In this lecture we study some puzzles in international financial markets, regarding the relationship between interest rates and exchange rates.
Consider a simple intertemporal model:

Assets:
- $M_t$ home money
- $B_t$ home nominal interest bearing assets, at rate $i$
- $B_t^*$ foreign nominal interest bearing asset, at rate $i^*$
- $F_t$ forward contract: purchase one unit of foreign currency next period in exchange for $f_t$ units of home currency ($f_t$ known in period $t$)
- $e_t$ (spot) exchange rate (home currency per foreign)

Household problem

$$\text{Max } E_t \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t} \right)$$

s.t. $P_tY_t + (1 + i_{t-1})B_{t-1} + e_t(1 + i_{t-1}^*)B_{t-1}^* + (e_t - f_{t-1})F_{t-1} + M_{t-1}$

$$= P_tC_t + B_t + e_tB_t^* + M_t$$
FOCS:
(lambda for lagrange multiplier on budget constraint.)

(1) Home bonds: \[ \beta E_t [\lambda_{t+1}](1+i_t) = \lambda_t \]

(2) Foreign bonds: \[ \beta E_t [e_{t+1}\lambda_{t+1}](1+i_t^*) = e_t\lambda_t \]

(3) Forward exchange: \[ E_t [(e_{t+1} - f_t)\lambda_{t+1}] = 0 \]
    Or \[ E_t [e_{t+1}\lambda_{t+1}] = fE_t [\lambda_{t+1}] \]

Consumption (lambda): \[ \lambda_t = \frac{U'_{c,t}}{P_t} \]

Now combine various equations to draw conclusions…
Covered interest rate parity (CIP): combine (1), (2) and (3):

\[ e_t \left( \frac{1 + i_t}{1 + i_t^*} \right) = f_t \]

Logic: suppose did not hold: \( e_t \left( \frac{1 + i_t}{1 + i_t^*} \right) > f_t \) or \( e_t (1 + i_t) \frac{1}{f_t} > 1 + i_t^* \), then:

1) borrow 1 euro in Frankfurt at rate \( 1 + i_t^* \)
2) convert 1 euro in spot exchange rate to \( e_t \) dollars
3) receive gross return \( e_t (1 + i_t) \) dollars at end of year
4) convert back to \( e_t (1 + i_t) / f_t \) euros, and repay loan of \( 1 + i_t^* \) euros with guaranteed profit left over.

This is an arbitrage opportunity. Demand for forward contracts would rise, and \( f_t \) would rise, restoring condition.

Empirical evidence supports this condition; seems it is how banks set their forward rates.
Sometimes see this written as an approximation in logs:

\[
\frac{e_t}{f_t} = \frac{1 + i_t^*}{1 + i_t}
\]

\[
\ln e_t - \ln f_t = \ln (1 + i_t^*) - \ln (1 + i_t) \approx i_t^* - i_t
\]

Terminology: Forward premium: \( \ln f_t - \ln e_t \)
Uncovered interest rate parity (UIP): use (2) and (1):

\[ e_t \left( \frac{1 + i_t}{1 + i_t^*} \right) = E_t \left[ e_{t+1} \frac{U'_{c,t+1}}{P_{t+1}} \right] / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

Can be rewritten:

\[ e_t \left( \frac{1 + i_t}{1 + i_t^*} \right) = \left\{ E_t \left[ e_{t+1} \right] E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] + \text{cov}_t \left[ e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}} \right] \right\} / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

\[ e_t \left( \frac{1 + i_t}{1 + i_t^*} \right) = E_t \left[ e_{t+1} \right] + RP_t \text{ where } \quad RP_t = \text{cov}_t \left[ e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}} \right] / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

- Note that this is not a riskless arbitrage opportunity; risk premium is a wedge between returns.

