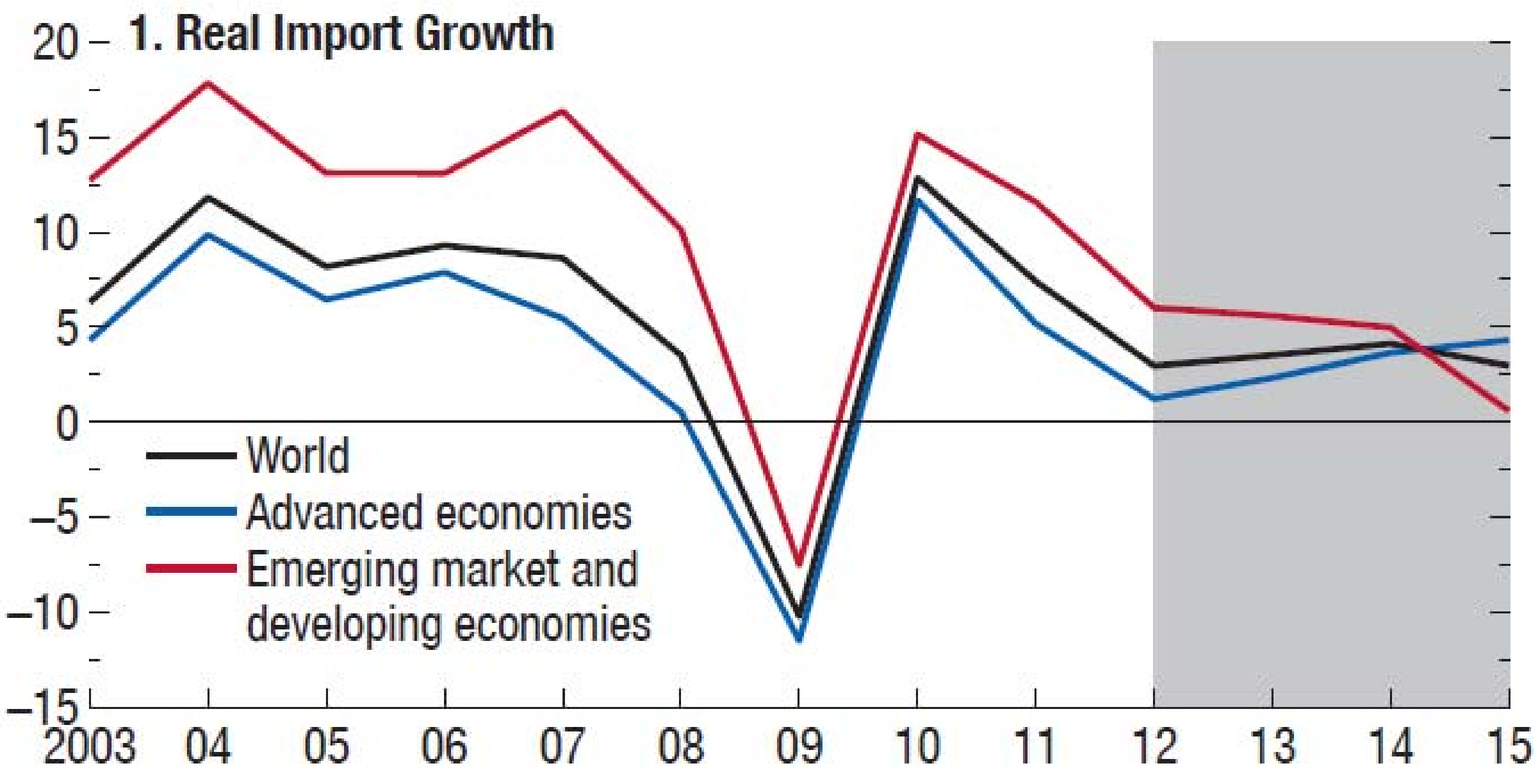
- Share of Durable Goods in trade, mean among countreis.

  Imports: 0.68
  Exports: 0.64

Especially relevant, given trade collapse observed during the Great recession 2007-9:

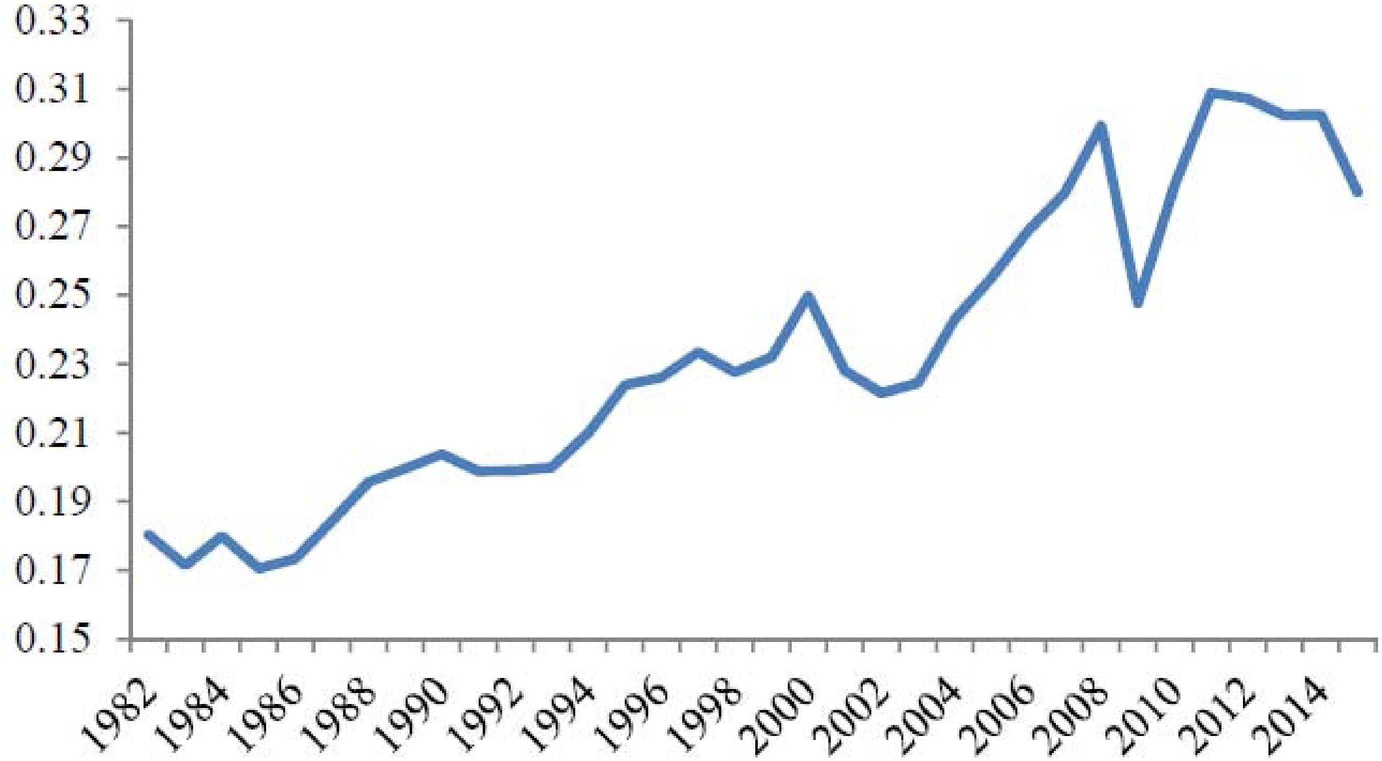
Figure 1. Historical Trends in Aggregate Trade, 1947-2009.





source: IMF WEO

Fig. 2. US goods trade volume (exports plus imports) as a share of US GDP



Source: Comtrade

Table 3: Share of Durable Goods in Trade

	Exclude Energy Products		Exclude Materials and Energy	
Country	Import	Export	Import	Export
Australia	0.70	0.56	0.71	0.45
Austria	0.69	0.67	0.70	0.69
Belgium	0.66	0.66	0.67	0.66
Canada	0.77	0.64	0.77	0.69
Czech Rep	0.72	0.75	0.73	0.77
Denmark	0.60	0.47	0.61	0.48
Finland	0.72	0.61	0.73	0.65
France	0.67	0.68	0.68	0.68
Germany	0.69	0.71	0.70	0.71
Hungary	0.74	0.77	0.75	0.78
Iceland	0.55	0.28	0.56	0.28
Ireland	0.73	0.59	0.73	0.59
Italy	0.65	0.64	0.66	0.64
Japan	0.57	0.89	0.58	0.89
Korea	0.76	0.78	0.77	0.78
Mexico	0.73	0.78	0.74	0.78
Netherland	0.68	0.60	0.69	0.61
New Zealand	0.66	0.26	0.66	0.26
Norway	0.70	0.59	0.71	0.61
Portugal	0.65	0.53	0.66	0.54
Spain	0.68	0.65	0.69	0.66
Sweden	0.68	0.73	0.69	0.76
Switzerland	0.65	0.69	0.66	0.69
UK	0.69	0.74	0.70	0.74
US	0.69	0.75	0.70	0.77
Mean	0.68	0.64	0.69	0.65
Median	0.69	0.66	0.70	0.68

Table 2: Dividing SITC Categories into Different Sectors

SITC	Description	Sector
0	Food and live Animals	Nondurable
1	Beverages and tobacco	Nondurable
2	Crude Materials, inedible, except fuels	Raw materials
3	Mineral fuels, lubricants and related materials	Energy products
4	Animal and vegetable oils, fats and waxes	Nondurable
5	Chemicals and related products, N.E.S.	Nondurable
6	Manufactured goods classified chiefly by material	
61	Leather, leather manufactures, n.e.s., and dressed furskins	Durable
62	Rubber manufactures, n.e.s.	Durable
63	Cork and wood manufactures (excluding furniture)	Nondurable
64	Paper, paperboard and articles of paper pulp, of paper or of paperboard	Nondurable
65	Textile yarn, fabrics, made-up articles, n.e.s., and related products	Nondurable
66	Non-metallic mineral manufactures, n.e.s.	Durable
67	Iron and steel	Durable
68	Non-ferrous metals	Durable
69	Manufactures of metals, n.e.s.	Durable
7	Machinery and transport equipment	Durable
8	Miscellaneous manufactured articles	
81	Prefabricated buildings; sanitary, plumbing, heating and lighting fixtures and fittings, n.e.s.	Durable
82	Furniture, and parts thereof; bedding, mattresses, mattress supports, cushions and similar stuffed furnishings	Durable
83	Travel goods, handbags and similar containers	Nondurable
84	Articles of apparel and clothing accessories	Nondurable
85	Footwear	Nondurable
87	Professional, scientific and controlling instruments and apparatus, n.e.s.	Durable
88	Photographic apparatus, equipment and supplies and optical goods, n.e.s.; watches and clocks	Durable
89	Miscellaneous manufactured articles, n.e.s.	Nondurable
9	Commodities and transactions not classified elsewhere in the SITC	
91	Postal packages not classified according to kind	Nondurable
93	Special transactions and commodities not classified according to kind	Nondurable
95	Coin, including gold coin; proof and presentation sets and current coin	Durable
96	Coin (other than gold coin), not being legal tender	Durable
97	Gold, non-monetary (excluding gold ores and concentrates)	Durable

