Currency Undervaluation and Comparative Advantage

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Abstract

This paper highlights a tradeoff implied by a policy of export-led growth through currency undervaluation. While undervaluation can foster domestic manufacturing in countries like China by sustaining trade surplus, it also can harm a country's comparative advantage by altering the composition of exports. Undervaluation may discourage specializing in high-value added manufacturing and instead favor specialization in non-differentiated goods with higher price elasticity. A dynamic general equilibrium model of two traded good sectors and capital account restrictions shows that undervaluation can either raise or lower welfare depending on two competing effects on comparative advantage: an elasticity effect versus an agglomeration effect working through firm entry and roundabout production.

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1. Introduction

The coincidence of large reserve accumulation and rapid GDP growth in some Asian economies, most notably China, has raised the question of whether a policy of capital controls, reserve accumulation and currency undervaluation can contribute to export-led growth by sustaining a prolonged trade surplus.[1] Potential mechanisms for how a sustained trade surplus can promote growth include a home market effect (Epifani and Gancia (2017), Corsetti, et al. (2007)) as well as learning-by-doing externalities (Rodrik (2008), Aizenman and Lee (2010), Korinek and Serven (2016), and Choi and Taylor (2022)). However, this paper argues theoretically that there is a potential drawback in such a policy of export-led growth, in terms of undesirable effects on a country's comparative advantage. Gains obtained by raising the overall level of a country's exports may be negated by implications of currency undervaluation for the composition of those exports.

The paper's main argument is that undervaluation may disproportionately encourage production and export of non-differentiated goods, which generally are characterized by a high elasticity substitution and hence greater sensitivity to exchange rates compared to goods classified as differentiated. The undervaluation thus puts at a relative disadvantage the production and export of differentiated goods, which tend to confer certain welfare benefits. As an example, an undervalued currency could help China raise exports and become the workshop of the world in terms of assembly activities. But if China wished to move up the product ladder and become a creator of branded goods, a strategy based on simple price competitiveness may be counterproductive.

To consider this tradeoff between the potential benefits of a net trade surplus and the composition of trade, this paper uses a two-country monetary model with two traded good sectors. One sector consists of differentiated goods, characterized by monopolistic competition, firm entry subject to a sunk cost, and roundabout production. The other sector consists of non-differentiated goods characterized by perfect competition. Given free entry in the differentiated goods sector, the model exhibits the production delocation externality and home market effect studied in the trade literature (see Ossa (2011)) and

[1] See for example, Dooley et al. (2004), Rodrik (2008), Aizenman and Lee (2010), Bacchetta, et al. (2013), and McMillan et al. (2014), Michaud and Rothert (2014)).

Epifani and Gancia (2017)), in which a large home market attracts greater entry of new firms and varieties of differentiated goods. This delocation effect is amplified by a roundabout production structure, in which differentiated goods producers use each other's output as inputs; this implies a virtuous circle of agglomeration, in which greater availability of locally produced inputs lowers production costs of firms, thus conferring even greater comparative advantage in this sector.

Drawing on the international monetary and macroeconomics literature of exchange rate regimes, the model also features a set of policy tools that policy makers can use to peg a nominal exchange rate while sterilizing effects on domestic monetary policies and price levels. This implies that one country can effectively peg its real exchange rate, and can engineer a depreciation in its terms of trade to confer price competitiveness to its exports. These tools include reserve accumulation in the presence of private capital market segmentation (full capital controls) as well as international monetary transfers.[2]

The main findings arising from this model are, first, that currency undervaluation can either encourage or discourage comparative advantage in differentiated goods depending on two competing mechanisms, and second, that the welfare implications of undervaluation depend upon which way comparative advantage shifts. On one hand, while the home net trade surplus implies a rise in demand for all home goods, this will tend to favor the non-differentiated sector, to the extent that we associate product differentiation with lower substitutability between home and foreign products. This would imply that demand for home non-differentiated goods should be more responsive to the given fall in relative price between home and foreign products induced by the undervaluation policy. We refer to this as an elasticity mechanism.

On the other hand, the presence of firm entry and agglomeration in the differentiated sector implies that the home trade surplus created by the undervaluation can create a home market effect that disproportionately promotes production of the differentiated good. In particular, as a rise in demand for home products induces more home firms to enter the differentiated sector and foreign firms to exit, home-produced

[2] In steady state, this policy will take the form of a transfer from the home government to foreign. The adverse wealth effect at home will induce a rise in endogenous labor supply, which lowers home wage and hence the relative price of home exports compared to foreign exports.

varieties occupy a larger share of this market. This firm delocation mechanism is amplified by agglomeration, in which a rise in the number of differentiated goods varieties produced locally further lowers home production costs, since differentiated goods use other differentiated goods as inputs. We refer to this as an agglomeration mechanism.

Model simulations indicate that either of these two mechanisms can dominate, depending on parameter values, so that an undervaluation can foster specialization either in differentiated or non-differentiated goods. In particular, the elasticity effect will tend to dominate in cases with a low share of intermediates in roundabout production and with a high elasticity of substitution between non-differentiated goods. In contrast, the agglomeration effect dominates in cases with a high share of intermediates. Further, we find that in cases with a high intermediates share, the implication of the elasticity mechanism flips, so that a higher elasticity for non-differentiated goods actually reinforces the agglomeration effect and further amplifies home specialization in differentiated goods during currency undervaluation. The reason is that a high degree of production specializing in differentiated goods becomes easier when home consumers can substitute imported foreign non-differentiated goods for home non-differentiated goods no longer produced at home.

The paper finds that the welfare implications of currency undervaluation depend crucially on the direction of the resulting shift in comparative advantage. As is usual in the trade literature studying firm delocation, specialization in differentiated goods confers welfare gains to households, in that consumers do not need to pay trade costs associated with differentiated goods when they are produced at home, thus lowering the consumption price index and raising overall consumption. Ossa (2011) named this the "firm relocation externality." Specialization in non-differentiated goods, on the other hand, implies greater imports of foreign differentiated goods and hence greater trade costs and a negative effect on home welfare. Model simulations indicate that if undervaluation confers a sufficiently strong specialization in differentiated goods, it can lead to a net rise in steady state welfare.

This work is related to the international trade literature studying production delocation and the home market effect. Exchange rate policy is an alternative to the usual tariff policy in enlarging the home market and generating production delocation. One significant difference between exchange rate policy and tariff policy in this regard is that while tariff policy can be targeted to a desired sector such as differentiated goods, an exchange rate undervaluation does not distinguish a-priori between sectors. This is why delocation created by exchange rate policy can either encourage or discourage firm entry in the desired sector, depending on the relative elasticities of substitution.

The paper is related to Epifani and Gancia (2017), which showed in an environment with one traded good (and one nontraded good) that transfers from one country to another can raise domestic welfare by encouraging firm delocation, and that such transfers can be interpreted as a currency depreciation. The present paper differs in several respects. Foremost, it shows important implications for comparative advantage by introducing more than one traded sector. Second, it develops a full dynamic macro environment that considers the dynamic implications of delocation, including the short run costs leading to gains in the steady state. Third, it explicitly models the asset market structure and foreign exchange intervention policy that are used to implement currency undervaluation.

More broadly, the paper relates to the literature discussing how currency undervaluation and capital market restrictions could have contributed to the growth success in China (such as Dooley, et al. (2004), Rodrik (2008), Aizenman and Lee (2010), Bacchetta, et al. (2013), Jeanne (2013), Michaud and Rothert (2014), Korinek and Serven (2016) and Choi and Taylor (2022)). Again, this literature tends to focus on economic environments with one traded good, while the present paper differs in showing side effects of such a growth strategy on comparative advantage between two traded sectors, and the resulting limitations of such a strategy to raise welfare.

This paper shares with Bergin and Corsetti (2020) a focus on comparative advantage between two traded sectors, and the present model draws on the goods market structure of that paper. However, the present paper differs fundamentally in studying the effect of a currency undervaluation, rather than the effect of monetary stabilization policy on risk premia. Consequently this paper abstracts from monetary policy and from a stochastic environment needed to model business cycles.

The next section presents the theoretical model. Section 3 studies some analytical relationships implied by the model, aiding intuition regarding the role of substitution elasticities in the two sectors. Section 4 presents simulations from a benchmark version of the model showing how undervaluation can have adverse effects on comparative advantage in the presence of a highly substitutable non-differentiated good. Section 5 shows how this result changes in the presence of agglomeration arising from roundabout production, and section 6 studies robustness and sensitivity to other key parameters. Section 7 draws implications for policy choices and concludes.

2. Model

Consider a dynamic two-country monetary model suitable for studying the effect of a sustained currency undervaluation on comparative advantage. The model features two tradable good sectors, which differ in terms of their substitutability between domestic and foreign versions. Comparative advantage is measured in terms of the share of each good in domestic production and exports. The model also features capital account restrictions segmenting the international asset market, which permits the home country to pursue a policy of currency undervaluation through reserve accumulation.