- Interpret a positive risk premium: foreign currency has high value (\( e \) high) in bad states (\( U' \) high); foreign currency assets good hedge and can offer lower \( i^* \).
Market efficiency condition: rewrite (3):

\[ f_t = E_t \left[ e_{t+1} \frac{U'_{c,t+1}}{P_{t+1}} \right] / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

Can be rewritten as for UIP above

\[ f_t = E_t[e_{t+1}] + \text{cov}_t \left[ e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}} \right] / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

\[ = E_t[e_{t+1}] + RP_t \]

Question: is the forward rate a good predictor of the expected future spot exchange rate?
b. Empirical literature

Early Tests of Interest parity and market efficiency

Framework:
The earliest tests of forward efficiency regressed the future spot rate on the forward rate in logs:

\[ e_{t+1} = a_0 + a_1 f_t + \varepsilon_t \]

Tested if \( a_0 = 0 \) and \( a_1 = 1 \). Null hypothesis is that the forward rate provides an unbiased forecast of the future spot exchange rate.

Note the equation tested here replaces the expected value of \( e_{t+1} \) in the original condition with the actual future value, due to lack of data on expectations.
Another problem is nonstationarity of exchange rate. Most researchers subsequently have tested the equation with $e_t$ subtracted on each side:

\[
(e_{t+1} - e_t) = a_0 + a_1 (f_t - e_t) + \varepsilon_t
\]

Related test: Since covered interest parity holds well, replace the right term with the interest rate differential. We then have a test of uncovered interest parity:

\[
(e_{t+1} - e_t) = a_0 + a_1 (i_t - i^*_t) + \varepsilon_t
\]

If the risk premium assumed to be constant, it would appear in the $a_0$ term, making it deviate from the hypothesized value of zero, but not affect the $a_1$ term. So the researchers focused on hypothesis that $a_1=1$. 
Results:
Froot (1990 JEPerspectives:) Summarize the literature of 75 papers on the subject. The average estimate of $a_1$ over 75 papers is -0.88, only a few find $a_1>0$, and none find $a_1>1$.

Conclusions:
In general, papers reject the hypothesis that $a_1=1$, and most find that it is actually negative.

This means that a country’s currency is expected to appreciate in future periods when its interest rate is high.

Rather than offsetting a high interest rate, future appreciation makes it even more profitable to buy a currency.

How can this be consistent with market equilibrium? How can we explain this finding?
Time-varying risk premia

- If the risk premium varies over time, we could write the equation from before:

$$(e_{t+1} - e_t) = a_0 + a_1(f_t - e_t) - RP_t + \varepsilon_t$$

- Where $RP_t$ and $e_t$ together are the error term in the regression. So the error term includes a component that may be correlated with regressors, biasing estimate of $a_1$.

- To solve this problem, some researchers have used ARCH models of the risk premium. (Domowitz-Hakkio 1995 JIE).

- They propose a separate regression for the risk premium itself as a function of the interest rate differential.

- But results were not encouraging. It appears that the risk premium not a simple function of interest rate differential.
Eichenbaum and Evans (QJE 1995):

Motivation

- This paper documents the failure of UIP conditional on monetary policy shocks. Shocks are identified here in a vector autoregression (VAR).

- It also provides evidence regarding the disagreement between Mussa and Stockman regarding whether monetary or real shocks drove exchange rates.
Methodology:

- Use a simple VAR. Included in the vector of variables is a variable representing policy.

- Apply a Cholesky decomposition, in which ordering of variables matters: variables, $X$, preceding it are observed contemporaneously.

- Define policy shock as the innovation to the policy variable (nonborrowed reserve ratio) that is orthogonal to the contemporaneous values of the observed variables.
Estimating the Effects of Shocks to the Economy

- Vector Autoregression for a $N \times 1$ vector of observed variables:

$$Y_t = B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + u_t,$$

$$E u_t u_t' = V$$

- $B$s, $u'$s and $V$ are Easily Obtained by OLS.
- Problem: $u'$s are statistical innovations.
  - We want impulse response functions to fundamental economic shocks, $e_t$.

$$u_t = C' e_t,$$

$$E e_t e_t' = I,$$

$$C C' = V$$

These notes from VAR review lecture notes of Lawrence Christiano
VAR: \[ Y_t = B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + C e_t \]

