Bergin - homework 2, August 2017

This assignment requires you to work in Dynare, which requires Matlab. You may use the laptops provided by the institute in our classroom or in the computer room of the residence. Or if you have Dynare and Matlab already on your own computer, copy the model file BKKPB2016.mod, downloadable from http://old.econ.ucdavis.edu/faculty/bergin/ECON260D/BKKPB2016.mod
into a directory where Matlab can read it, such as the Matlab directory.

Consider the following two country problem, representing a simplified version of the BKK model from class.

$$Max \;\; E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{1}{2} U\left(C_{H,s}, \left(1 - L_{H,s}\right)\right) + \frac{1}{2} U\left(C_{F,s}, \left(1 - L_{F,s}\right)\right) \right]$$

$$\text{s.t. } C_{H,s} + C_{F,s} + I_{H,s} + I_{F,s} = Y_{H,s} + Y_{F,s}$$

$$Y_{i,s} = A_{i,s} K_{i,s-4}^{\theta} L_{i,s}^{1-\theta} \qquad \text{for } i = H, F$$

$$K_{i,s} = \left(1 - \delta\right) K_{i,s-1} + s_{i,s}$$

$$I_{i,s} = \frac{1}{4} \sum_{j=1}^{4} s_{i,s-j+1}$$

$$NX_{i,s} = \left(Y_{i,s} - C_{i,s} - I_{i,s}\right) / Y_{i,s}$$

$$where \;\; U\left(C_{i,s}, L_{i,s}\right) = \frac{\sigma}{\sigma - 1} \left[C_{i,s}^{\mu} \left(1 - L_{i,s}\right)^{1-\mu} \right]^{\frac{\sigma-1}{\sigma}}$$

$$\begin{bmatrix} \left(A_{H,s} - 1\right) \\ \left(A_{F,s} - 1\right) \end{bmatrix} = \begin{bmatrix} \rho_{HH} & \rho_{HF} \\ \rho_{FH} & \rho_{FF} \end{bmatrix} \begin{bmatrix} \left(A_{H,s-1} - 1\right) \\ \left(A_{F,s-1} - 1\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{H,s} \\ \varepsilon_{F,s} \end{bmatrix}, \qquad E_t \left[\begin{pmatrix} \varepsilon_{H,s} \\ \varepsilon_{H,s} \end{pmatrix} \begin{pmatrix} \varepsilon_{H,s} & \varepsilon_{H,s} \end{pmatrix} \right] = V$$

1) Derive first order conditions for consumption, labor, and capital. Confirm the set of model equations (including first order conditions, constraints, and definitions) match the equations in the Dynare program.

2) Run the Dynare program by entering the commands:
   addpath c:\dynare\4.4.3\matlab (This command might be updated depending on how the institute set up the laptops)
   dynare BKKPB2016

3) Interpret the impulse responses to a home productivity shock for output, labor, investment, consumption, and net exports. Explain the economic logic, in terms of what the agents are doing and why.

4) Report the standard deviations and cross-country correlations for output and consumption. How well do these match the standard deviations in data, as described in the paper by Backus et al. (1992)? Does this model imply the consumption correlation puzzle? Explain the economic logic driving the result.

5) Try running the simulation with alternative values for the intertemporal elasticity (σ) within the usual range of (0.5 , 2). How does this affect results in part 4 above? Does this make a difference for the consumption correlation puzzle?

6) Now consider a more general utility function, where γ is the intratemporal elasticity of substitution between consumption and leisure, and σ remains the intertempral elasticity of substitution.

$$U\left(C_{i,s}, L_{i,s}\right) = \frac{\sigma}{\sigma - 1}\left[\mu C_{i,s}^{\frac{\gamma-1}{\gamma}} + (1-\mu)\left(1 - L_{i,s}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)}$$

Re-derive the first order conditions for consumption and labor, and edit the Dynare code (including the definition of utility on lines 31-32). Simulate the model under various choices of γ. I suggest a starting calibration of γ =1.001 (with σ =0.5 still), and confirm that the results are the same as for the previous model with Cobb-Douglas preferences between consumption and leisure.

Next, try cases with γ =3.5 (try a lower value if your computer can't find a solution with 3.5) and then with γ =0.2. Describe what you find for the consumption correlation. Explain why this happens, in terms of economic logic. I suggest you look at the impulse responses for the variables, and think about the meaning of the international risk sharing condition.