2.1 Goods market structure

The goods market specification largely follows that of Bergin and Corsetti (2020), whereby households consume goods produced in two sectors, one consisting of differentiated goods and the other non-differentiated. The differentiated goods come in many varieties, produced by an endogenous number of monopolistically competitive firms in the home and foreign country, n_t and n_t^*. Each variety is an imperfect substitute for any other variety in this sector, of either home or foreign origin, with elasticity ϕ. The non-differentiated goods come in a home and foreign version, which are imperfect substitutes with elasticity η. However, within each country, all goods in this sector are perfectly substitutable with each other, and are produced in a perfectly competitive environment. The differentiated sector will be denoted with a *D* and the non-differentiated sector with a *N*.

The overall consumption index is specified as $C_t \equiv C_{D,t}^{\theta} C_{N,t}^{1-\theta}$, where

$C_{D,t} \equiv \left(\int_0^{n_t} c_t(h)^{\frac{\phi-1}{\phi}} dh + \int_0^{n_t^*} c_t(f)^{\frac{\phi-1}{\phi}} df \right)^{\frac{\phi}{\phi-1}}$ is the index over the endogenous number of home and foreign varieties of the differentiated manufacturing good, $c_t(h)$ and $c_t(f)$, and

$C_{N,t} \equiv \left(\nu^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\nu)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$ is the index over goods differentiated only by country of origin, $C_{H,t}$ and $C_{F,t}$, with $\nu \in [0,1]$ accounting for the weight on domestic goods. The corresponding welfare-based consumption price index is

$$P_t \equiv \frac{P_{D,t}^{\theta} P_{N,t}^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}, \tag{1}$$

where
$$P_{D,t} = \left(n_t p_t(h)^{1-\phi} + n_t^* p_t(f)^{1-\phi} \right)^{\frac{1}{1-\phi}} \tag{2}$$

is the index over the prices of all varieties of home and foreign differentiated goods, $p_t(h)$ and $p_t(f)$, and

$$P_{N,t} = \left(\nu P_{H,t}^{\ 1-\eta} + (1-\nu) P_{F,t}^{\ 1-\eta} \right)^{\frac{1}{1-\eta}} \tag{3}$$

is the index over the prices of home and foreign non-differentiated goods.

The relative demand functions for domestic residents implied from our specification of preferences are as follows:

$$C_{D,t} = \theta P_t C_t / P_{D,t} \text{ and } C_{N,t} = (1-\theta) P_t C_t / P_{N,t} \tag{4a,b}$$

$$c_t(j) = \left(p_t(j) / P_{D,t} \right)^{-\phi} C_{D,t} \text{ for varieties } j = \{h, f\} \tag{5a,b}$$

$$C_{H,t} = \nu \left(P_{H,t} / P_{N,t} \right)^{-\eta} C_{N,t} \text{ and } C_{F,t} = (1-\nu)\left(P_{F,t} / P_{N,t} \right)^{-\eta} C_{N,t} \tag{6a,b}$$

2.2 Households

The representative home household derives utility from consumption (C_t) and from holding real money balances (M_t/P_t), and disutility from labor (l_t). The household derives income from working at the wage rate W_t and from profits rebated from home firms (Π_t). Home households are precluded by government policy from international asset trade, so they are limited to holdings of home currency debt (B_{Ht}) which pays interest rate i_{t-1}, and which can only be traded domestically. Households pay lump-sum taxes (T_t).

Household optimization for the home country may be written:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, l_t, \frac{M_t}{P_t} \right)$$

where utility is defined by

$$U_t = \frac{1}{1-\sigma} C_t^{1-\sigma} + \ln \frac{M_t}{P_t} - \frac{1}{1+\psi} l_t^{1+\psi} ,$$

subject to the budget constraint:

$$P_t C_t + \left(M_t - M_{t-1} \right) + \left(B_{Ht} - B_{Ht-1} \right) = W_t l_t + \Pi_t + i_{t-1} B_{Ht-1} - T_t .$$

Household optimization implies an intertemporal Euler equation:

$$\beta \left(1 + i_t \right) E_t \left[\frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}} \right] = 1 , \tag{7}$$

which in the absence of international asset trade defines the domestic interest rate. Optimization also implies a labor supply condition:

$$\frac{W_t}{P_t} = l_t^{\psi} C_t^{\sigma} , \tag{8}$$

and a money demand condition:

$$\frac{M_t}{P_t} = C_t^{\sigma} \left(\frac{1 + i_t}{i_t} \right). \tag{9}$$

Due to the full asset market segmentation, there is no interest rate parity condition.

The problem and first order conditions for the foreign household are analogous. While they do not face an explicit prohibition on international asset trade from their government, since neither home government nor home households are able to sell home currency bonds internally, foreign households are effectively limited to holding foreign currency bonds (B_{Ft}^{*}) which pay interest rate i_t^{*} .

2.3 Firms in the differentiated goods sector

In the manufacturing sector, the production of each differentiated variety follows

$$y_t \left(h \right) = \alpha_D \left[G_t(h) \right]^{\zeta} \left[l_t \left(h \right) \right]^{1-\zeta} , \tag{10}$$

where $l_t(h)$ is the labor employed by firm *h*, and $G_t(h)$ is a composite of differentiated goods used by firm *h* as an intermediate input. $G_t(h)$ is specified as an index of home and foreign differentiated varieties that mirrors the consumption index specific to differentiated goods ($C_{D,t}$).[3] Productivity, α_D, is common to all firms in the sector.

There is free entry in the sector, but, once active, firms are subject to an exogenous death shock with probability δ. The number of firms active in the differentiated sector, n_t, at the beginning of each period evolves according to:

$$n_{t+1} = (1-\delta)(n_t + ne_t), \tag{11}$$

where ne_t denotes new entrants.

To set up a firm, managers incur a one-time sunk cost, *K*, and production starts with a one-period lag. Entry costs are in units of differentiated goods, allocated over varieties analogously to demands for consumption of differentiated good in equation (5).

Total home demand for a domestic differentiated goods firm is:

$$d_t(h) = c_t(h) + d_{G,t}(h) + d_{K,t}(h) \tag{12}$$

which includes the demand for consumption ($c_t(h)$) by households, and the demand by firms for intermediate inputs ($d_{G,t}(h)$), and firm entry investment ($d_{K,t}(h)$). We assume iceberg trade costs τ_D for exports, so that market clearing for a firm's variety is:

$$y_t(h) = d_t(h) + (1+\tau_D)d_t^*(h), \tag{13}$$

where $d_t^*(h)$ is the analogous foreign demand for home variety *h*. Firm profits are computed as:

$$\pi_t(h) = p_t(h)d_t(h) + e_t p_t^*(h)d_t^*(h) - mc_t y_t(h), \tag{14}$$

where $mc_t = \zeta^{-\zeta}(1-\zeta)^{\zeta-1} P_{D,t}{}^{\zeta} W_t^{1-\zeta} / \alpha_{D,t}$ is marginal cost.[4]

Thus the value function of firms that enter the market in period *t* may be represented as the discounted sum of profits of domestic sales and export sales:

[3] This specification of roundabout production, involving the differentiated goods sector but not the non-differentiated sector, goes back to the specification of Krugman and Venables (1995), which was designed to create agglomeration specifically in the manufacturing sector, which is central to this paper's result as well.

[4] This marginal cost is implied by the optimal combination of inputs derived from cost minimization using the production function in equation (10).

$$v_t(h) = E_t\left\{\sum_{s=0}^{\infty}\left(\beta(1-\delta)\right)^s \frac{\mu_{t+s}}{\mu_t}\pi_{t+s}(h)\right\}, \tag{15}$$

where we assume firms use the discount factor of the representative household, who owns the firm, to value future profits. With free entry, new producers will invest until the point that a firm's value equals the entry sunk cost:

$$v_t(h) = P_{D,t}K, \tag{16}$$

where K is specified as an index of differentiated varieties that mirrors the consumption index specific to differentiated goods ($C_{D,t}$). By solving for cost minimization we can express the relative demand for labor and intermediates as a function of their relative costs:

$$\frac{P_{D,t}G_t(h)}{W_t l_t(h)} = \frac{\zeta}{1-\zeta}. \tag{17}$$

And we can solve for the optimal price setting by the firm:

$$p_t(h) = \frac{\phi}{\phi-1}mc_t. \tag{18}$$

where mc is marginal cost defined above. The good price in foreign currency moves one-to-one with the exchange rate:

$$p_t^*(h) = (1+\tau_D)p_t(h)/e_t, \tag{19}$$

where recall the nominal exchange rate, e, measures home currency units per foreign.

