Topic 3: International Risk Sharing and Portfolio Diversificatin

Part 1) Working through a complete markets case

- In the previous lecture, I claimed that assuming complete asset markets produced a perfect-pooling equilibrium. We begin this lecture on asset markets by demonstrating this claim.

Model setup:

- Two countries, denoted home and foreign
- Only one good in the world.
- Two periods: denoted 1 and 2.
- Output in period 1 is known.
- Output in period 2 varies by state of nature, s=1,2,...,S.
- Assume output is an endowment.
- Worldwide asset market in Arrow-Debreu securities, with period 2 payoffs that vary according to state of nature.

Notation:

Y_1 is home output endowment in period 1

$Y_2(s)$ is home output endowment in period 2 if state s occurs

Y^*_1 is home output endowment in period 1

$Y^*_2(s)$ is home output endowment in period 2 if state s occurs

C_1 is home consumption in period 1

$C_2(s)$ is home consumption in period 2 if state s occurs

C^*_1 is home consumption in period 1

$C^*_2(s)$ is home consumption in period 2 if state s occurs

$B_2(s)$ is home net purchase of state s A-D securities in period 1 (for payoff in period 2 if state s occurs)

$p(s)$ is world price of one of state-s security.

$\pi(s)$ is probability of state s occurring, where $\sum_{s=1}^{S} \pi(s) = 1.$

Home Problem:

Household maximizes the intertemporal (two period) expected utility:

$$U = u(C_1) + \beta \sum_{s=1}^{S} \pi(s) u(C_2(s))$$

subject to the home sequence of $S+1$ budget constraints:

$$\sum_{s=1}^{S} p(s) B_2(s) = Y_1 - C_1 \quad \text{in period 1}$$

$$C_2(s) = Y_2(s) + B_2(s) \quad \text{in period 2, for each state } s=1...S$$

Or combining these into the intertemporal budget constraint:

$$C_1 + \sum_{s=1}^{S} p(s) C_2(s) = Y_1 + \sum_{s=1}^{S} p(s) Y_2(s)$$

First order conditions:

$$p(s)u'(C_1) = \pi(s)\beta u'(C_2(s))$$ for each s=1...S

or rewriting this: $$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = p(s)$$

This is a form of intertemporal consumption smoothing.

It also implies consumption smoothing across states:

$$\frac{\pi(s)u'(C_2(s))}{\pi(s')u'(C_2(s'))} = \frac{p(s)}{p(s')}$$

An analogous problem applies to the foreign household, and will produce analogous first order conditions.

Note that the security prices in these conditions $p(s)$ will be identical, since same securities being traded globally.

Implications:

So we have the following implications:

$$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = p(s) = \frac{\pi(s)\beta u'(C^*{}_2(s))}{u'(C^*{}_1)}$$

and

$$\frac{\pi(s)u'(C_2(s))}{\pi(s')u'(C_2(s'))} = \frac{p(s)}{p(s')} = \frac{\pi(s)u'(C^*{}_2(s))}{\pi(s')u'(C^*{}_2(s'))}$$

This indicates that that home and foreign marginal rates of substitution in consumption are equal – across time and states.

If we assume a standard CRRA utility function

$$U(C) = \frac{1}{1-\rho}C^{1-\rho}$$

and define world output: $Y^W = Y + Y^*$

The first order conditions imply (across states in period 2):

$$\frac{C_2(s)}{C_2(s')} = \frac{C^*{}_2(s)}{C^*{}_2(s')} = \frac{Y^W{}_2(s)}{Y^W{}_2(s')}$$

and (across periods)

$$\frac{C_2(s)}{C_1} = \frac{C^*{}_2(s)}{C^*{}_1} = \frac{Y^W{}_2(s)}{Y^W{}_1}$$ for all states.

This means

$$\frac{C_2(s)}{Y^W{}_2(s)} = \frac{C_2(s')}{Y^W{}_2(s')}$$

and the same for foreign consumption.