Model features:

- Two symmetric countries, H and F.
- Two production sectors, nondurable and durable.
- Nondurable only used for domestic consumption.
- Durables traded and used for consumption and investment.
- Durable means utility depends positively on stock of goods accumulated from past expenditures. (opposite of habits)
- Quadratic adjustment/installation costs for capital and durables to moderate volatility that enter budget constraints.
- Iceberg trade costs (to generate endogenous home bias): a fraction of goods disappears in international shipment.
- Note: define real exchange rate in terms of nominal prices (but money not enter model)

Model Equations:

Production: $Y^j_{Ht} = A^j_{Ht}(K^j_{Ht})^{\chi}(L^j_{Ht})^{1-\chi}$

Aggregation for capital: $K^j_{Ht} = \left(\alpha^{\frac{1}{\gamma}}(K^{jH}_{Ht})^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}}(K^{jF}_{Ht})^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$

Household utility:

$$u_t = \frac{\left[\left(\mu^{\frac{1}{\zeta}} D_{Ht}^{\frac{\zeta-1}{\zeta}} + (1-\mu)^{\frac{1}{\zeta}} C_{Ht}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} - \rho L^{\nu}_{Ht}\right]^{1-\sigma}}{1-\sigma}$$

Aggregation for consumption: $D_{Ht} = \left[\psi^{\frac{1}{\theta}}(D^H_{Ht})^{\frac{\theta-1}{\theta}} + (1-\psi)^{\frac{1}{\theta}}(D^F_{Ht})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$

Law of motion for durables: $D^k_{Ht+1} = (1-\delta_D)D^k_{Ht} + d^k_{Ht}$

Adjustment cost for durables: $\Delta_{Ht}^{k} = \frac{1}{2}\phi_1 \left(d_{Ht}^{k} - \delta_D D_{Ht}^{k}\right)^2 / D_{Ht}$

And for capital: $K_{Ht+1}^{jk} = (1-\delta)K_{Ht}^{jk} + I_{Ht}^{jk}$ $\quad \Lambda_{Ht}^{jk} = \frac{1}{2}\phi_2 \left(I_{Ht}^{jk} - \delta K_{Ht}^{jk}\right)^2 / K_{Ht}^{j}$

Iceberg cost (tau) enters market clearing condition for home durable good:

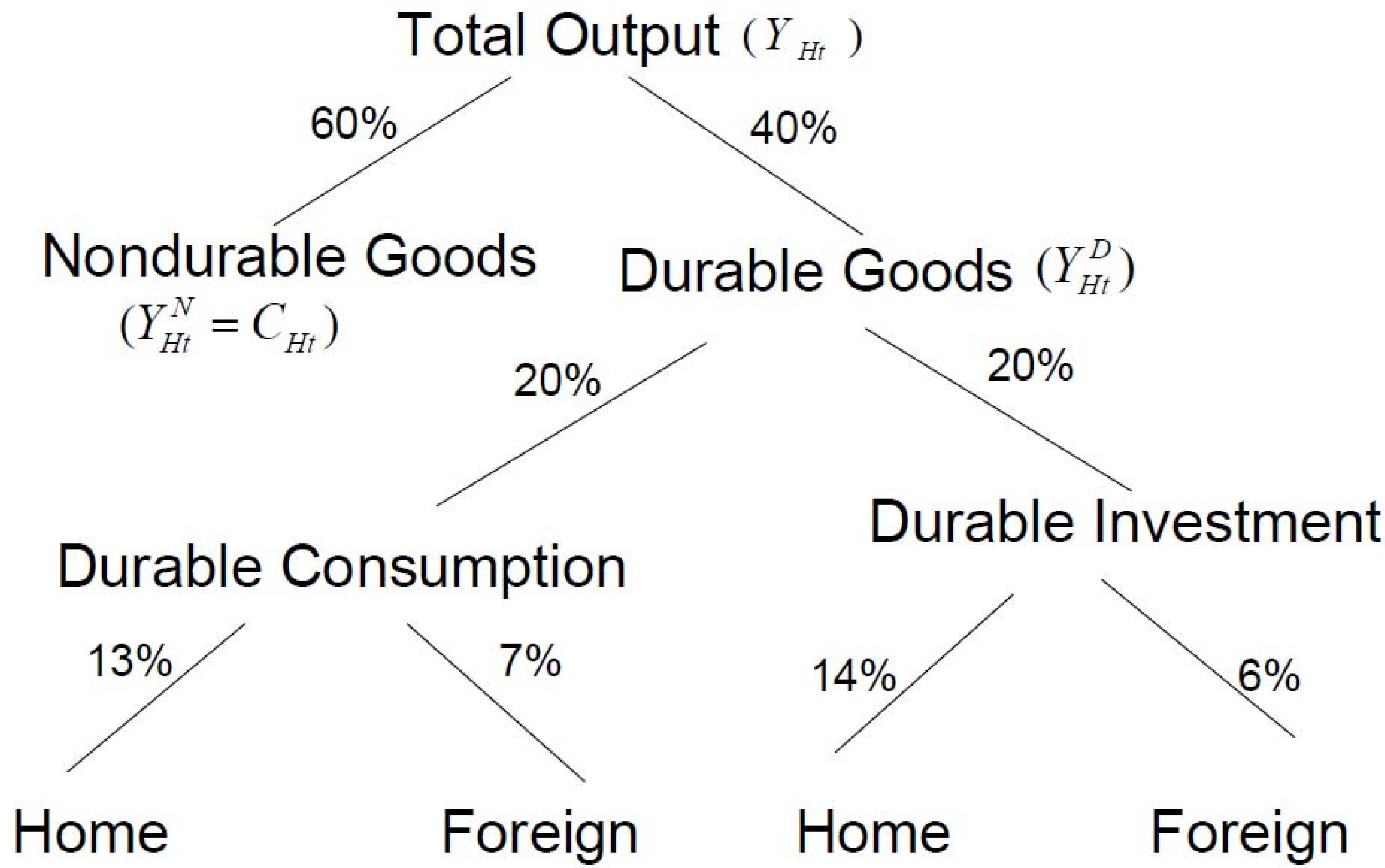
$$\begin{aligned} Y_{Ht}^{D} =& d_{Ht}^{H} + \Delta_{Ht}^{H} + I_{Ht}^{NH} + \Lambda_{Ht}^{NH} + I_{Ht}^{DH} + \Lambda_{Ht}^{DH} + \frac{1}{2}\Phi B_{Ht+1}^{2} \\ &+ \frac{d_{Ft}^{H} + \Delta_{Ft}^{H} + I_{Ft}^{NH} + \Lambda_{Ft}^{NH} + I_{Ft}^{DH} + \Lambda_{Ft}^{DH} + \frac{1}{2}\Phi B_{Ft+1}^{2}}{1-\tau} \end{aligned}$$