Note that, since households own firms, they receive firm profits but also finance the creation of new firms. In the household budget, the net income from firms may be written:

$$\Pi_t = n_t\pi_t(h) - ne_t P_{D,t}K.$$

In reporting quantitative results, we will refer to the overall home gross production of differentiated goods defined as: $y_{D,t} = n_t y_t(h)$, using the fact that all firms are the same size.

2.4 Firms in the non-differentiated sector

In the second sector firms are assumed to be perfectly competitive in producing a good differentiated only by country of origin. The production function for the home non-differentiated good is linear in labor:

$$y_{H,t} = \alpha_N l_{H,t} . \tag{20}$$

It follows that the price of the homogeneous goods in the home market is equal to marginal cost:

$$P_{H,t} = W_t / \alpha_N . \tag{21}$$

An iceberg trade cost specific to the non-differentiated sector implies prices of the home good abroad are

$$P^*_{H,t} = P_{H,t}\left(1+\tau_N\right)/ e_t . \tag{22}$$

Analogous conditions apply to the foreign non-differentiated sector.

2.5 Government policies

The home government issues money (M_t) and home currency bonds (B^s_{Ht}), and levies lump sum taxes on domestic households (T_t). The home government has the ability to purchase foreign currency bonds in the international asset market, to hold as foreign currency reserves (R_{Ft}). The model also allows for inter-governmental transfers (X_t), defined in foreign currency units, and defined as positive when home is the giver. The home government faces the following budget constraint:

$$T_t + \left(M_t - M_{t-1}\right) + \left(B^s_{H,t} - \left(1+i_{t-1}\right)B^s_{H,t-1}\right) = e_t\left(R_{F,t} - \left(1+i^*_{t-1}\right)R_{F,t-1}\right) + e_t X_t , \tag{23}$$

The corresponding budget constraint for the foreign government is:

$$T^*_t + \left(M^*_t - M^*_{t-1}\right) + \left(B^{s*}_{F,t} - \left(1+i^*_{t-1}\right)B^{s*}_{F,t-1}\right) + X_t = 0 ,$$

where $B^{s*}_{F,t}$ is the issuance of foreign currency bonds by the foreign government.

The home government policy of international asset controls and sterilization of foreign exchange operations is similar to the model in Chang, Liu and Spiegel (2015), designed to represent Chinese-style capital account polices.[5] As in their case, the home country's net foreign assets are equal to its reserves, and the level of reserves completely determines the trade balance and the real exchange rate.

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[5] The model simplifies several details relative to Chang et al. (2015), such as assuming the capital market is completely closed, the home government issues no bonds, and monetary policy and sterilization work through direct transfers to domestic households rather than bond issuance.

The closed capital market allows the home government to adjust reserves as needed to target a desired nominal exchange rate. Given that capital controls and sterilization of foreign exchange operations free monetary policy in each country to prevent changes in domestic price indexes, we can view the exchange rate policy in the home country as targeting a desired real exchange rate:

$$rer_t = \overline{rer}, \tag{24}$$

where the real exchange rate is defined as usual: $rer_t = e_t P_t^* / P_t$. Currency devaluation will imply accumulation of foreign currency reserves by the home government. The closed capital account prevents private asset trades from undoing the effects of reserves accumulation on the exchange rate.

We specify the government fully sterilizes the foreign exchange operations to insulate the domestic money supply, which is held constant:

$$M_t = \overline{M}. \tag{25}$$

Given the lack of nominal frictions in the model, the specification of monetary policy is irrelevant to the results reported below.[6] We further assume that the home government holds constant its supply of domestic currency bonds:

$$B_{H,t}^s = \overline{B_H^s}. \tag{26}$$

Given the fixed money and bond supplies, the home government budget constraint implies that the purchase of reserves is paid for by taxes on home households.

Since the exchange rate policy of the home country in simulation experiments will imply indefinite accumulation of reserves to maintain an undervalued currency, a mechanism must be specified to ensure stationarity of reserves levels. The model follows Korinek and Serven (2016) in specifying that a portion of debt claims in reserves will be forgiven. We model this in the form of international transfers set by a policy rule responding to the level of reserves:[7]

$$X_t = \psi R_{Ft-1}. \tag{27}$$

[6] It is nonetheless useful to use money as a numeraire in the model, given the fact there are multiple traded goods.

[7] Results are robust to alternative specifications of this rule, such as transfers fully deferred to a distant future period. This could be interpreted as a partial default by the foreign country on claims against it held by the home country as reserves.

The activity of the foreign government is modeled as simply as possible. The foreign government holds foreign money supply and government issued foreign-currency bonds constant ($M_t^* = \overline{M^*}$, $B_{Ft}^{s*} = \overline{B_F^{s\,*}}$).

2.6 Market clearing

The market clearing condition for the differentiated goods market is given in equation (13) above. Market clearing for the non-differentiated goods market requires:

$$y_{H,t} = C_{H,t} + \left(1+\tau_N\right)\left(C_{H,t}^*\right). \tag{28}$$

Labor market clearing requires:

$$l_t = n_t l_t\left(h\right) + l_{H,t}. \tag{29}$$

Given the prohibition on home households purchasing foreign bonds or exporting domestic bonds, bond market clearing requires:

$$B_{Ht} = B_{Ht}^s \tag{30}$$

for the home bond, and

$$B_{Ft}^* + R_{F,t} = B_{Ft}^{s*} \tag{31}$$

for the foreign bond.

Combining household, firm and government budget constraints along with the goods market clearing condition implies a balance of payments constraint:

$$\int_0^{n_t} e_t p_t^*\left(h\right) d_t^*\left(h\right) dh + e_t P_{Ht}^* C_{H,t}^* - \int_0^{n_t^*} p_t\left(f\right) d_t\left(f\right) df - P_{F,t} C_{F,t} = e_t\left(R_{Ft} - \left(1+i_{t-1}^*\right) R_{Ft-1}\right) + e_t X_t. \tag{32}$$

This states that a home trade surplus will imply an accumulation of home reserves or net transfers.

2.7. Model equilibrium

Equilibrium is defined as sequences of the following 35 home-country variables -- P_t, $P_{D,t}$, $P_{N,t}$, $P_{H,t}$, $P_{H,t}^*$, $p_t\left(h\right)$, $p_t^*\left(h\right)$, $C_{D,t}$, $C_{N,t}$, $C_{H,t}$, $C_{F,t}$, $c_t(h)$, $c_t(f)$, $d_{G,t}(h)$, $d_{G,t}(f)$, $d_{K,t}(h)$, $d_{K,t}(f)$, C_t, l_t, i_t, $l_t(h)$, $G_t(h)$, $y_t(h)$, $\pi_t(h)$, $v_t(h)$, n_t, ne_t, $d_t(h)$, $y_{H,t}$, $l_{H,t}$, W_t, B_{Ht}, M_t, T_t, $B_{H,t}^s$ -- along with their 35 foreign-country counterparts, as well as $R_{F,t}$, X_t and the nominal exchange rate, e_t, satisfying the following 35 home-country equilibrium conditions -- price indexes (1, 2, 3), price setting rules (21, 22, 18, 19), demand conditions (4a, 4b, 5a, 5b, 6a, 6b), demand conditions analogous to (6a) and (6b) for differentiated varieties used in intermediate input, demand conditions analogous to (6a) and (6b) for differentiated varieties used in the entry cost, consumption Euler (7), labor supply (8), money demand (9), production function (10), choice between production factors (17), market clearing for differentiated variety (13), definition of firm profit (14) and firm value (15), firm entry condition (16), firm number law of motion (11), definition of home demand facing a variety (12), production function for non-differentiated good (20), market clearing for non-differentiated good (28), labor market clearing (29), government budget constraint (23), money supply rule (25), government bond supply rule (26), home bond market clearing condition (30) -- along with their foreign counterparts, plus the home transfer rule (27), exchange rate policy rule (24), and the balance of payments condition (32).

See the Appendix for a full listing and discussion of equilibrium conditions of the dynamic system, definition of steady state, model solution methodology.

2.8. Parameterization for numerical experiments

See Table 1 for a summary of parameter values. Risk aversion (reciprocal of time preference) is set at $\sigma = 2$ as is common in the business cycle literature. Labor supply elasticity is $1/\psi = 1.9$ following Hall (2009). Time preference is set at $\beta = 0.96$, consistent with an annual frequency.