<u>This means home and foreign consumption are always a constant fraction of world output, regardless of state:</u>

$$\frac{C^*{}_2(s)}{Y^W{}_2(s)} = \mu = \frac{C^*{}_1}{Y_1^W} \qquad \frac{C_2(s)}{Y^W{}_2(s)} = 1 - \mu = \frac{C_1}{Y_1^W}$$

Or taking a ratio of the conditions above:

$$C_1 = \left(\frac{1-\mu}{\mu}\right) C_1^*$$

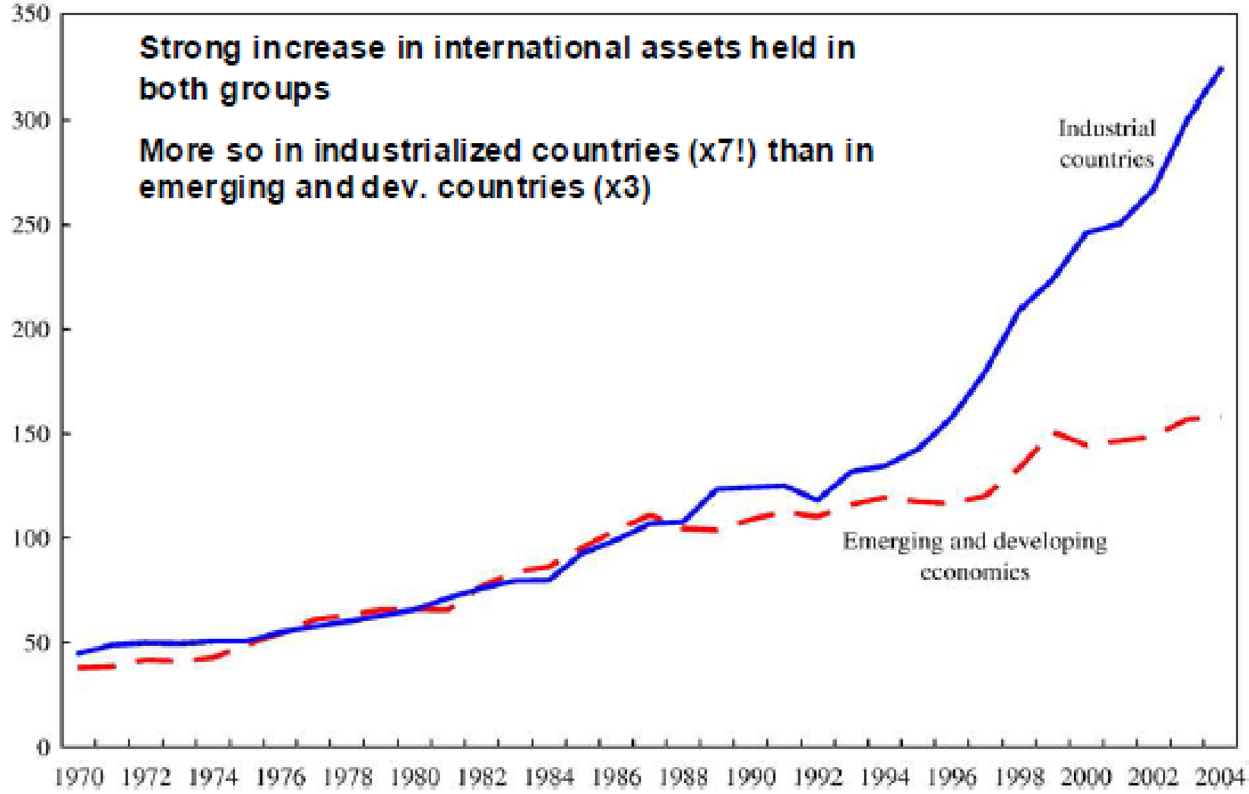
This property of complete assets markets helps explain why models like Backus et al (1992) have such a hard time reproducing low consumption correlations seen in the data.

Part 2) Financial Globalization and Portfolio Diversification Puzzle

- Financial globalization: International asset flows have increased dramatically.