- Impulse Response to \(i^{th}\) Shock:

\[ Y_t - E_{t-1} Y_t = C_i e_{it}, \]

\[ E_t Y_{t+1} - E_{t-1} Y_{t+1} = B_1 C_i e_{it} \]

\[ \ldots \]

- To Compute Dynamic Response of \(Y_t\) to \(i^{th}\) Element of \(e_t\) We Need

\[ B_1, \ldots, B_p \text{ and } C_i. \]
Identification Problem

\[ Y_t = B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + u_t \]

\[ u_t = C e_t, \ E u_t u'_t = C C' = V \]

- We know \( B' s \) and \( V \), we need \( C \).
- Problem
  - \( N^2 \) Unknown Elements in \( C \),
  - Only \( N(N+1)/2 \) Equations in

\[ C C' = V \]

- Identification Problem: Not Enough Restrictions to Pin Down \( C \)
- Need More Identifying Restrictions!
Using VAR to Estimate Impulse Response Functions Under Recursiveness Assumption

- Vector autoregression:

\[ Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + B_g Y_{t-g} + u_t \]

\[ u_t = C e_t. \]

- To think about recursiveness assumption, it is convenient to work with

\[ A_0 \equiv C^{-1} \]

so that:

\[ A_0 Y_t = A_0 B_1 Y_{t-1} + A_0 B_2 Y_{t-2} + \ldots + A_0 B_g Y_{t-g} + e_t, \]

\[ A_0^{-1} (A_0^{-1})' = V \]
Using VAR to Estimate Impulse Response Functions Under Recursiveness Assumption ..

- Consider:

\[
Y_t = \begin{pmatrix}
X_{1t} \\
R_t \\
X_{2t}
\end{pmatrix}_{(k_1 \times k_1)} \quad A_0 = \begin{bmatrix}
a_{11} & 0 & 0 \\
(k_1 \times k_1) & (k_1 \times 1) & (k_1 \times k_2) \\
a_{21} & a_{22} & 0 \\
(1 \times k_1) & (1 \times 1) & (1 \times k_2) \\
a_{31} & a_{32} & a_{33} \\
(k_2 \times k_1) & (k_2 \times 1) & (k_2 \times k_2)
\end{bmatrix}
\]

where

- \( R_t \) interest rate (middle equation is policy rule)
- \( X_{1t} \sim k_1 \) variables whose current and lagged values do appear in policy rule
- \( X_{2t} \sim k_2 \) variables whose current values do no appear in the policy rule.

- Zero restrictions on \( A_0 \) are implied by recursiveness assumption:
  - Zero in middle row: current values of \( X_{2t} \) do not appear in policy rule
  - Zeros in first block of rows ensure that monetary policy shock does not affect \( X_{1t} \)
    - First block of zeros: prevents direct effect, via \( R_t \)
    - Second block of zeros: prevents indirect effect, via \( X_{2t} \)
Data:

- Data on bilateral exchange rates of five countries with the US dollar. Monthly data, starting in 74.1.

- **Five variables** in first system estimated:
  1) US-IP: industrial production (output)
  2) US-CPI: price level
  3) NBR/TR, non-borrowed reserve ratio to total reserves, measures degree of liquidity
  4) Gap in 3-mo T-bils: foreign-home,
  5) exchange rate (first nominal and then real)

- Measure monetary policy action as a change in nonborrowed reserves not explained as a response to changes in contemporaneous price or output.
Results: Show figure 1: impulse responses to NBR/TR shock

1) A U.S. monetary contraction leads to a rise in initial period in U.S. interest rate rel to foreign (fall in $R_{for} - R_{US}$).

2) Appreciates real and nominal exchange rates for U.S.

3) Exchange rate effect reaches maximal after 2-3 years. This “delayed overshooting” is inconsistent with UIP; reflects wrong sign in UIP tests above.

(Discuss Overshooting)

4) Plot also the “excess return” on holding dollar assets: return from dollar appreciation plus higher interest rate.

5) Tables 1a and 1b: Reject that the maximal impact happens in initial period.