Parameters for the differentiated and non-differentiated sectors are taken from Bergin and Corsetti (2020), which in turn are based on estimates from Rauch (1999). We choose θ so that differentiated goods represent 55 percent of trade in value. This is the average estimate over time periods in Table 2 of Rauch (1999), showing the shares of differentiated goods in the value of overall trade in a dataset consisting of 63 countries, taken from the World Trade Database. Rauch (1999) classifies commodities at the three and four-digit SITC level into three categories -- organized exchange, reference priced, and differentiated -- based on examination of commodity market handbooks and yearbooks. To set the elasticities of substitution among the differentiated goods we draw on the estimates by Broda and Weinstein (2006), classified by sectors based on Rauch (1999). Their estimate of the elasticity of substitution between differentiated goods varieties is ϕ=5.2 (the sample period is 1972-1988).

We will consider a range of parameterizations for the elasticity of substitution between non-differentiated goods. One parameterization is taken from Broda and Weinstein (2006), whose estimate is η = 15. Another common parameterization comes from the real business cycle literature, in particular Backus et al. (1992), which uses a lower elasticity of substitution between home and foreign goods, η =1.5. Finally, we will also consider the effects of a higher degree of substitutability by doubling the Broda-Weinstein estimate, η = 30, which is at the upper bound of values of this parameter for which we can compute a numerical model solution.

The firm death rate is set at $\delta = 0.1$, which is four times the standard rate of 0.025 to reflect the annual frequency. The mean sunk cost of entry is normalized, $\overline{K}$ =1.[8] The benchmark share of intermediates in differentiated goods production is set to ζ =1/3, from Bergin and Corsetti (2020), but, sensitivity analysis will consider a range of values.[9]

The two countries are of equal size with no exogenous home bias, $\nu = 0.5$, but the model allows trade costs to determine home bias ratios.[10] Regarding trade costs, τ_D is set so that exports represent 26% of GDP, as is the average in World Bank national accounts data for both China and the OECD average from 2001-2019.[11] In model simulation, this requires a value of τ_D =0.33.[12] This is similar to the value of trade costs typically assumed by macro research, such as 0.25 in Obstfeld and Rogoff, 2001. But it is small compared to some trade estimates, such as 1.7 suggested by Anderson and van Wincoop (2004), and adopted

[8] Changing the value of sunk entry cost simply rescales levels of variables in a way that leaves the percent changes in responses to undervaluation unaffected.

[9] There is a wide range of views regarding the appropriate calibration for this parameter. Jones (2007) suggests a value of 0.43 for the share of intermediates, and it is common in the related literature to use a value at 1/2. We will consider a range of values for this parameter in sensitivity analysis, but we use a modest value for our benchmark model, as this facilitates successful numerical solution for the broad range of values for substitution elasticities, which are the main focus of our numerical analysis.

[10] Maintaining symmetry between countries helps make transparent the effect of asymmetric policies on the equilibrium; an asymmetric calibration would introduce other factors shaping comparative advantage in trade by implying a larger home market effect for one country.

[11] See https://data.worldbank.org/indicator/NE.EXP.GNFS.ZS?locations=OE. The value for China is 25.7, and for OECD 25.6.

[12] To coincide with standard accounting definitions, differentiated goods used as intermediates are included in the measure of exports, and excluded in the measure of GDP.

by Epifani and Gancia (2017). We follow Bergin and Corsetti (2020) in setting the benchmark trade cost in the non-differentiated sector as $\tau_N = 0$. Sensitivity analysis will consider alternative calibrations of trade costs in both sectors.

Calibration of the parameter governing international transfers, ψ, is adjusted in each experiment to consistently imply a steady state ratio of reserves to GDP of 0.81, which is the average ratio for Chinese data over the 2006-2019 span, which is the range of data available from IMF International Financial Statistics.

For simplicity, and without loss of generality, the money and government bond supplies are set at: $\overline{M} = \overline{M^*} = 0$ and $\overline{B_H^s} = \overline{B_F^{s*}} = 0$.

3. Some analytical relationships

This section derives analytical relationships from the model above, to help develop intuition for the key mechanics driving simulation results to follow. To facilitate transparency of the central mechanism and analytical tractability, the model is simplified to abstract from roundabout production, firm entry, and entry costs (ς=0, $\overline{K}$=0). A statistic of particular interest in simulations below is the share of differentiated goods in a country's exports. Let us define a ratio of a country's differentiated exports to non-differentiated exports -- $\Omega_{Ht} \equiv \dfrac{n_t\, p_t^*(h) c_t^*(h)}{P_{H,t}^* C_{H,t}^*}$ for home, and $\Omega_{Ft} \equiv \dfrac{n_t^*\, p_t(f) c_t(f)}{P_{Ft} C_{F,t}}$ for foreign -- and then compute the ratio between the two countries: $\Omega_t = \Omega_{Ht} / \Omega_{Ft}$. A value of this ratio exceeding unity indicates that home specializes in differentiated goods more than the foreign country.

Substituting into the ratio the demand equations from above (Eqn. (5) for $c_t(f)$, Eqn. (6) for $C_{F,t}$, and Eqn. (4) for $C_{D,t}$ and $C_{N,t}$, along with their foreign counterparts):

$$\Omega_t = \frac{\nu n\left(p_t^*(h) / P_{D,t}^*\right)^{1-\phi} \theta P_t^* C_t^*}{(1-\nu)\left(P_{H,t}^* / P_{N,t}^*\right)^{1-\eta} (1-\theta) P_t^* C_t^*} \Bigg/ \frac{\nu n^*\left(p_t(f) / P_{D,t}\right)^{1-\phi} \theta P_t C_t}{(1-\nu)\left(P_{F,t} / P_{N,t}\right)^{1-\eta} (1-\theta) P_t C_t}.$$

Cancel common terms within a country across sectors, and across sectors within a country:

$$\frac{n\left(p_t^*(h) / P_{D,t}^*\right)^{1-\phi}}{\left(P_{H,t}^* / P_{N,t}^*\right)^{1-\eta}} \Bigg/ \frac{n^*\left(p_t(f) / P_{D,t}\right)^{1-\phi}}{\left(P_{F,t} / P_{N,t}\right)^{1-\eta}}.$$

Further simplification becomes possible if we assume a case with no trade costs or home bias ($\tau_D = \tau_N = 0,\ \nu = 0.5$), so that sector price indexes cancel from the expression above ($P^*_{D,t} = P_{D,t} / e_t$, $P^*_{N,t} = P_{N,t} / e_t$):[13]

$$\frac{n\left(e_t p^*_t(h)\right)^{1-\phi}}{\left(e_t P^*_{H,t}\right)^{1-\eta}} \Bigg/ \frac{n^* p_t(f)^{1-\phi}}{P_{F,t}^{1-\eta}}.$$

Substitute in for prices from price-setting equations (Eqns. (18-19) for $p^*_t(h)$, and Eqns. (21-22) for $P^*_{H,t}$), which are all functions of wages in their respective countries, then normalize technology terms ($\alpha_{D,t} = \alpha_{N,t} = 1$) and rearrange:

$$\Omega = \frac{n}{n^*}\left(\frac{eW^*}{W}\right)^{\phi-\eta}. \tag{33}$$

This condition makes transparent several lessons about factors in this model that determine a country's export specialization. First, the condition above indicates that one key determinant of this specialization is the relative wage across countries, eW^*/W, sometimes referred to as the "terms of labor."[14] A fall in the home relative wage rate leads to a proportional fall in the home price of both differentiated and non-differentiated goods compared to their foreign counterparts, as shown in the price setting rules above, thus lowering the home terms of trade. In the context of this model, this terms of labor can be viewed as a policy variable targeted by the exchange rate and monetary policy rules – if the home reserves policy rule targets the nominal exchange rate, while the monetary policies in each country target domestic nominal wages.[15] An alternative but equivalent way to understand how policy targets the terms of labor is that governments set the international transfer, X_t, to target it: a transfer from home to foreign country raises home labor supply

[13] The assumption of no trade costs is not strictly necessary for this result, but is useful for tractability; it is not intended to show that trade costs have no impact on our result. Simulation results to follow will quantify the contribution of trade costs. Further, while this assumption implies Purchasing Power Parity holds and so the CPI-based real exchange rate cannot vary, nonetheless, the terms of trade between countries can vary. The government reserves and transfers policy can still lower the relative price of home exports to home imports, as explained below. The mechanism in this case is that the transfer from home to foreign raises home labor supply due to the negative wealth effect, which in turn lowers the home wage; the opposite occurs abroad.
[14] Ghironi and Melitz (2005).
[15] Given home capital controls, the home government has the ability to use taxes to sterilize the effect of reserves accumulation of domestic nominal variables such as the home wage.

due to the adverse wealth effect, which puts downward pressure on home wages; the opposite occurs abroad in response to the receipt of the transfer.[16]

A second lesson is that which sector a change in the terms of labor confers specialization upon depends crucially on the relative elasticity of the two sectors, $\phi - \eta$. While a rise in eW^*/W confers a proportional price advantage to both home differentiated and non-differentiated goods compared to foreign counterparts, it will have a larger effect on the demand of differentiated goods, and hence raise the home share of differentiated exports, if the substitution elasticity in this sector is greater than that in the non-differentiated sector ($\phi > \eta$). But given that we usually think of product differentiation as associated with less substitutability with competing products, we are more likely to see the opposite situation ($\phi < \eta$), which implies that a home terms of trade depreciation will favor specialization in exports of the non-differentiated good, thus lowering the Ω ratio.