Calibration:

Table 4: Calibration

Parameter	Value	Description
α	0.5	Share of Home Goods in Capital When Trade Cost Is Zero
χ	0.36	Capital Share in Production
γ	9.1	(Long-run) Elasticity of Substitution between Home and Foreign Capital
τ	0.1	(Iceberg) International Trade Cost
β	0.99	Subjective Discount Factor
δ	0.013	Depreciation Rate of Capital
δ_D	0.05	Depreciation Rate of Durable Consumption
μ	0.23	Share of Durable Consumption Stock in Consumption Bundle
ν	1.65	Preference Parameter of Labor Supply
ψ	0.5	Share of Home Goods in Durable Consumption When Trade Cost Is Zero
ρ	5.83	Preference Parameter
σ	2	Preference Parameter
θ	6.85	(Long-run) Elasticity of Substitution b/t Home and Foreign Durable Consumption
ζ	1.1	Elasticity of Substitution b/t Durable and Nondurable Consumption
ϕ_1	$1.4^{\dagger}$	Durable Consumption Adjustment Cost
ϕ_2	$8.5^{\dagger}$	Capital Adjustment Cost
Φ	0.00001	Bond Holding Cost
Ξ_1	0.87	AR(1) Coefficient of Technology Shock in Nondurable Good Sector
Ξ_2	0.9	AR(1) Coefficient of Technology Shock in Durable Good Sector
$\sigma(\varepsilon_{Ht}^N)$	0.0096	Standard Deviation of Productivity Shock in Nondurable Good Sector
$\sigma(\varepsilon_{Ht}^D)$	0.036	Standard Deviation of Productivity Shock in Durable Good Sector

Steady state calibration:



Results:

Consider standard RBC model (BKK 1992 model, but with bonds only asset market) (also considers a monetary model which we will study later)

- Volatility of exports and imports too low, lower than GDP.
- But does replicate positive comovement of exports and imports

Model with highly volatile real exchange rate:

- While it induces greater trade volatility, imports and exports move in opposite directions.
- We will discus monetary models and the trick for getting real exchange rate volatility later on(Chari et al paper)

Benchmark model results:

Succeeds in replicating facts:

- Procyclical imports and exports, while net exports still countercyclical.
- Volatility: exports and imports 3 times as volatile as GDP
- Shortcoming: like other RBC models, underpredicts exchange rate volatility:

Intuition:

- Story: positive technology shock raises wealth and demand.
- Procyclical imports: Usual macro story: rise in investment and durables raises imports.
- Procyclical exports: rise in home output depreciates terms of trade, leading to rise in foreign demand for home goods.
- Volatility: While standard RBC model has this for investment, this model extends the story to consumption via durable consumption good.
- This amplifies effect, generating larger volatility in exports and imports.

Table 5: Performance of Standard Models

Panel A: Standard Deviations Relative to That of Real GDP

	Consumption	Investment	Real Import	Real Export	$\frac{RealNetExport}{RealGDP}$	Real ER[♯]
Data[†]	0.798	2.890	3.335	2.626	0.250	2.432
HP[‡]	0.462	2.663	0.727	0.608	0.087	0.385
DSGE[‡]	0.545	2.830	0.826	0.835	0.077	0.375
GHH[‡]	0.613	2.697	0.935	0.947	0.173	0.284
Lo-elast.[‡]	0.401	2.767	1.651	1.625	0.467	1.216
UIP[‡]	0.925	2.875	3.477	3.466	1.016	1.458