6) Variance decomp: monetary shocks account for 18-43% of exchange rate volatility.
FIGURE I
Dynamic Response Functions: Benchmark Specification
Real exchange rate: (Table 1a)

<table>
<thead>
<tr>
<th></th>
<th>Max impact</th>
<th>Std. error</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.032</td>
<td>-2.679</td>
<td>-2.474</td>
</tr>
<tr>
<td></td>
<td>1.033</td>
<td>1.226</td>
<td>1.031</td>
</tr>
<tr>
<td></td>
<td>0.025</td>
<td>0.014</td>
<td>0.008</td>
</tr>
<tr>
<td>(9)</td>
<td>Max month</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.650</td>
<td>32.070</td>
<td>36.498</td>
</tr>
<tr>
<td></td>
<td>0.023</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>(10)</td>
<td>31–36 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.016</td>
<td>42.917</td>
<td>38.122</td>
</tr>
<tr>
<td></td>
<td>13.640</td>
<td>15.713</td>
<td>15.481</td>
</tr>
<tr>
<td></td>
<td>0.092</td>
<td>0.006</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Variance decompositions
nominal exchange rate (table 1b)

<table>
<thead>
<tr>
<th></th>
<th>8) Max impact</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-1.913</td>
<td>-2.961</td>
<td>-2.950</td>
<td>-3.000</td>
</tr>
<tr>
<td></td>
<td>Std. error</td>
<td>0.952</td>
<td>1.532</td>
<td>1.815</td>
<td>1.330</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
<td>0.022</td>
<td>0.027</td>
<td>0.052</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9) Max month</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>24.654</td>
<td>35.304</td>
<td>37.990</td>
<td>37.478</td>
</tr>
<tr>
<td></td>
<td>Std. error</td>
<td>11.818</td>
<td>10.412</td>
<td>7.360</td>
<td>6.918</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
<td>0.018</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10) 31–36 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>22.084</td>
<td>41.021</td>
<td>38.767</td>
<td>38.474</td>
</tr>
<tr>
<td></td>
<td>Std. error</td>
<td>13.901</td>
<td>16.271</td>
<td>15.135</td>
<td>15.879</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
<td>0.112</td>
<td>0.012</td>
<td>0.010</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Variance decompositions
Overshooting

- Suppose U.S. monetary contraction raises US interest rate relative to foreign rate.

- According to UIP, if people are still willing to hold foreign assets despite the lower interest rate, they must expect to be compensated by appreciation in value of the foreign currency over time \((e_{$/€} \text{ rising over time})\).

- Now consider role of PPP, which seems to hold in long run: A fall in U.S. money supply should make the dollar more valuable \((e_{$/€} \text{ lower})\) in future than initially.

- If we put these two conclusions together, we get the following type of path for the exchange rate following a U.S. monetary contraction:
- The exchange rate falls on impact, but it overshoots its long run level. This is so it can gradually move upward in subsequent periods, and still end up lower than initially.

- This is one characterization of the “Overshooting” theory of Dornbusch.
Conclusions:
- Reject UIP; instead see “delayed overshooting”
- Monetary shocks are important, but hey explain less than half of exchange rate fluctuations

Critiques: What do you think about the identification scheme for identifying monetary policy shocks?

Question: do you see a way to make money from the finding of this paper?
c. **Benigno et. al (2012 NBER Macro Annual)**

Main idea:

Want to explain why a fall in \( i \) predicts a currency depreciation, rather than appreciation as required by interest rate parity.

Must be a large fall in \( RP \).

\[
i_{t+1} - i^*_t = E[e_{t+1}] - e_t + RP_t
\]
A simple way to view this is in terms of theory presented earlier in lecture. Recall a general expression for the risk premium:

\[ RP_t = E_t \left[ e \frac{U'_{c,t+1}}{P_{t+1}} \right] / E_t \left[ \frac{U'_{c,t+1}}{P_{t+1}} \right] \]

Recall also that under a model with perfect risk sharing, we have seen:

\[ \frac{U'_{c}^{*}}{P^{*}} = e \frac{U'_{c}}{P} \]

So we have:

\[ RP_t = E_t \left[ \frac{C_{t+1}^{* - \rho}}{P_{t+1}^{*}} \right] / E_t \left[ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right] \]
Rewrite this using: $\log E[X] = E[\log X] + \frac{1}{2} \text{var}[\log X]$