A third lesson is that another factor favoring home specialization in differentiated goods is a larger number of firms in this sector compared to the foreign country. Since consumers do not distinguish between home and foreign suppliers, but prefer to spread purchases over all producers (assuming equal prices), a greater number of home producers leads to home occupying a larger share of the global differentiated goods market. Potentially if a home country exchange rate policy stimulates domestic firm entry, condition (33) shows that it can raise home specialization in differentiated goods. However, the analytical result above takes the number of firms as exogenous. Numerical solutions to follow will solve for the endogenous number of firms, and show exactly how it is affected by exchange rate policy.

4. Simulation of benchmark model without intermediates

The experiment consists of the home country pegging its real exchange rate at 1.01, which is an undervaluation by 1% relative to the ratio of 1.00, which otherwise would be

[16] This mechanism, by which transfers induce changes in labor supply, is discussed in Corsetti et al. (2013) in relationship to the transfer problem. Epifani and Gancia (2017) also discuss how such an endogenous labor supply rise in response to a home transfer can work to amplify the firm delocation mechanism in an extension to their model, section 4.4.2.

the equilibrium in the symmetric steady state.[17] We initially study a case with no roundabout production and hence no use of differentiated goods as intermediates ($\varsigma = 0$); the subsequent section will consider a case with intermediates.

4.1 Implications for comparative advantage

Begin by considering the effect of the undervaluation policy on steady state. As a baseline for comparison, the first column of Table 2 reports the case when the exchange rate is pegged at 1.00, which implies complete symmetry in steady states across the two countries. Significant asymmetries appear when the home country instead pegs at 1.01, representing home undervaluation. Columns (2-5) report the percent change in steady state values for a set of variables due to currency undervaluation relative to the symmetric steady state in column (1).[18] Columns differ from each other in terms of the calibration of the elasticity of substitution between home and foreign non-differentiated goods. [19]

We begin by analyzing the case in column (2) with elasticity η=1.5, taken from Backus et al. (1992). Undervaluation raises home production of both goods (a positive change from the symmetric case reported for y_D and y_H), while production of both goods falls in the foreign country. This rise in overall home production follows from the home trade surplus generated by the currency undervaluation. However, home production rises more for the differentiated good sector than for non-differentiated. The share of differentiated goods in overall home GDP rises 1.1% compared to the symmetric case of no currency undervaluation; the foreign share falls 1.2%. This asymmetry reflects a home

[17] There is no consensus in the empirical literature regarding the relevant magnitude of undervaluation in China, due in part to the difficulty in estimating the equilibrium real exchange rate. For example, a prominent estimate in Cheung, et al. (2007) suggests an undervaluation of 50%, though an alternate estimate using revised data in Cheung, et al. (2011) suggest a smaller value around 10%. We study a small undervaluation in the numerical experiments, because it makes it possible to obtain a model solution under a range of values for the elasticities of interest. Robustness checks to follow will consider undervaluations of larger magnitude.

[18] The table does not report values for interest rates i and i^* since they are constant at $1/\beta - 1$ in steady state. Although not reported in the table due to space constraints, the currency undervaluation implies a rise in home reserves as a ratio to GDP by 81 percentage points, as required by the calibration explained above. It also implies a rise in home taxes as a share of GDP by 3.29 percentage points, in order to pay steady state international transfers.

[19] The values reported in column (1) are the same for all values of elasticity η, since the steady state under an exchange rate of 1.0 is symmetric across countries, so there is no difference between prices of home and foreign goods to make this price elasticity matter.

comparative advantage in differentiated goods, which is also reflected in a rise in the share of differentiated goods in home exports by 4.8% and corresponding fall for foreign.[20]

The shift in specialization between sectors does not come from the usual comparative advantage mechanism involving differences in relative prices of goods across sectors and countries. The undervalued currency makes all home exports, both in differentiated and non-differentiated sectors, cheaper by the same percentage. So if we compute a ratio of home to foreign differentiated export prices, relative to home to foreign non-differentiated prices $\left(\left(p_t^*(h)/P_{H,t}^*\right)/\left(p_t(f)/P_{F,t}\right)\right)$, the table shows that this ratio does not change for an undervalued exchange rate.

Instead, the rise in home specialization in differentiated goods arises from a mechanism described in the trade literature as "firm delocation." or a "home market effect" (see Epifani and Gancia (2017) for a recent application).[21] The positive home trade balance creates a rise in the overall demand facing home producers, which encourages more firm entry in the home market, since the benefit of entry in terms of profits exceeds the sunk entry cost. The home country thus represents a greater share of the total varieties of differentiated goods in global production and trade. The simulation results show a clear firm delocation, with a rise in the number of home firms by 5.2% and a corresponding fall in the number of foreign firms (5.1%). One way to see the central role of firm entry in driving our results for comparative advantage above is to note that the rise in overall home production of differentiated goods, 4.6%, is more than fully accounted for by the rise in number of home firms, 5.2%.[22]

A novel result seen in the simulations is that the impact of currency undervaluation on comparative advantage depends critically on the degree of substitutability between home and foreign non-differentiated goods, η. In particular, a higher value for this

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[20] The share of differentiated goods in home exports is computed as $\omega_{Ht} \equiv \dfrac{n_t\, p_t^*(h) d_t^*(h)}{n_t\, p_t^*(h) d_t^*(h) + P_{H,t}^* C_{H,t}^*}$; and $\omega_{Ft} \equiv \dfrac{n_t^*\, p_t(f) d_t(f)}{n_t^*\, p_t(f) d_t(f) + P_{Ft} C_{F,t}}$ for foreign.

[21] Earlier discussion of this mechanism include, among many others, Corsetti, et al. (2007) and Ossa (2011), and reach back to Krugman (1980).

[22] The rise in firm numbers is larger even than the rise in sales because the sunk cost of new firm entry is in units of differentiated goods, whose average price falls with more domestic varieties available.

elasticity dampens, and potentially even inverts, the effect of undervaluation on our metrics of home comparative advantage in differentiated goods.

To demonstrate this point, we next study the case of a higher elasticity, η=15, based on estimates in Broda and Weinstein (2006). Simulation results in column (3) of Table 2 show that undervaluation still raises the number of home firms, but by a smaller percentage than under the lower elasticity: by 5.0% versus 5.2% when comparing columns (3) and (2). This results from the much greater stimulation of demand for non-differentiated goods comparing the two elasticities, by 5.4% rather than 0.45%. The share of differentiated goods in exports rises by half as much (2.2% versus 4.8%), and the share of differentiated goods in production actually falls rather than rises (falling by 0.62% rather than rising 1.1%).

One can consider yet higher elasticities. In principal, letting the elasticity approach infinity, so that the home and foreign goods become perfect substitutes, mimics the homogeneous good structure typically employed in trade models. For purposes of comparison, Table 2 reports results for the case of an elasticity, η= 30 (column (4)), which is the highest value for which I can compute a numerical solution for equilibrium. The bias of demand for non-differentiated goods is even stronger than the previous two cases, and now both the home shares of differentiated goods in exports and in production move in a negative direction. Figure 1 shows how both of these two measures of home specialization in differentiated goods fall progressively as the substitution elasticity increases.

The non-monotonicity in the effect of currency undervaluation can be attributed to two competing mechanisms. While the firm delocation mechanism discussed above favors specialization in differentiated goods, there is also a second mechanism favoring the opposite specialization, in non-differentiated goods. The logic for this second offsetting mechanism is simple, but not standard to the literature. While a currency undervaluation makes both differentiated and non-differentiated home goods cheaper than their foreign counterparts, if we assume that a higher degree of substitutability is associated with less product differentiation, this means that the demand for home non-differentiated goods shifts more strongly from foreign to home goods in response to a given drop in home price. This implies that, in the absence of a firm delocation mechanism, a currency undervaluation should in general foster specialization in the non-differentiated good, and the strength of this mechanism grows as the cross-border price elasticity for non-differentiated goods rises relative to that for differentiated goods.

One way to partly disentangle the distinct effects of these two mechanisms is to consider a case in which firm entry is suspended, and the number of firms is fixed at the level from the symmetric equilibrium. Column (5) reports the case for elasticity $\eta = 15$. Compared to the case with free firm entry (column (3)), the effect of the undervaluation on home export specialization in differentiated goods flips sign to become negative, and the effect on production specialization becomes more strongly negative, by an order of magnitude. Without firm entry to facilitate production delocation, the elasticity mechanism clearly dominates.