- Why matters:
  - Welfare gains of international risk sharing
  - Transmission of shocks across countries
  - Affect design of monetary and fiscal policies

International financial openness, 1970–2004
(Domestic assets held by foreigners + Foreign assets held by domestic agents)/ GDP
source Lane and Milesi-Ferreti (2007)



a) <u>Documenting Equity Puzzle:</u>

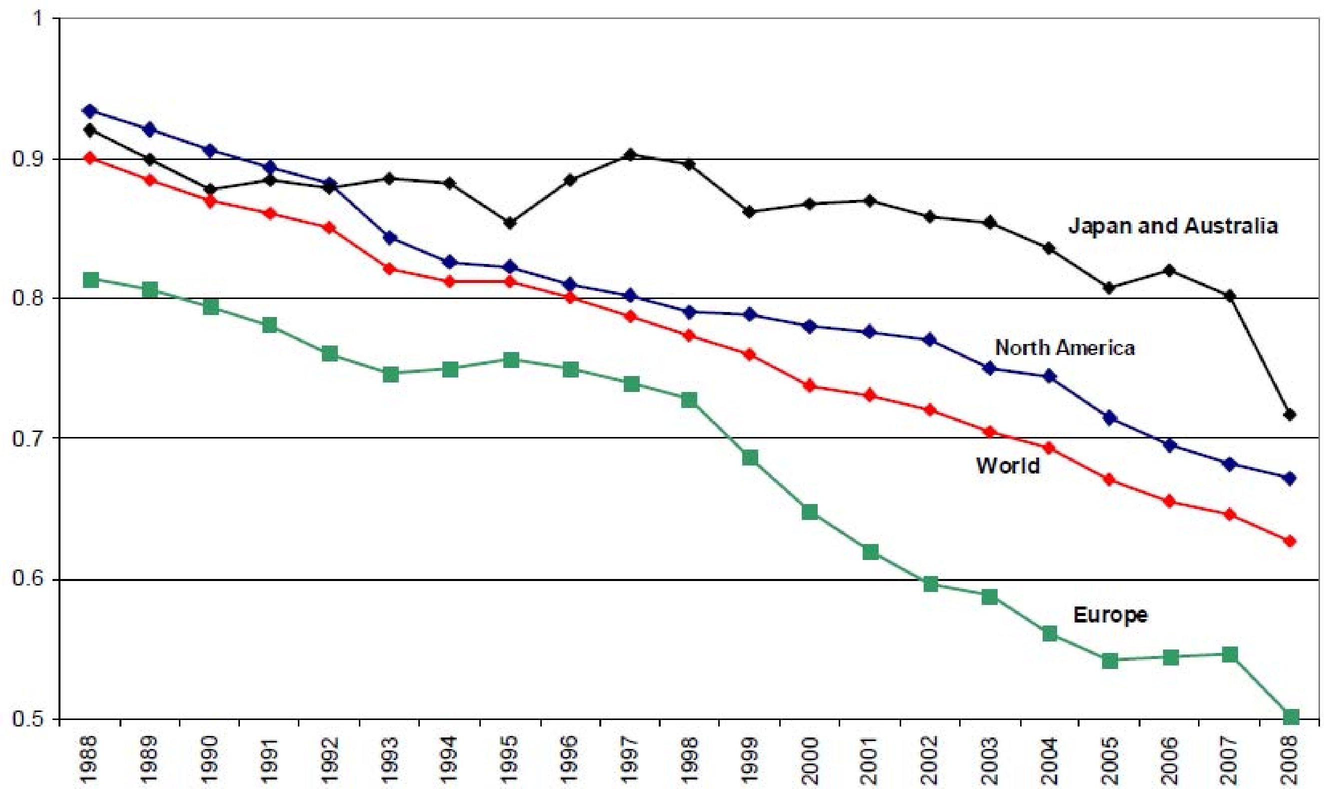
However, while international equity trade has grown over time, diversification remains far below what benchmark theory says is optimal

Why is this a puzzle: One might think that if the a country is 10% of the world equity market, it should hold only 10% of its equity portfolio in home equities; with 90% in foreign assets.

$$HB = 1 - \frac{\text{Share of Foreign of Equity Holdings}}{\text{Share of Foreign Stocks in World Market Capitalization}}$$