Assume that the shocks are log normally distributed, then:  
(lower case letters represent logs, overbars are means)

$$\log RP_t = (\overline{p} - \overline{p}^*) + \rho(\overline{c} - \overline{c}^*)$$
$$+ \frac{1}{2}(\text{var}[p^*] + \rho^2 \text{var}[c^*] + 2\rho \text{cov},[p^*,c^*])$$
$$- \frac{1}{2}(\text{var}[p] + \rho^2 \text{var}[c] + 2\rho \text{cov},[p,c])$$

Under symmetry and perfect risk sharing:

$$RP_t = \frac{1}{2} \left( \sigma_{p^*t}^2 - \sigma_{pt}^2 \right) + \rho \left(\sigma_{cp^*t}^2 - \sigma_{cpt}^2 \right)$$

where: $\sigma_{pt}^2 = \text{var}_t(p_{t+1})$, and $\sigma_{cpt}^2 = \text{cov}_t(c_{t+1}, p_{t+1})$
Interpretation:

- The first bracketed term above is a Jensen inequality term.

- The second bracketed term represents how a home nominal asset acts as a hedge against consumption risk.

- If consumption and the price positively correlated, home nominal assets pay off best in real terms (low price) in bad states of the world (low consumption)

- So home assets are a hedge and risk premium is low.

- Says that a rise in the volatility of money supply can lower the risk premium.
Intuition:

- In a sticky price model, a money supply rise only partly translates into a rise in price.

- To the degree that price doesn’t rise fully, there is a rise in demand which raises production and consumption.

- So a rise in monetary volatility raises the correlation of price and consumption.

- This makes home assets more attractive as a hedge, and lowers the risk premium.

- Suggests that we study the effect of shocks to monetary policy volatility instead of shocks to money supply.
Empirical: estimate a vector auto-regression based on Eichenbaum and Evans (1995) studied above, but include measures of volatility to monetary policy shocks (as well as shocks to inflation target and GDP growth rate).

Measure policy shocks: change in futures for forecasted treasury bill rate. Measure standard deviation of these.

Data in VAR:
1) monetary of monetary policy shocks
2) us interest rate; 3) foreign interest rate
4) real exchange rate
5) aggregate price index
6) aggregate output index
Report response for:
1) US interest rate (FFR)
2) slope of term premium (i-long term – i-short term)
3) real exchange rate
4) failure in UIP: \( i_t^* - i_t + E_t(s_{t+1} - s_t) \)

Results:
1) show volatility in monetary policy (ie in 2007-8 crisis)

2) replicate EE(1995) result for shocks to monetary policy: rise in money supply leads to delayed overshooting depreciation in dollar

3) show that a rise in variance of monetary policy shocks leads to rise in UIP deviation, excess return on foreign assets (dollar becomes more attractive as a hedge).
Responses to U.S. monetary policy contraction

Figure 2: Dynamic responses to an orthogonalized innovation to the Federal Funds Rate. Each column reports, for each country pair, the responses of the US Federal Funds rate ($i$), the slope of the US term structure ($i_{sd}$), the Real Exchange Rate ($q$), the foreign currency risk premium ($exr$). $x$-axes: months, $y$-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.
Responses to rise in U.S. monetary policy volatility

Figure 3: Dynamic responses to an orthogonalized innovation to the volatility of shocks to the monetary policy instrument. Each column reports, for each country pair, the responses of the US Federal Funds rate \( (i) \), the slope of the US term structure \( (i_{sl}) \), the Real Exchange Rate \( (q) \), the foreign currency risk premium \( (e_x) \). \( x \)-axes: months, \( y \)-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.
d. Engel (AER 2016):

- Discusses an additional stylized fact: High real interest rate countries tend to have currencies that are strong in real terms.

- While this fact is known in the literature, Engel claims it conflicts with the usual explanation of the UIP puzzle in using a time-varying risk premium: The high interest rate country tends to have high expected returns on its short term asset because it has a high currency risk premium.