This section concludes with a discussion of how the firm delocation mechanism differs when fostered by a policy of currency undervaluation, compared to the more familiar context of tariff-induced delocation. A tariff can target a particular sector, say by imposing a tariff specifically on foreign differentiated goods and not non-differentiated goods. This degree of sector specificity is not possible for currency undervaluation, which impacts the cross-country relative price of all sectors equally. It is perhaps surprising, then, that a strong delocation effect is still possible for the cases of currency undervaluation. The assumption that free firm entry applies only to the case of differentiated goods in our model provides a rationale for why a common drop in price can disproportionately benefit production in the differentiated goods sector, and shift specialization in this direction. This assumption is generally thought reasonable, given that the profits arising from imperfect competition in the differentiated goods are logically justified in terms of the need to pay a sunk cost of firm entry to create a differentiated variety.

4.2 Welfare implications

Next turn to welfare implications of the undervaluation policy. Table 2 shows that the steady state level of consumption is lower following currency undervaluation, and labor is higher (leisure lower) for all elasticities, both of which would suggest a lower steady state level of utility. This is confirmed in Table 2, where steady state utility is reported as a percent change in consumption units, that is, the number of units of consumption a household would forego in order to move from a steady state where the exchange rate is unity to one where the exchange rate is 1.01.[23] Column (2) shows that home utility falls 3.4 percent for the case of a low elasticity (η=1.5). The utility loss grows larger for higher elasticities of substitution, which we saw above are associated with a deterioration in home specialization in differentiated goods.

In order to provide a more complete measure of the total welfare effects of undervaluation over time, one can conduct a dynamic simulation of a policy change. Assume the economy starts at a steady state where the exchange rate was pegged at 1.0, but beginning in period 1, policy is changed to peg at a value of 1.01, implying a 1% undervaluation. The simulation tracks the evolution of state variables until the new steady state is approached, and we compute welfare as the present discounted sum of utility over 200 periods. Figure 2a shows the dynamics of household welfare over these periods, indicating an initial fall in home utility relative to the steady state under the old policy, followed by a gradual rise in home utility to a new steady state level, which still remains lower than the steady state under the old policy. The bottom of row of Table 2 reports the present discounted sum of utility over the periods of the simulation, converted to units of steady state consumption. It shows a fall in welfare of 3.5% consumption units for the case of elasticity η=1.5, and a fall of 7.8% for an elasticity of η=15.

Part of the home welfare loss can be attributed to international transfers associated with sustaining currency undervaluation, which amount to 6.7% of GDP each year paid by the home government to foreign. This transfer reflects the need for the home country to purchase international reserves to maintain the undervalued real exchange rate, and then to forgive a fraction of the debt obligation in order to prevent explosive growth in debt. This inter-governmental transfer then implies a rise in home taxes and fall for foreign taxes, which lowers the wealth of home households. A second channel by which undervaluation affects welfare is the effect on firm delocation, which is the main focus of this paper. Ossa (2011) coined the "production relocation externality", in which specialization in the differentiated goods sector reduces the domestic price index by ensuring that less of the goods consumed by domestic consumers are subject to trade costs. We attempt to disentangle these two channels. When we filter out the effect of firm delocation, by holding

[23] This comes from solving for the value of Δ satisfying $(1-\beta)\sum_{t=0}^{\infty}\beta^t U_t^{e=1.01} = \frac{1}{1-\sigma}\left(\bar{C}(1-\Delta/100)\right)^{1-\sigma} - \bar{l}^{1+\psi}/(1+\psi)$.

constant n at the initial steady state value, Figure 2a shows a constant fall in welfare for all periods starting the period after the change in exchange rate target. This is the effect of the international transfer, which under the benchmark specification in equation (27) rises in the period immediately after reserves increase, and then remains constant, as does the level of reserves.[24] Relative to this constant utility loss from transfers, the figure shows that introducing firm dynamics initially lowers welfare further in the initial period. This reflects the fact that investment in new firms is costly in terms of lost consumption, especially since capital controls prevent households from borrowing abroad to finance the investment expenditure. But as the rise in firm number allows a rise in production, consumption and hence utility rise. This rebound is not enough, however, under this parameterization to fully compensate for the loss in utility from the transfer, and the level of utility in the new steady state remains below that in the initial steady state.

5. Results with intermediates and agglomeration

Consider next a version of the economic environment where differentiated goods are used as intermediates in roundabout production of other differentiated goods. This introduces a type of firm agglomeration: as more firms locate domestically, domestic producers of differentiated goods enjoy lower production costs, as they avoid the trade costs associated with imported intermediates. Recall that the specification of production allows for a share, ζ, of differentiated goods used as inputs. The benchmark calibration in Table 2 sets $\zeta = 0$; Table 3 considers a calibration of $\zeta = 0.33$ (taken from Bergin and Corsetti (2020)).

Table 3 shows that undervaluation now robustly leads to a rise in home specialization in differentiated goods. For all elasticities shown in Table 3, the rise in number of home firms is larger than the corresponding case without intermediates in Table 2. For example, for $\eta=1.5$, home firm number rises by 6.6% instead of the 5.2% noted above. Further, increasing the elasticity of substitution (η) now increases the degree of

[24] The onset of this permanent fall in welfare can be postponed if we postpone the onset of the transfer payments with a transfer rule with a larger lag. For example, simulation experiments with transfer rule $X_t = \psi e_t R_{Ft-100}$ imply an equilibrium path in which transfers and the associated welfare loss do not begin until period 100.

firm delocation: as one moves from column (2) to (4) in Table 3, the percentage rise in home firms progressively increases, as does the effect on the shares of differentiated goods in both home exports and production.[25] Remarkably, this result is the opposite of what was observed in Table 2 in the absence of intermediates, where higher elasticity dampened delocation. Under this calibration of intermediates share, there now is no elasticity under which devaluation moves in a negative direction any of the metrics of home comparative advantage in differentiated goods. Figure 3 shows how these measures of firm delocation rise progressively with the substitution elasticity. Firm delocation also rises with a higher share of intermediates (ζ), as shown in Figure 4.

The reason the effect of the substitution elasticity on firm delocation flips sign in the presence of intermediates is that substitution between home and foreign non-differentiated goods has two distinct channels for affecting firm delocation. On one hand, a high degree of substitutability between home and foreign non-differentiated goods makes this sector more responsive to the drop in home export prices, thus conferring home comparative advantage to the non-differentiated sector rather than the differentiated sector. But on the other hand, a high degree of substitutability can promote delocation by making it easier for the home country to specialize in the non-differentiated sector, and letting home consumers import foreign non-differentiated goods in place of domestic versions.

The presence of intermediates strengthens the pro-delocation effect through agglomeration, since it creates a positive feedback cycle of delocation and comparative advantage. As firm production of differentiated goods shift to home, the price index of differentiated goods drops at home, which then lowers the production costs of differentiated goods that rely on other differentiated goods as inputs. Table 3 shows that the home export price of differentiated goods, $p_t^*(h)$, now falls more than that for non-differentiated goods ($P_{H,t}^*$), creating a comparative advantage in differentiated goods seen in $\left(\left(p_t^*(h)/P_{H,t}^*\right)/\left(p_t(f)/P_{F,t}\right)\right)<1$, which was not observed for any column in Table 2.

The stronger delocation and agglomeration effect observed under a high elasticity improves the home welfare implications of undervaluation. While a policy of

[25] When reporting the ratio of differentiated goods production to GDP, the latter follows national accounts practice and is computed as final goods, excluding the value of differentiated goods used as intermediates.

undervaluation still reduces steady state utility for the case of elasticity of substitution of 1.5 (column (2)), the change in steady state utility becomes positive for an elasticity of η =15, and strongly so for an elasticity of 30 (columns (3) and (4)). As an illustration of the essential role of firm delocation in this result, column (5) shows the effect of holding constant the number of firms. With no firm delocation possible, the presence of intermediates fails to generate a steady state utility gain from undervaluation.

An additional reason why home experiences a net welfare gain in steady state is that a higher elasticity lowers the size of the international transfer. The reason again is tied to a stronger delocation effect under the higher elasticity. As home specializes in differentiated goods and consumers pay less in transport costs on their consumption bundles, the home price index (*P*) falls, as observed in Table 3. The lower home price index, and higher foreign price index, imply a deprecation in the home real exchange rate for a given nominal exchange rate. This means it takes a smaller nominal exchange rate devaluation for the home government to achieve its target of a 1% undervaluation of the home real exchange rate. As a result, the home government does not need to purchase as many reserves each period to push down the real exchange rate value, and so a smaller transfer is required each period to maintain the stable steady state reserve ratio. Since smaller transfers imply a smaller welfare loss associated with the undervaluation policy, the gains from delocation dominate and lead to a positive steady state welfare gain on net.