Panel B: Correlation with Real GDP

	Real Import	Real Export	$\frac{RealNetExport}{RealGDP}$	$corr(RIM_t, REX_t)^{\sharp}$
Data[†]	0.827	0.415	-0.467	0.194
HP[‡]	0.929	0.588	-0.551	0.628
DSGE[‡]	0.801	0.663	-0.214	0.809
GHH[‡]	0.894	0.278	-0.497	0.252
Lo-elast.[‡]	-0.647	0.973	0.852	-0.799
UIP[‡]	0.286	0.069	-0.112	-0.894

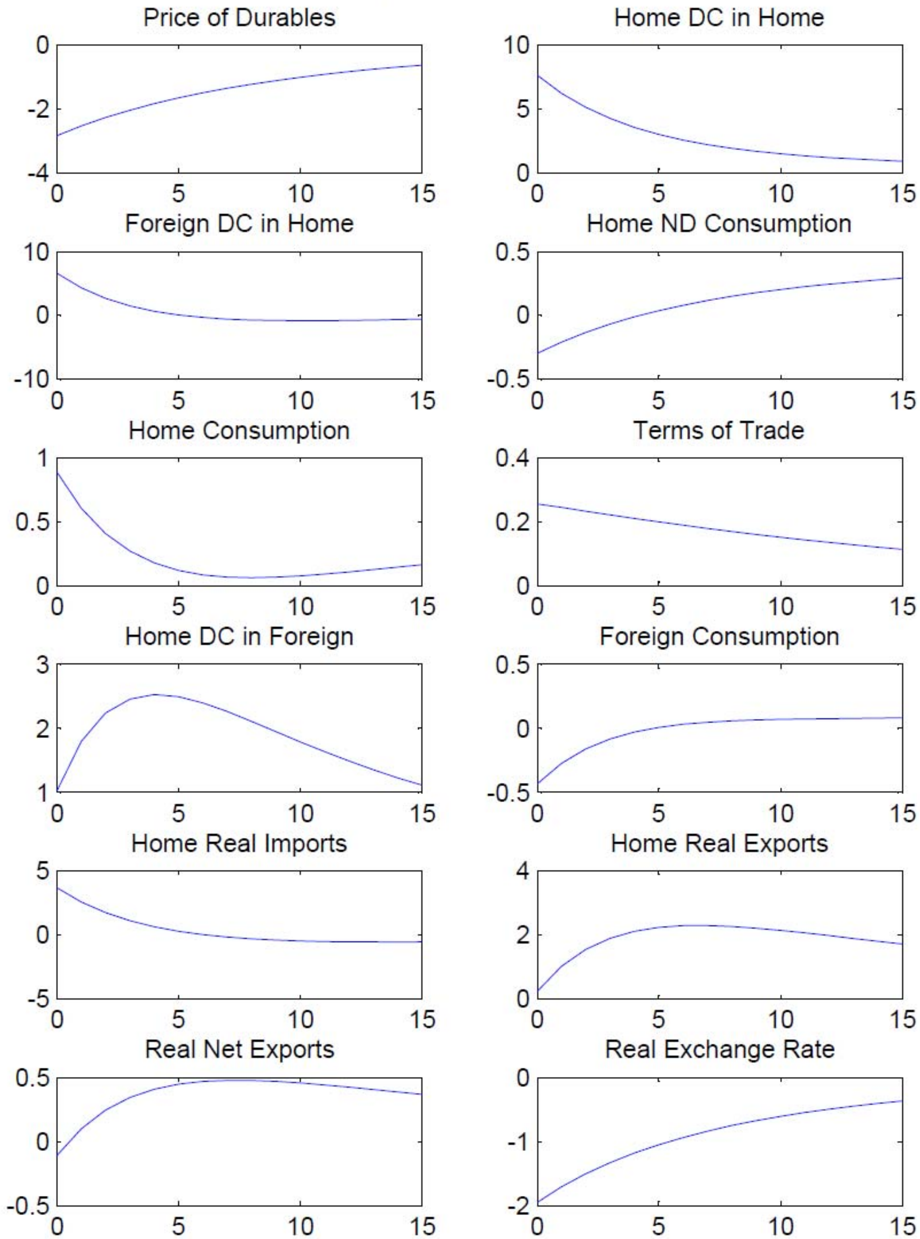
Panel C: Correlation with Real GDP

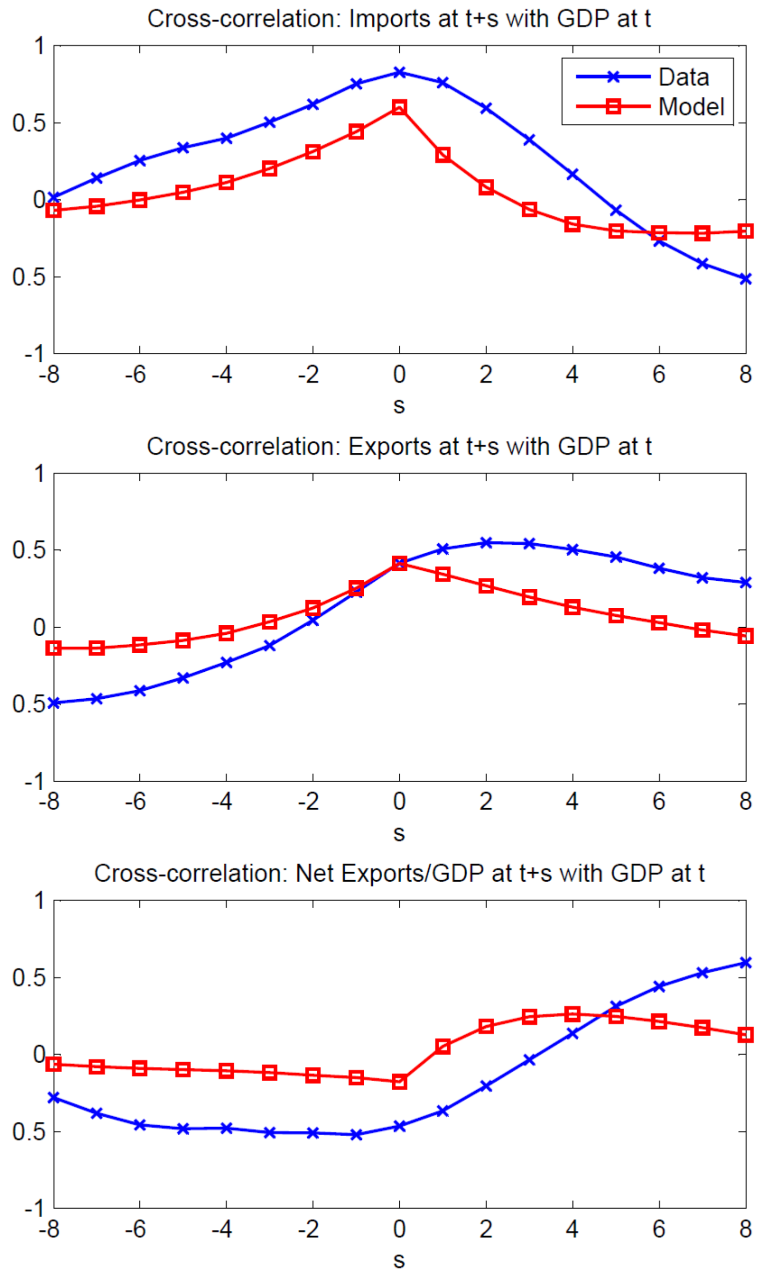
	Real Import	Real Export	$\frac{RealNetExport}{RealGDP}$	$corr(RIM_t, REX_t)^{\sharp}$
HP[‡]	0.999	0.500	-0.819	0.491
DSGE[‡]	0.988	0.601	-0.552	0.634
GHH[‡]	0.985	0.241	-0.608	0.152
Lo-elast.[‡]	0.369	0.984	0.848	0.212
UIP[‡]	0.569	0.070	-0.181	-0.749

Table 6: Simulation Results of Benchmark Model

	Panel A: Standard Deviations Relative to That of Real GDP							
	C	I	DC	L	RIM	REX	RNX	Q
Data†	0.798	2.890	2.983	0.670	3.335	2.626	0.250	2.432
Benchmark‡	0.878	2.594	2.473	0.547	2.633	2.678	0.337	1.262
High Spillover	0.948	2.905	2.738	0.539	1.826	1.775	0.322	1.297
Medium Spillover	0.917	2.894	2.754	0.549	2.652	2.615	0.393	1.271
High Correlation	0.920	2.750	2.680	0.549	2.880	2.936	0.402	1.058
High Correlation 2	0.874	2.666	2.381	0.544	2.558	2.596	0.266	0.435
No Correlation	0.902	2.757	2.658	0.549	2.619	2.678	0.596	1.470
High Persistence	0.922	2.840	2.473	0.539	2.423	2.411	0.580	1.282
Technology Costs	0.961	2.828	2.551	0.535	2.726	2.779	0.355	1.041