- Puzzle as summarized by Chris Telmer: Right now the Brazilian Real (BRL) is overvalued in real terms. Brazilian real interest rates are also high. The latter suggests more BRL appreciation. But we know that eventually PPP must kick in, with the BRL depreciating.
• This new fact is different from the UIP puzzle above in that it relates to:
  o Real rather than nominal interest and exchange rate
  o Level of exchange rate rather than change

• But we can show the two conditions are linked:

• First convert nominal interest rate and exchange rates into real counterparts by adjusting for inflation:
  
  \[ UIP: \quad i_{t+1} - i^*_{t+1} = Ee_{t+1} - e_t \]

• Subtract \( E\pi_{t+1} - E\pi^*_{t+1} = (E p_{t+1} - p_t) - (E p^*_{t+1} - p^*_t) \) from both sides
\[ (i_{t+1} - E\pi_{t+1}) - (i^*_{t+1} - E\pi^*_{t+1}) = E\left(e_{t+1} + p^*_{t+1} - p_{t+1}\right) - \left(e_t + p^*_t - p_t\right) \]

\[ (i_{t+1} - E\pi_{t+1}) - (i^*_{t+1} - E\pi^*_{t+1}) = E\left(e_{t+1} + p^*_{t+1} - p_{t+1}\right) - \left(e_t + p^*_t - p_t\right) \]

So \[ r_{t+1} - r^*_{t+1} = Eq_{t+1} - q_t \]

Where \( r \) is the real interest rate and \( q \) is the real exchange rate.

- Second, rearrange and iterate forward:
  \[
  q_t = \left(r^*_{t+1} - r_{t+1}\right) + Eq_{t+1}
  = \left(r^*_{t+1} - r_{t+1}\right) + \left(r^*_{t+2} - r_{t+2}\right) + Eq_{t+2}
  = R_t + \lim_{j \to \infty} Eq_{t+j}
  \]
Where \( R_t = \sum_{i=0}^{\infty} E \left( r^*_t - r^t_{t+i} \right) \) represents prospective real interest differentials

- Now assume also long run PPP: \( \lim_{j \to \infty} Eq_{t+j} = \bar{q} \)

Then, \( q_t - \bar{q} = R_t \)

Note this predicts that if \( \uparrow r \rightarrow \downarrow R_t \) then this should imply \( \downarrow q \), that is, a real appreciation.

- For reference, we can also define a measure of the excess return on the foreign deposit inclusive of currency appreciation, that is, the deviation from UIP:
  \[ \rho_{t+1} = i^*_t + e_{t+1} - e_t - i_t \]
Empirical: 1) confirm failure of UIP in real terms

Recall equation: \( q_{t+1} - q_t = -\left(r^*_{t+1} - r_{t+1}\right) \)

Surprisingly, beta is >0 and often >1.