Source Country	Domestic Market in % of World Market Capitalization (1)	Share of Portfolio in Domestic Equity in % (2)	Degree of Equity Home Bias $= EHB_i$ (3)
Australia	1.8	76	0.76
Brazil	1.6	99	0.98
China	7.8	99	0.99
Canada	2.7	80	0.80
Euro Area	13.5	57	0.50
Japan	8.9	74	0.71
South Africa	1.4	88	0.88
South Korea	1.4	89	0.88
Sweden	0.7	44	0.43
Switzerland	2.3	51	0.50
United Kingdom	5.1	54	0.52
United States	32.6	77	0.66

Table (1): Home Bias in Equities in 2008 for selected countries (source IMF and FIBV)



Home Bias in Equities measures across developed countries

b) Demonstrate that the "world portfolio" is optimal in one good model:

Model setup:

- Like model above: one world good, endowment economy, two periods.
- Many countries: n=1...N
- Asset market allows trade in shares of endowment process in other countries (equities), as well as a noncontingent bond

Note: the equities are like a share in a mutual fund that pays its owner $Y^n{}_2(s)$ in state *s.*

Notation: same as model above for most variables:

$Y^n{}_1$	income of country *n* in period 1.
$Y^n{}_2(s)$	income of country *n* in period 2 in state s.
$C^n{}_1$	consumption in country n in period 1.
$C^n{}_2(s)$	consumption in country n in period 2.
$\pi(s)$	the probability of state *s* occurring.

New notation for new variables:

$x^n{}_m$	fractional shares of country *m*'s future output bought by residents in country *n*.
$V^n{}_1$	the market value in period 1 of country *n*'s uncertain output in period 2. The price of the shares in $\boldsymbol{x^n{}_m}$.
$B^n{}_2$	non-contingent bonds purchased in period 1, which pay off at rate (1+r) in period 2

Household Problem

The resident maximizes the two-period utility:

$$U = u(C_1^n) + \beta E_1\left[u(C_2^n)\right] = u(C_1^n) + \beta \sum_{s=1}^{S} \pi(s) u(C_2^n(s))$$

subject to the budget constraint for period 1:

$$Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m$$

and subject to the budget constraint for period 2:

$$C_2^n(s) = (1+r)B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s)$$

First order conditions

- With respect to bond holdings ($B^n{}_2$):

$$u'(C_1^n) = (1+r)\beta \sum_{s=1}^{S} \pi(s) u'(C_2^n(s))$$

Which is the usual consumption-smoothing Euler equation.

- With respect to portfolio shares ($x^n{}_m$):

$$V_1^m u'(C_1^n) = \beta \sum_{s=1}^{S} \pi(s) u'(C_2^n(s)) Y_2^m(s) \quad m = 1 \ldots N$$

The left-hand side is the marginal utility cost of buying country *m*'s risky future output on date 1, and the right-hand side is the expected marginal utility gain from this.

Solution:

We will assume a CRRA utility function: $u(C) = \frac{1}{1-\rho} C^{1-\rho}$

We conjecture that the solution is for all agents to hold shares in the same fully-diversified global portfolio, where the share of each country in this global portfolio is the country's share in world wealth.

We conjecture also consumption levels that divide up total world production (Y^w) between the countries according to their shares in world wealth.

Define country *n*'s share of initial world wealth as:

$$\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n} (Y_1^m + V_1^m)}$$

So our guess for the portfolio shares is μ^n:

$$x^n{}_m = \mu^n \quad for \quad m = 1 \ldots N$$

and our guess is that the country share of world consumption in each period and state is also μ^n:

$$C_1^n = \mu^n \sum_{m=1}^{N} Y_1^m = \mu^n Y_1^w$$

$$C_2^n(s) = \mu^n \sum_{m=1}^{N} Y_{21}^m(s) = \mu^n Y_2^w(s) \quad for \quad s = 1 \ldots S$$

Also conjecture that $B^n{}_2 = 0$.

Now we need to verify that this solution does actually satisfy the budget constraints and first order conditions.