Traded Nondurable	0.748	2.950	2.892	0.571	2.048	2.082	0.302	1.113
Low Durable Share	0.748	2.612	2.628	0.569	0.960	0.933	0.240	0.803
	Correlation with GDP							
	RIM	REX	RNX	$corr(RIM, REX)$	Elasticity§	σ_{Y,Y^*}	σ_{C,C^*}	
Data†	0.827	0.415	-0.467	0.194	0.90 (0.12)	0.68	0.60	
Benchmark‡	0.606	0.411	-0.187	0.421	1.05 (0.20)	0.01	−0.17	
High Spillover	0.576	0.405	-0.129	0.160	0.69 (0.13)	−0.03	0.23	
Medium Spillover	0.599	0.324	-0.228	0.171	0.89 (0.19)	−0.01	−0.14	
High Correlation	0.630	0.337	-0.288	0.265	1.19 (0.19)	0.03	−0.20	
High Correlation 2	0.801	0.554	-0.177	0.577	1.89 (0.27)	0.56	0.39	
No Correlation	0.564	0.375	-0.135	0.215	1.07 (0.26)	−0.02	−0.23	
High Persistence	0.618	0.333	-0.180	0.097	0.95 (0.19)	0.16	0.03	
Technology Costs	0.560	0.232	-0.292	0.386	1.41 (0.13)	0.08	−0.09	
Traded Nondurable	0.714	0.388	-0.331	0.550	0.69 (0.11)	0.002	−0.08	
Low Durable Share	0.828	0.220	-0.374	0.228	0.70 (0.13)	−0.04	−0.08	

Figure 2: Impulses Response Functions





Cross-correlation: Imports at t+s with GDP at t
Data
Model
1
0.5
0
0.5
-1
-8
-6
-4
-2
0
2
4
6
8
s
Cross-correlation: Exports at t+s with GDP at t
1
0.5
0
0.5
-1
-8
-6
-4
-2
0
2
4
6
8
s
Cross-correlation: Net Exports/GDP at t+s with GDP at t
1
0.5
0
0.5
-1
-8
-6
-4
-2
0
2
4
6
8
s

Implications for Backus-Smith puzzle:

- In the risk sharing condition, marginal utility is of consumption services, including from previously purchased durables.