<table>
<thead>
<tr>
<th>Country</th>
<th>( \hat{\zeta}_q )</th>
<th>95% interval</th>
<th>90% interval</th>
<th>( \hat{\beta}_q )</th>
<th>95% interval</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.030 (-0.182, 0.208)</td>
<td>(-0.141, 0.171)</td>
<td>0.722 (-1.103, 3.065)</td>
<td>(-0.670, 2.665)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.111) (-0.151, 0.200)</td>
<td>(-0.118, 0.162)</td>
<td>(0.768) (-1.004, 2.749)</td>
<td>(-0.673, 2.492)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>-0.071 (-0.321, 0.124)</td>
<td>(-0.274, 0.072)</td>
<td>1.482 (-0.237, 3.283)</td>
<td>(0.076, 3.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.186) (-0.312, 0.107)</td>
<td>(-0.266, 0.061)</td>
<td>(1.089) (-0.834, 3.881)</td>
<td>(-0.353, 3.514)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>-0.040 (-0.274, 0.099)</td>
<td>(-0.232, 0.065)</td>
<td>1.733 (0.321, 4.896)</td>
<td>(0.643, 4.531)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.183) (-0.257, 0.087)</td>
<td>(-0.229, 0.058)</td>
<td>(1.112) (0.246, 4.740)</td>
<td>(0.546, 4.405)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.069 (-0.222, 0.278)</td>
<td>(-0.153, 0.255)</td>
<td>0.431 (-1.154, 2.542)</td>
<td>(-0.881, 2.227)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.186) (-0.182, 0.262)</td>
<td>(-0.122, 0.244)</td>
<td>(0.971) (-1.478, 2.633)</td>
<td>(-1.125, 2.196)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.110 (-0.018, 0.367)</td>
<td>(0.024, 0.332)</td>
<td>2.360 (0.593, 4.595)</td>
<td>(0.985, 4.320)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.195) (-0.007, 0.363)</td>
<td>(0.023, 0.331)</td>
<td>(0.946) (0.297, 4.958)</td>
<td>(0.815, 4.558)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.165 (-0.521, 0.029)</td>
<td>(-0.447, 0.028)</td>
<td>1.850 (0.288, 4.055)</td>
<td>(0.654, 3.771)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.211) (-0.527, 0.016)</td>
<td>(-0.492, 0.024)</td>
<td>(0.886) (0.176, 4.144)</td>
<td>(0.465, 3.913)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G6</td>
<td>-0.050 (-0.238, 0.127)</td>
<td>(-0.194, 0.091)</td>
<td>1.983 (0.394, 4.335)</td>
<td>(0.644, 3.969)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.143) (-0.218, 0.110)</td>
<td>(-0.190, 0.078)</td>
<td>(0.976) (0.091, 4.241)</td>
<td>(0.570, 3.934)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Empirical: 2) levels equation: \[ q_t - \bar{q} = \sum_{i=0}^{\infty} E(r^*_t - r_{t+i}) \]
So beta should be >0. Estimates support this.

Table 4—Regression of \( q_t \) on \( \hat{r}^*_t - \hat{r}_t \): \( q_t = \zeta_Q + \beta_Q (\hat{r}^*_t - \hat{r}_t) + u_{q,t+1} \)

<table>
<thead>
<tr>
<th>Country</th>
<th>( \beta_Q )</th>
<th>95% interval</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>46.996 (8.688)</td>
<td>(25.157, 95.390)</td>
<td>(31.793, 90.162)</td>
</tr>
<tr>
<td>France</td>
<td>20.372 (10.854)</td>
<td>(–1.998, 46.182)</td>
<td>(3.549, 42.051)</td>
</tr>
<tr>
<td>Germany</td>
<td>52.410 (12.415)</td>
<td>(0.616, 91.078)</td>
<td>(28.470, 87.010)</td>
</tr>
<tr>
<td>Italy</td>
<td>38.359 (8.042)</td>
<td>(10.971, 73.668)</td>
<td>(15.560, 68.766)</td>
</tr>
<tr>
<td>Japan</td>
<td>19.650 (6.582)</td>
<td>(–2.817, 47.262)</td>
<td>(3.032, 42.822)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>15.744 (7.875)</td>
<td>(–0.793, 36.283)</td>
<td>(4.006, 32.573)</td>
</tr>
<tr>
<td>G6</td>
<td>43.702 (10.124)</td>
<td>(17.664, 80.124)</td>
<td>(23.549, 75.480)</td>
</tr>
</tbody>
</table>

Parameter Estimates and Confidence Intervals
Empirical: 3) cumulative excess returns regression
So beta should be <0. Estimates support this.

\[
\tilde{\beta}_\rho \sum_{j=0}^\infty (\rho_{t+j+1} - \tilde{\rho}) \text{ on } \tilde{\gamma}_t^* - \tilde{\gamma}_t: \tilde{\beta}_\rho \sum_{j=0}^\infty (\rho_{t+j+1} - \tilde{\rho}) = \zeta_\rho + \beta_\rho (\tilde{\gamma}_t^* - \tilde{\gamma}_t) + u_{\rho t}
\]

**Table 5—Regression of \(\hat{E}_t \sum_{j=0}^\infty (\rho_{t+j+1} - \bar{\rho})\) on \(\hat{\gamma}_t^* - \hat{\gamma}_t\): \(\hat{E}_t \sum_{j=0}^\infty (\rho_{t+j+1} - \bar{\rho}) = \zeta_\rho + \beta_\rho (\hat{\gamma}_t^* - \hat{\gamma}_t) + u_{\rho t}\)**