Nonetheless, despite the rise in home steady state utility, home welfare computed as the discounted stream of utility over the full simulation remains negative after the adoption of undervaluation. Figure 2b shows how home welfare drops with the initial adoption of the undervaluation policy, but eventually rises, and it shows that for an elasticity of 15 or 30, it reaches a new steady state above that of the original policy. However, given the discounting of distant future gains, the short run loss dominates, and the net effect on welfare is negative. In fact the figure shows that the fall in utility in initial periods is larger for a large elasticity. This is because a higher elasticity induces a larger trade surplus in response to a given real exchange rate depreciation, and a larger drop in home consumption relative to home production.

Let us here summarize implications from the simulations. While it is possible that a policy of reserve accumulation and undervaluation can promote home welfare in steady state by a mechanism of firm delocation and agglomeration, this result depends crucially on implications for comparative advantage between the two sectors. Specialization in differentiated goods tends to confer welfare gains by helping home consumers avoid trade costs associated with importing differentiated goods from abroad. Production delocation and agglomeration, arising from the need for imported intermediates, provide an engine by which a trade surplus created by undervaluation can promote comparative advantage in these differentiated goods. But in the absence of agglomeration, an elasticity effect instead dominates, favoring the opposite comparative advantage, with specialization in non-differentiated goods. Further, even if the equilibrium implies a rise in home welfare in steady state, this does not guarantee a rise in welfare in a dynamics sense, since the gains from firm delocation require up-front sunk investment in firm creation, which is costly in the short-run transition.

6. Robustness

Table 4 reports sensitivity analysis for several alternative economic environments. For simplicity, each column reports the percentage change in the undervaluation equilibrium from the symmetric equilibrium for that respective environment (when the real exchange rate is pegged at unity), without reporting the values in the symmetric equilibrium. Unless stated otherwise, all cases assume production using intermediates (with share ζ=0.33) and an elasticity between non-differentiated goods of η=15, with trade costs τ_D = 0.33 and τ_N = 0.

While the benchmark model features two tradable sectors, we can consider what might be regarded as the more standard case of a single tradable sector, by setting the share of differentiated goods in consumption to θ = 1. Results are reported in the first two columns of Table 4. In general, while we confirm that firm delocation can operate in an environment with one traded sector if there is a net trade surplus, the second traded sector amplifies the firm delocation effect. One can gauge the firm delocation effect here by looking directly at changes in the number of firms, since measures of comparative advantage like the differentiated goods export and production shares have no meaning in a one-good environment. Undervaluation raises the number of home firms by 3.0% in column (1), which reports the case of no intermediates, which is smaller than the corresponding value of 5.0% in the two-good benchmark model (shown in column (3) in Table 2). Similarly, in the case with intermediates in production, column (2) shows that undervaluation raises firm numbers by 3.7%, which is smaller than the value of 6.4% in the corresponding two-good environment (column (3) in Table 3).

The remaining four columns in the table demonstrate the role of trade costs. In general, higher trade costs limit the scope for the firm delocation effect by implying less openness. However, since savings on trade costs are the source of welfare gains from firm delocation, higher trade costs imply larger welfare gains for a given level of firm delocation.

Columns (3) and (4) of Table 4 consider the effect of a higher trade cost for differentiated goods, calibrated at τ_D =1.7 as in Epifani and Gancia (2017). For the low elasticity calibration (η = 1.5), the rise in frim number (0.44%) is smaller than in the benchmark calibration (6.4% from column (2) of Table 3).[26] For the higher elasticity case (η=15), the number of home differentiated goods firms falls (by 2.0%) rather than rises due to undervaluation. For this higher elasticity, the elasticity effect raises comparative advantage of non-differentiated goods so much, that the home production of differentiated goods falls in absolute terms, despite the effect of a trade surplus which would otherwise tend to raise production of both goods.

The opposite case of a low trade cost (τ_D = 0.1) is considered in column (5). Comparing column (5) to the counterpart result for the benchmark calibration in column (3) of Table 3, the rise in firm number is higher (18.9% versus 6.6%). Again, the reason is that a higher degree of openness, implying a higher share of trade in GDP, provides greater scope for countries to specialize.

The last column (6) considers the effect of a trade cost on non-differentiated goods that is the same as that on differentiated goods ($\tau_N = \tau_D$ = 0.33). While firm delocation operates similarly as in the benchmark specification in Table 3, the effect on home steady state welfare flips sign from positive (0.30% in column (3) of Table 3) to negative (-3.6%). This supports the claim that the home welfare gains from specialization in differentiated

[26] The rise in differentiated goods share as a percentage of the steady state value is actually higher in the high trade cost case than the lower trade cost benchmark, but this mainly reflects the low steady state value of the differentiated share when its trade cost is so high (7.4%).

goods in the benchmark model derives from savings on paying trade costs for imports of differentiated good, so that when shifting production from one sector to the other does not affect average trade costs paid by a country, this basis for welfare gains is eliminated.

Additional robustness checks are discussed in Appendix D, which show that our result is robust to an alternative asset market specification that allows some degree of international trade in bonds by private home agents, as well as robust to an alternative specification of roundabout production that uses non-differentiated goods material input as well as differentiated goods. Finally, it also considers a CES aggregator over the differentiated and non-differentiated goods, showing that a higher degree of substitutability between the two sectors amplifies the firm delocation effect by allowing for greater home specialization in the differentiated goods sector.[27]

The numerical experiments above were limited to a modest devaluation of 1 percent, as this was necessary to find numerical solutions to the model for the wide range of elasticities and other parameters of interest. However, Figure 5 shows results for larger degrees of undervaluation, where we instead limit the calibration to a small elasticity of substitution (η=1.5, while retaining the intermediate input share ς =0.33). The leftmost portion of the x-axis reproduces the benchmark results from a 1 percent undervaluation. The figure shows that progressively scaling up the percentage of currency undervaluation does not affect the sign of percent changes in key variables, and that the percent changes in these variables are scaled up nearly proportionately to the size of the currency undervaluation.

7. Conclusions

China's growth success has prompted research on the potential benefits of a policy of reserve accumulation and currency undervaluation. This paper argues that while undervaluation may promote net exports, it nonetheless may have the drawback of

[27]The benchmark model here makes the standard assumption in the trade literature on production relocation (as in Ossa, 2011; and Bergin and Corsetti, 2020), that all differentiated goods are traded. However, another alternative model that could be considered would specify that the sunk cost permits only domestic sales, but an additional fixed cost is needed each period for a firm to export, so that only a fraction of home differentiated goods sell in the foreign market. This specification would tend to weaken the firm relocation externality, since with a smaller fraction of home differentiated firms that export, it is harder to displace foreign firms and force them to exit their own market. See discussion in Bergin and Corsetti (2020) on this point.

fostering comparative advantage and specialization of exports in disadvantageous sectors. The model developed in this paper highlights a tradeoff between two forces shaping comparative advantage over differentiated and non-differentiated goods. On one hand, the enlargement of the home market implied by a sustained trade surplus encourages investment in new firm and product creation in the differentiated goods sector. This effect can drive firm delocation from the foreign country and promote home comparative advantage in differentiated goods. This comparative advantage is reinforced by an agglomeration effect, in which the delocation of firms from foreign to the home market makes cheaper intermediate inputs available to domestic producers, as using a local supplier avoids trade costs associated with importing inputs from abroad. One may conceive that Chinese producers have benefited from the agglomeration of manufacturing of related goods nearby. Such a comparative advantage confers welfare gains in terms of lower trade costs incurred by consumers, implying a lower price index and higher level of consumption.

On the other hand, to the degree that non-differentiated goods are associated with a high degree of substitutability, their production may respond more strongly to the price competitiveness fostered by a currency undervaluation. If this elasticity effect dominates the home market effect, the undervaluation can shift home comparative advantage away from differentiated goods, and reverse the welfare gains implied by differentiated goods production.

This result does not negate other potential benefits from undervaluation and trade surplus, such as learning by doing or technological development that promote growth in the home economy. But it does add to the list of effects working counter to such benefits. Even in the absence of adverse comparative advantage, sustaining a trade surplus with undervaluation and reserve accumulation is a costly policy in pecuniary terms, with implicit taxes on households to subsidize foreign consumers. Our model highlights an additional mechanism augmenting this loss, in terms of discouraging comparative advantage in a sector that may be important for development of new goods.

The model presents the benefits of domestic production of differentiated goods in terms of lower trade costs associated with importing such goods, but this may be viewed as a proxy for a wider range of benefits, such as higher profit margins associated with branded products, higher quality and value added, and greater opportunities for technological improvement. Future research could explore how this tradeoff changes over time as an economy like China develops. A strategy of undervaluation that promotes growth in early stages of industrialization could become less desirable at later stages of development, when the sophistication of a country's products takes on greater importance.