- But data measure consumption expenditure, as expenditure on new durable goods and nondurables.

- Correlation in model between real exch. rate and total consumption expenditure as measured in data is negative, as in data.

- But correlation with ratio of just nondurable consumption is positive, indicating risk sharing as predicted by theory.

Miyamoto and Nguyen (JIE 2017)

Adds three mechanisms to explain how goods trade can generate positive international output comovement.

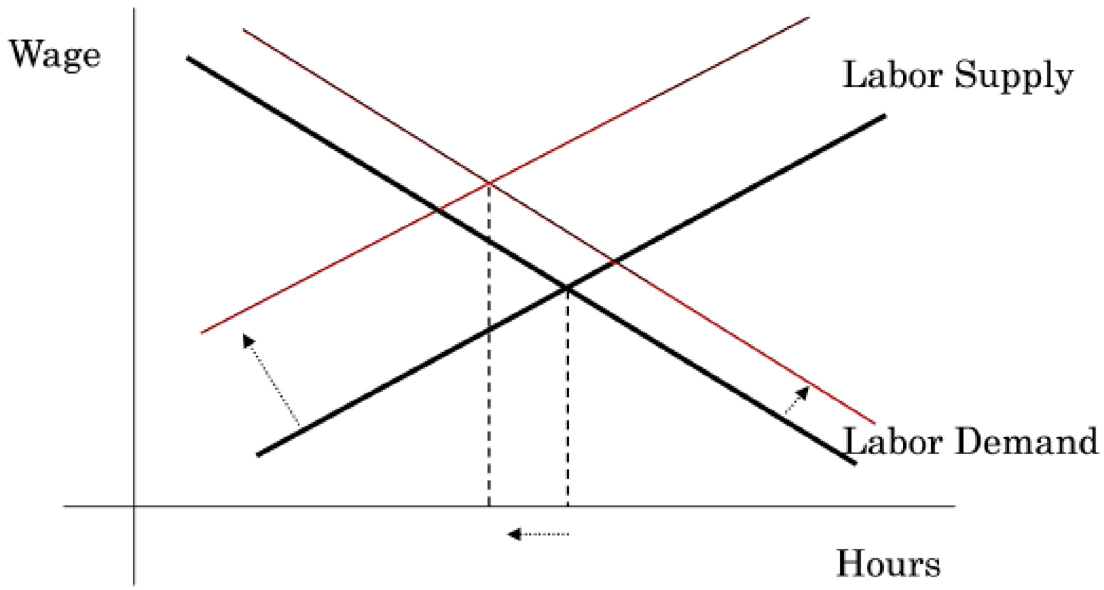
1) Labor supply without wealth effects (Jaimovich-Rebelo preferences)

2) imported intermediates inputs used for production

3) variable capital utilization

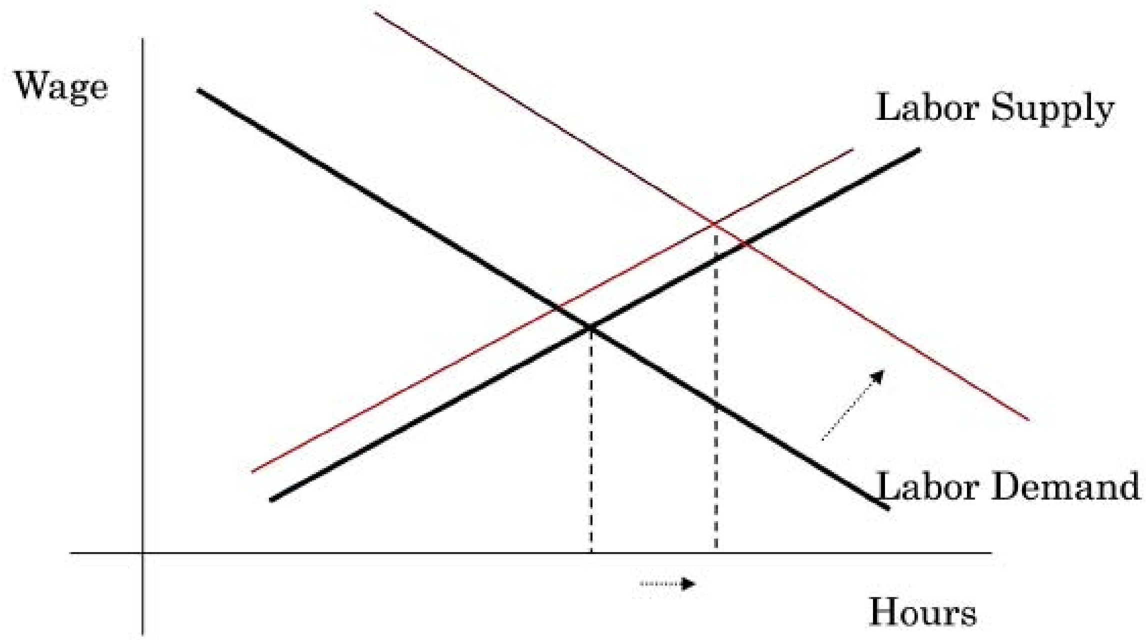
In context of a two country model where one county is large (US) and other is a small (Canada)