<table>
<thead>
<tr>
<th>Country</th>
<th>(\hat{\beta}_\rho)</th>
<th>95% interval</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-24.762</td>
<td>(-60.281, -10.757)</td>
<td>(-52.700, -15.414)</td>
</tr>
<tr>
<td></td>
<td>(5.523)</td>
<td>(-98.321, -10.812)</td>
<td>(-68.054, -14.849)</td>
</tr>
<tr>
<td>France</td>
<td>-13.983</td>
<td>(-39.998, 3.105)</td>
<td>(-34.960, 0.200)</td>
</tr>
<tr>
<td></td>
<td>(8.268)</td>
<td>(-45.244, 8.814)</td>
<td>(-40.468, 4.248)</td>
</tr>
<tr>
<td>Germany</td>
<td>-33.895</td>
<td>(-62.299, -5.924)</td>
<td>(-58.804, -10.621)</td>
</tr>
<tr>
<td></td>
<td>(10.365)</td>
<td>(-87.170, 3.844)</td>
<td>(-73.809, -4.432)</td>
</tr>
<tr>
<td>Italy</td>
<td>-26.556</td>
<td>(-54.355, -4.446)</td>
<td>(-49.863, -10.649)</td>
</tr>
<tr>
<td></td>
<td>(6.206)</td>
<td>(-64.174, -4.848)</td>
<td>(-57.032, -9.335)</td>
</tr>
<tr>
<td>Japan</td>
<td>-15.225</td>
<td>(-41.927, 2.218)</td>
<td>(-37.617, -2.177)</td>
</tr>
<tr>
<td></td>
<td>(6.487)</td>
<td>(-42.394, -0.325)</td>
<td>(-38.379, -3.176)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-10.717</td>
<td>(-31.865, 3.436)</td>
<td>(-27.130, 1.060)</td>
</tr>
<tr>
<td></td>
<td>(8.565)</td>
<td>(-42.105, 13.602)</td>
<td>(-37.710, 9.599)</td>
</tr>
<tr>
<td>G6</td>
<td>-30.890</td>
<td>(-59.899, -9.893)</td>
<td>(-56.359, -14.642)</td>
</tr>
<tr>
<td></td>
<td>(8.352)</td>
<td>(-68.593, -9.665)</td>
<td>(-60.065, -13.478)</td>
</tr>
</tbody>
</table>
Explain why contradictory

This finding that \( \text{cov} \left( E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, r_t^* - r_t \right) < 0 \) is surprising in light of the well-known uncovered interest parity puzzle. We have documented that when \( r_t^* - r_t \) is above average, foreign deposits tend to have expected excess returns relative to US deposits. That seems to imply that the high interest rate currency is the riskier currency. But the estimates from equation (9) deliver the opposite message—the high interest rate currency has the lower cumulative anticipated risk premium. Since we have found \( \text{cov} \left( E_t \rho_{t+1}, r_t^* - r_t \right) > 0 \) and \( \text{cov} \left( E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, r_t^* - r_t \right) < 0 \), we must have \( \text{cov} \left( E_t \rho_{t+j}, r_t^* - r_t \right) < 0 \) for at least some \( j > 0 \). That is, we must have a reversal in the correlation of the expected one-period excess returns with \( r_t^* - r_t \) as the horizon extends.
Figure 2. Slope Coefficients and 90 Percent Confidence Interval of the Regression:

$$\tilde{E}_t(\rho_{t+j}) = \zeta_j + \beta_j(i^*_t - i_t) + u^j_t$$

$$\rho_{t+1} = (i^*_t - i_t) + (e_{t+1} - e_t)$$
Conclude:

- Models that explain UIP failure in terms of a rise in the currency risk premium when a country’s interest rate is high for short horizons.

- Cannot explain why the implied risk premium switches sign at longer horizons.

- This poses a new challenge to theoretical models trying to explain failure in UIP.