1) Budget constraint for second period:

Plugging the conjectured asset allocation ${x^n}_m = \mu^n$ and ${B^n}_2 = 0$ into period 2 budget constraint, we find the consumption equation for period 2 above.

So we know this consumption allocation is consistent with the period2 budget constraint for each country.

2) Bond Euler equation:

For the CRRA utility, the bond FOC becomes

$$1+r=\frac{(C_1^n)^{-\rho}}{\beta\sum_{s=1}^{S}\pi(s)u'(C_2^n(s))^{-\rho}}$$

The consumption plans above will satisfy this Euler equation for each country if the equilibrium real interest rate is:

$$1+r=\frac{(Y_1^w)^{-\rho}}{\beta\sum_{s=1}^{S}\pi(s)u'(Y_2^w(s))^{-\rho}}$$

So the real interest rate is lower if the level of world output is high today relative to what it is expected to be next period.

3) <u>Equities Euler equation</u>:
Similarly satisfied for equilibrium equity share prices:

$$V_1^m = \sum_{s=1}^{S} \pi(s)\beta \left[\frac{Y_2^w(s)}{Y_1^w} \right]^{-\rho} Y_2^m(s) \quad m = 1...N$$

4) <u>Budget constraint for period 1</u>:

$$Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m$$

Sub in for $\boldsymbol{C^n{}_1}$, $B^n{}_2$ and $x^n{}_m$:

$$Y_1^n + V_1^n = \mu^n Y_1^w + \sum_{m=1}^{N} \mu^n V_1^m$$

Simplify and sub in the definition of μ^n:

$$Y_1^n + V_1^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n} (Y_1^m + V_1^m)} Y_1^w + \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n} (Y_1^m + V_1^m)} V_1^w$$

multiply by the reciprocal of μ^n:

$$\sum_{m=1}^{n} (Y_1^m + V_1^m) = Y_1^w + V_1^w$$

Which is true by definition.

This verifies that in a simple model it is optimal for people to hold equity portfolios that are fully diversified internationally.

In conclusion, this underscores that it is a puzzle that in practice people have a bias toward domestic equities.

c) Summary of Theoretical Explanations

1) transaction and information costs (coupled potentially with low gains from international risk sharing;

- See Lewis (2000) for a survey on the gains from international risk sharing);

- See Heathcote and Perri (2004), Bhamra, Coeurdacier and Guibaud (2012), Martin and Rey (2004) among others for theoretical work.

- See Veldkamp and Van Nieuwerburgh (2008) for information costs.

2) Real exchange rate fluctuations

- People in different countries face different consumption price indices (because they consume different basket of goods -trade costs and non-traded goods or because of local currency pricing...). Might be a reason to hold different portfolios.

- see Coeurdacier (2009), Kollmann (2006), Obstfeld (2007), Baxter, Jermann and King (1998), Collard, Dellas, Diba and Stockman (2007).

3) Non diversifiable labor income

- Investors have some labor income that cannot be diversified away.

- Baxter and Jermann (1997), Botazzi, Pesenti and VanWincoop (1996), Heathcote and Perri (2007), Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2010).

Part 3: Heathcote and Perri (JPE 2013)

- Main idea: Starts with workhorse BKK model from last lecture (two country, two-good RBC model), and adds equity trades to study portfolio decision.

- Equilibrium portfolios biased toward domestic assets because endogenous international rel price fluctuations make dom. assets good hedge against labor income risk.

Background literature:

Lucas (1982):

- Two-country stochastic endowment (tree) model, where agents in each country have common preferences.
- Shows that perfect risk sharing of idiosyncratic output shocks can be achieved by economizing on the number of assets-— equity only are enough.
- Pooling of risk involves agents of each country owning half the claims to the home endow-ment and half the claims to the foreign endowment.

Baxter and Jermann (1997)

- Extend Lucas's model in one direction by introducing nondiversifiable labor income.
- Show that if asset returns and labor income are highly correlated within a country, then agents can hedge nondiversifiable labor income risk with a large short position in domestic assets.
- So implies aggressive diversification.