- Key: Canadian hours in response to US productivity shocks



- US productivity goes up, appreciation for Canada, hours go down in Canada
  - Labor supply: $MRS_t = W_t$ where W_t is real wage in consumption unit
  - Labor demand: $\frac{\partial F}{\partial H} p_t^D = W_t$ where p_t^D is domestic goods price

- With **three key features**



- Labor supply: Low wealth elasticity of labor supply
- Labor demand: Imported intermediate inputs and utilization

Role of preferences and labor supply:

Use more general preferences:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \quad \frac{\left[C_{1t} - \phi_1^H \frac{1}{1+\frac{1}{v}} X_{1t} H_{1t}^{1+\frac{1}{v}}\right]^{1-\sigma} - 1}{1-\sigma}$$

where

$$X_{1t} = (C_{1t})^{\kappa_1} X_{1t-1}^{1-\kappa_1}.$$

This preference specification is introduced by Jaimovich and Rebelo (2009). The parameter κ_1 governs the wealth elasticity of labor supply. When $\kappa_1 = 1$, the preference is the common King et al. (1988) (KPR) utility function. As $\kappa_1 \to 0$, the utility function is the Greenwood et al. (1988) (GHH) preferences. In that case, there is no wealth effect on labor supply.

FOC for labor, for case of kappa = 1:

$$H_t^{\frac{1}{\nu}} = \tfrac{1}{\phi} w_t$$

So consumption does not enter labor supply condition.

This is different from BKK case above, with preferences:

$$U_{it} = \left(C_{it}^{\mu} (1 - L_{it})^{1-\mu} \right)^{1-\frac{1}{\sigma}} \quad i = h, f$$

which implies labor supply:

$$\frac{-U'_{L_h,t}}{U'_{C_h,t}} = F'_{L_h,t} \qquad \text{or} \qquad \frac{1-\mu}{\mu} \frac{C_t}{1-L_t} = w_t$$

where a rise in consumption lowers labor supply.

Solve for hours, in log deviations form:

$$\widehat{H_{1t}} = \frac{1}{\alpha + \frac{1}{\nu}} \left[-\widehat{C}_{1t} - \left(1 - \omega_1^C\right) \widehat{TOT}_t + (1-\alpha)\widehat{Z}_{1t} + \alpha \widehat{K}_{1t} \right], \qquad (14)$$

This equation allows us to decompose the movement of hours in the domestic economy into four components: the wealth effect from the change in consumption, the terms of trade effect, the effect from the domestic technology change and the capital accumulation effect. To

So if use version of preferences where consumption not enter, can eliminate negative effect of consumption in equation above.

Role of intermediate inputs:

Idea: production of home goods requires foreign goods in a round-about production structure (M).

Effect: When the terms of trade appreciate, imported intermediate goods become relatively cheaper.

Since domestic firms are able to use cheaper imported inputs from the foreign country to produce the goods with higher prices,

the change in the terms of trade shifts the labor demand curve further.

The intermediate good producer in country 1 specializes in the production of home goods Y_t^D by combining capital service, $u_{1t}K_{1t}$, labor, H_{1t}, and imported and domestic intermediate inputs, M_{21t} and M_{11t}, respectively, using the production function:

$$Y_{1t} = \left((u_{1t}K_{1t})^{\alpha}(Z_{1t}H_{1t})^{1-\alpha}\right)^{1-\alpha_{11}-\alpha_{21}} (M(M_{11t}, M_{21t}))^{\alpha_{11}+\alpha_{21}}, \qquad (4)$$

where $\alpha_{11} > 0$ and $\alpha_{21} > 0$ are the shares of domestic and imported intermediate inputs in gross output, respectively, $\alpha(1-\alpha_{11}-\alpha_{21}) > 0$ is the capital share, and $M(M_{11t}, M_{21t})$ is the composite of home and imported intermediate good. Roundabout production is introduced to capture the role of intermediate inputs in production and cross-border trade. The functional form of $M(.)$ is given as follows:

$$M_{1t} = \left((\alpha_{11})^{\frac{1}{\gamma_1}}(M_{11t})^{\frac{\gamma_1^{-1}}{\gamma_1}} + (\alpha_{21})^{\frac{1}{\gamma_1}}(M_{21t})^{\frac{\gamma_1^{-1}}{\gamma_1}}\right)^{\frac{\gamma_1}{\gamma_1^{-1}}}, \qquad (5)$$

Role of variable capacity utilization (u):

idea: Can use capital more intensively to raise its marginal productivity, but at cost of higher rate of depreciation.

Effect: So economy responds to terms of trade improvement by increasing utilization of capital.

Which further increases marginal product of labor and demand for labor.

Household is assumed to own capital K_{1t}, which evolves over time under the following law of motion:

$$K_{1t+1} = (1 - \delta(u_{1t}))\, K_{1t} + I_{1t}\left(1 - S\left(\frac{I_{1t}}{I_{1t-1}}\right)\right), \tag{2}$$

where $\delta(u_t)$ is the depreciation rate of capital which depends on the capital utilization u_{1t} and I_{1t} is the gross investment. We assume that increasing the intensity of capital utilization comes with a larger depreciation rate. The functional form for $\delta(u_{1t})$ is given by:

$$\delta\left(u_{1t}\right) = \delta_0 + \delta_{11}\left(u_{1t} - 1\right) + \frac{\delta_{21}}{2}(u_{1t} - 1)^2,$$

Results:

Canadian output and hours rise more in response to US productivity shock with the new features, compared to BKK.