Cole and Obstfeld (1991)

- Show that in a special case of the Lucas model, trade in assets is not required to achieve risk sharing.
- If the fruits yielded by the two trees are imperfect substitutes, then changes in relative endowments induce offsetting changes in the terms of trade.
- When preferences are log separable between the two goods, terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk sharing.

Distinction of this model:

- Like Baxter and Jermann, model implies labor income risk,
- but it allows imperfect substitutability between home and foreign traded goods. So relative prices provide some insurance against country-specific shocks, as in Cole and Obstfeld.
- Home bias arises because relative returns to domestic stocks move inversely with relative returns to labor in response to productivity shocks.
- This covariation arises due jointly to international relative price movements and to the presence of capital

<u>Model: like BKK in goods and production:</u>

- Two countries
- Firms in each country use country-specific capital and labor to produce an intermediate good.
- Home intermediate good is labeled *a*, and foreign *b*, subject to country specific productivity shocks
- combined to produce country-specific final consumption and investment goods.
- The final goods biased toward using a larger fraction of the locally produced intermediate

Asset market structure different:

- Only internationally traded asset are shares in the domestic and foreign intermediate goods–producing firms.
- firms make investment and employment decisions and distribute non-reinvested earnings to shareholders.
- denote by s^t the history of events each period, with probability $\pi(s)$

- Preferences log separable in consumption and leisure

$$U(c(s^t), n(s^t)) = \log(c(s^t)) - V(n(s^t))$$

where V is disutilty of labor

- production function:

$$F(z(s^t), k(s^{t-1}), n(s^t)) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta}$$

where z is productivity shock, k capital and n labor

- Cobb-Douglas aggregation of home and foreign goods:

$$G(a(s^t), b(s^t)) = a(s^t)^{\omega} b(s^t)^{(1-\omega)},$$

$$G^*(a^*(s^t), b^*(s^t)) = a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega},$$

where $\omega > 0.5$ determines the size of the local input bias

- Letting q_a and q_b denote the prices of goods a and b rel to the domestic final good
- The terms of trade p is the price of good b relative to good a.

$$p(s^t) = \frac{q_b(s^t)}{q_a(s^t)} = \frac{q_b^*(s^t)}{q_a^*(s^t)}$$

- The real exchange rate, e, is defined as the price of foreign consumption relative to domestic consumption. By the law of one price, it can be expressed as

$$e(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} = \frac{q_b(s^t)}{q_b^*(s^t)}$$

ie: $e = P^*/P = (p_a/P) / (p_a^*/P^*) = q_a/q_a^*$ since $p_a = p_a^*$

The household budget constraint:

$$c(s^t) + P(s^t)[\lambda_H(s^t) - \lambda_H(s^{t-1})] + e(s^t)P^*(s^t)[\lambda_F(s^t) - \lambda_F(s^{t-1})]$$
$$= l(s^t) + \lambda_H(s^{t-1})d(s^t) + e(s^t)\lambda_F(s^{t-1})d^*(s^t) \quad \forall t \geq 0, s^t.$$

where

$P(s^t)$ is the price at state s of shares in home firm in domestic consumption units

$\lambda_H(s^t)$ is the fraction of the domestic firm purchased by the domestic agent,

$d(s^t)$ is the domestic dividend payment per share

$l(s^t) = q_a(s^t)w(s^t)n(s^t)$ is domestic labor earnings

Firm problem:

Firms maximize discounted stream of dividends:

$$\sum_{t=0}^{\infty} \sum_{s^t} Q(s^t) d(s^t)$$

where

$Q(s^t)$ is the price firms use to value future dividends rel to consumption, using household preferences

$$Q(s^t) = \frac{\pi(s^t)\beta^t U_c(s^t)}{U_c(s^0)}$$

dividends are given by:

$$d(s^t) = q_a(s^t)[F(z(s^t), k(s^{t-1}), n(s^t)) - w(s^t)n(s^t)] - [k(s^t) - (1-\delta)k(s^{t-1})],$$

Equilibrium portfolio solution:

Portfolio exhibits a level of diversification, $1-\lambda$:

$$\lambda_F(s^t) = 1 - \lambda_H(s^t) = 1 - \lambda$$
$$= \left(\frac{1-\theta}{1-\omega} + 2\theta\right)^{-1}$$

Depends on :

θ capital share in production function

ω home bias in goods demands

- nests Lucas (1982): $\omega = 0.5$: $1-\lambda$=0.5
- Implies: equity diversification is increasing with degree of trade openness $(1-\omega)$.
- (and higher capital share raises diversification if θ>0.5)

Trade openness prediction has strong support in data:

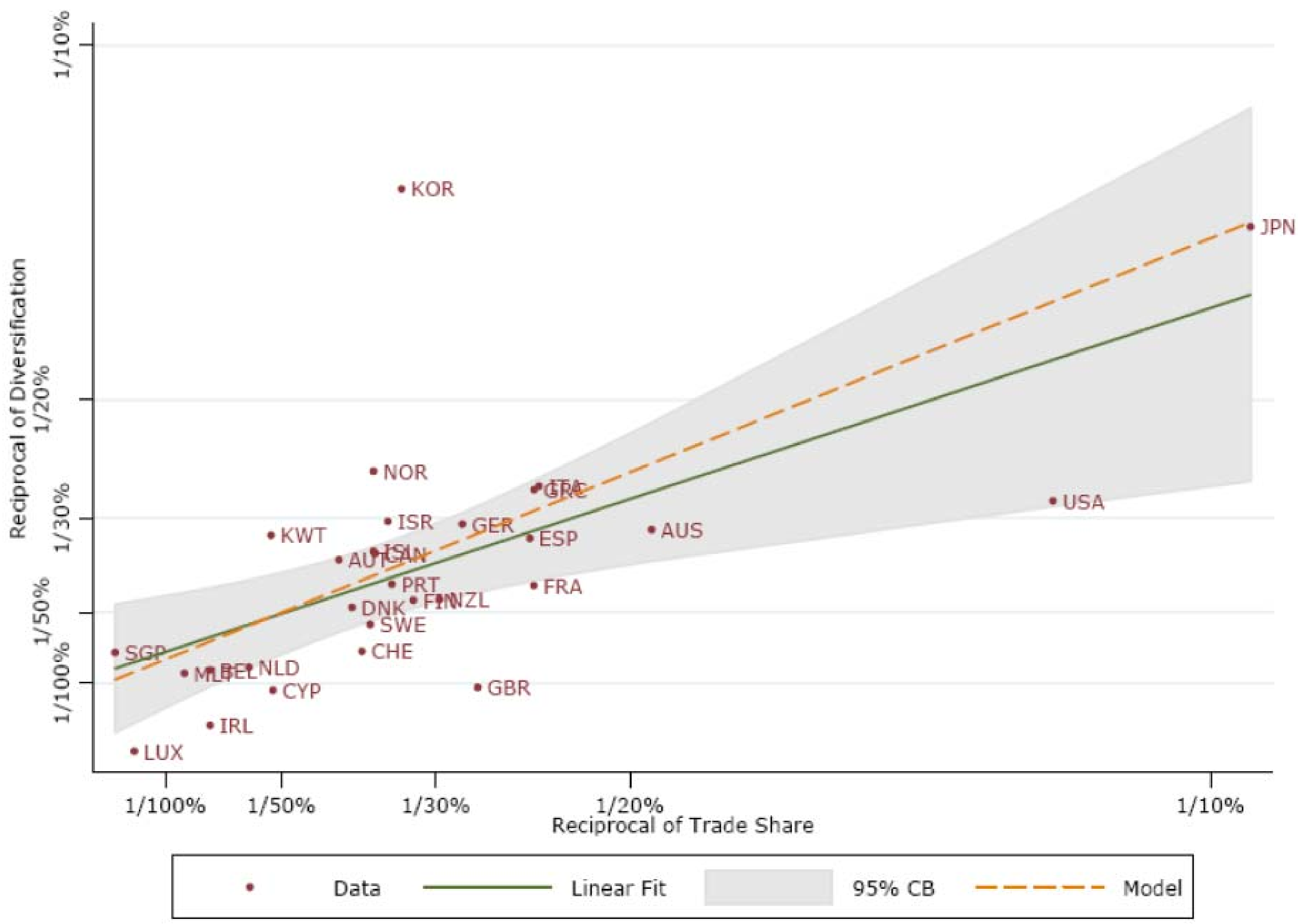


TABLE 1

DIVERSIFICATION AND TRADE

	Dependent Variable: Diversification				
	Data				
	OLS			LAD	Model
	(1)	(2)	(3)	(4)	(5)
Trade openness	1.26	1.52	1.08*	1.42*	.78
	(.19)	(.22)	(.15)	(.31)	(.05)
Log GDP per capita		−.13			
		(.24)			
Log population		.05			
		(.05)			
Predicted median diversification	.53	.51	.47*	.45*	.34
	(.06)	(.05)	(.05)	(.04)	(.01)
Observations	19	19	19	19	19
R^2	.37	.40	.41	.37	.97

Column (1): stat significant positive relationship

Note that for median country, predicts diversification ratio of 0.53, similar to data.

Col (2) : include regressors for size

Col (3): use alternative measure of diversification.

Equilibrium portfolio solution, representation 2:

The diversification can also be represented in terms of relative incomes across counties and covariances:

$$1 - \lambda \approx \frac{1}{2}\left[1 + \left(\frac{1-\theta}{\theta}\,\frac{\rho+\delta}{\rho}\right)M\right]$$

where

$$M(s^t) \approx M = \left[2\left(\frac{1-\theta}{1-\omega} + 2\theta\right)^{-1} - 1\right]\left(\frac{\theta}{1-\theta}\,\frac{\rho}{\rho+\delta}\right) \quad \forall t, s^t,$$

$M(s^t) = \mathrm{Cov}\left(\Delta\hat{l}(s^t)\ \Delta\hat{d}(s^t)\right)/\mathrm{Var}(\Delta\hat{d}(s^t))$ is the ratio of the equilibrium conditional covariance between relative log earnings and relative log dividends to the variance of relative log dividends

$$\Delta\hat{l}(s^t) = \log(l(s^t)) - \log(e(s^t)) - \log(l^*(s^t))$$

Main intuition:

- When Home productivity is high, so are wages and labor income.
- But dividends net of investment decrease. Generates a negative covariance between Home excess returns on stocks and Home returns to labor.
- So home ownership of equities is a good hedge against labor income fluctuations, so holding home equities is desirable for home agents.

Intuition for risk sharing under home equity bias:

- Home bias is consistent with international consumption risk sharing:

- When Home productivity is high, Home agents receive a positive transfer through their labor income but they have to finance higher investment, which makes it possible to equalize consumption despite higher labor income.

- Expectations of higher future Home productivity raises investment in the Home country relative to Foreign.

- A change in investment has a direct, and an indirect effect on relative consumption:

- D<u>irect effect</u>: a high home bias in portfolio means that most of the investment is financed by Home households, at the cost of their consumption relative to Foreign consumption. This effect depends on the share of investment financed by each country (equity ownership).

- <u>The indirect effect</u> works through relative prices and output.
  - With home bias, an increase in investment raises demand for Home output, causing appreciation. Everything else equal, this raises Home consumption.
  - The Home appreciation is however contained by (a) a positive response of output and (b) the fact that part of profits are paid to Foreign shareholders, whose spending is biased towards their own goods.

- The solution to the portfolio problem will imply that the direct and indirect effects exactly compensate each other, and will thus maintain the risk sharing condition $P_t C_t = \mathcal{E}_t P_t^* C_t^*$.

Conclusions:

- Financial globalization points out the need to understand increasing crossborder asset positions and their various macro implications.

- Open Economy Financial Macroeconomics first step in this direction

- Still many unresolved questions.
  - consumption/portfolio discrepancies;
  - portfolio/asset prices discrepancies;
  - welfare implications?
  - modelling heterogeneous investors/countries;