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Table 1. Benchmark Parameter Values

<u>Preferences</u>	
Risk aversion	$\sigma = 2$
Time preference	β =0.96
Labor supply elasticity	$1/\psi = 1.9$
Differentiated goods share	$\theta = 0.61$
Non-differentiated goods home bias	$\nu = 0.5$
Differentiated goods elasticity	ϕ = 5.2
Non-differentiated goods elasticity	$\eta = 1.5, 15, 30$
<u>Technology</u>	
Firm death rate	$\delta = 0.1$
Intermediate input share	$\zeta = 0, 0.33$
Differentiated goods trade cost	τ_D =0.33
Non-differentiated goods trade cost	τ_N =0
Firm sunk entry cost	$\overline{K} = 1$
Productivities	$\alpha_D = \alpha_N = 1$
<u>Policy</u>	
Monetary policy	$\overline{M} = \overline{M^*} = 1$
Exchange rate policy	$\overline{rer} = 1, 1.01$

Table 2. Steady State Values in Model without Intermediates

	(1) Initial symmetric steady state (e = 1.00)	(2) Effect of undervaluation policy: e = 1.01 (reported as %Δ from column (1))	(3)	(4)	(5)
		η=1.5	η=15	η=30	η=15 n, n^* fixed
n	0.733	5.212	4.972	4.579	0.000
n^*	0.733	-5.081	-4.796	-4.275	0.000
diff. export share, ω_H	0.456	4.792	2.174	-2.085	-5.319
diff. export share, ω_F	0.456	-4.906	-2.176	2.961	7.667
y_D	0.607	4.648	4.466	4.168	4.311
y_D^*	0.607	-4.605	-4.387	-3.988	-3.972
y_H	0.415	0.454	5.436	14.144	23.027
y_F^*	0.415	-0.436	-5.350	-13.872	-22.080
diff. prod. share	0.647	1.145	-0.623	-3.575	-6.217
diff. prod. share*	0.647	-1.236	0.638	4.048	7.449
p_D	0.507	0.995	2.035	3.836	6.819
p_D^*	0.507	-1.050	-2.125	-4.009	-6.825
$p^*(h)$	0.666	-2.126	-3.243	-5.189	-8.947
$p(f)$	0.666	2.144	3.269	5.229	9.384
p^*_H	0.405	-2.126	-3.243	-5.189	-8.947
p_F	0.405	2.143	3.269	5.229	9.384
$(p^*(h)/p(f))/(p^*_H/p_F)$	1.000	0.000	0.000	0.000	0.000
home utility	-1.755	-3.389	-5.573	-9.195	-13.257
foreign utility	-1.755	3.578	6.102	10.739	16.544
C	0.928	-0.655	-1.163	-2.023	-3.374
C^*	0.928	0.685	1.237	2.226	3.788
P	0.464	1.324	2.367	4.173	7.106
P^*	0.464	-1.357	-2.429	-4.308	-7.167
l	1.022	2.944	4.860	8.220	11.914
l^*	1.022	-2.912	-4.778	-8.003	-11.328
W/P	0.872	0.212	0.158	0.070	-0.937
W^*/P^*	0.872	-0.189	-0.118	0.013	1.115
GDP	1.068	2.713	4.292	7.066	10.189
GDP*	1.068	-2.688	-4.201	-6.811	-9.557
X^*/GDP	0.000	6.699	8.556	11.659	14.683
trade balance/GDP	0.000	0.033	0.051	0.083	0.113
PDV home utility		-3.534	-7.768	-14.364	-4.906
PDV foreign utility		3.825	8.862	18.421	5.348

Table 3. Steady State Values with Intermediates

	(1) Initial symmetric steady state (e = 1.00)	(2)	(3)	(4)	(5)
		Effect of undervaluation policy: e = 1.01 (reported as %Δ from column (1))			
		η=1.5	η=15	η=30	η=15 n, n^* fixed
n	0.408	6.360	6.636	7.276	0.000
n^*	0.408	-6.233	-6.557	-7.158	0.000
diff. export share, ω_H	0.548	3.295	6.356	13.211	-7.212
diff. export share, ω_F	0.548	-3.462	-6.397	-11.925	10.000
y_D	0.388	5.517	5.725	6.209	5.162
y_D^*	0.388	-5.489	-5.737	-6.199	-4.658
y_H	0.566	-0.540	-7.098	-20.480	28.261
y_F^*	0.566	0.566	7.166	20.810	-26.822
diff. prod. share	0.753	1.243	2.910	6.474	-5.323
diff. prod. share*	0.753	-1.353	-3.053	-6.398	6.315
p_D	2.094	0.249	-1.191	-4.182	8.682
p_D^*	2.094	-0.296	1.227	4.321	-8.667
$p^*(h)$	2.395	-1.125	0.447	3.664	-10.747
$p(f)$	2.395	1.095	-0.439	-3.601	11.292
p^*_H	0.472	-0.737	0.872	4.176	-11.042
p_F	0.472	0.702	-0.866	-4.087	11.699
$(p^*(h)/p(f))/(p^*_H/p_F)$	1.000	-0.779	-0.849	-0.993	0.699
home utility	-2.767	-2.829	0.298	7.245	-15.955
foreign utility	-2.767	2.980	-0.296	-6.525	20.984
C	0.585	-0.284	0.446	2.015	-4.194
C^*	0.585	0.307	-0.444	-1.920	4.843
P	1.171	0.570	-0.887	-3.910	8.947
P^*	1.171	-0.611	0.895	3.953	-9.025
l	1.367	2.768	0.156	-5.144	14.927
l^*	1.367	-2.766	-0.159	5.259	-14.035
W/P	0.403	0.872	0.977	1.217	-1.239
W^*/P^*	0.403	-0.860	-0.970	-1.172	1.510
GDP	0.967	3.410	2.019	-0.774	9.992
GDP*	0.967	-3.410	-2.049	0.802	-9.160
X^*/GDP	0.000	5.797	3.994	0.136	13.181
trade balance/GDP	0.000	0.024	0.006	-0.033	0.098
PDV home utility		-3.388	-6.918	-14.217	-15.378
PDV foreign utility		3.675	7.685	17.905	20.036

Table 4. Sensitivity Analysis
Effect of undervaluation policy
(%Δ in steady state values for *e=1.01* versus *e=1.00*)

	(1)	(2)	(3)	(4)	(5)	(6)
	$\theta = 1$		τ_D =1.7		τ_D =0.1	τ_D =0.33
	$\zeta = 0$	$\zeta = 0.33$	η=1.5	η=15		τ_N =0.33
n	2.990	3.709	0.444	-2.006	18.885	9.179
n^*	-2.931	-3.637	-0.435	2.145	-18.652	-8.940
diff. export share, ω_H	--	--	6.144	-11.444	7.788	0.658
diff. export share, ω_F	--	--	-5.819	17.992	-9.116	-0.859
y_D	2.698	3.244	0.363	-1.508	17.568	7.927
y_D^*	-2.673	-3.213	-0.358	1.603	-17.660	-7.886
y_H	0.343	-0.252	1.053	23.955	-13.575	-1.944
y_F^*	-0.328	0.274	-1.040	-23.089	13.900	2.003
diff. prod. share	--	--	-0.351	-5.843	7.328	1.988
*diff. prod. share**	--	--	0.348	6.183	-8.866	-2.227
p_D	1.124	0.881	0.208	5.525	0.555	0.080
p_D^*	-1.155	-0.931	-0.211	-5.471	-0.508	-0.148
$p^*(h)$	-1.855	-1.470	-1.746	-7.471	-1.021	-1.307
$p(f)$	1.865	1.444	1.777	7.832	0.978	1.257
p^*_H	-1.855	-1.242	-1.707	-7.702	-0.477	-0.745
p_F	1.865	1.215	1.738	8.115	0.377	0.685
$(p^*(h)/p^*_H)/(p(f)/p_F)$	0.000	-0.456	-0.078	0.513	-1.139	-1.129
home utility	-3.006	-3.080	-0.846	-10.482	-4.315	-3.553
foreign utility	3.156	3.255	0.853	12.399	4.660	3.798
C	-0.557	-0.438	-0.263	-2.778	-0.409	-0.371
C^*	0.582	0.469	0.265	3.048	0.441	0.407
P	1.124	0.881	0.527	5.796	0.824	0.746
P^*	-1.155	-0.931	-0.527	-5.829	-0.875	-0.808
l	2.698	3.005	0.625	9.176	4.301	3.483
l^*	-2.673	-2.994	-0.618	-8.773	-4.309	-3.486
W/P	0.284	0.683	-0.198	-1.009	1.406	1.064
W^*/P^*	-0.264	-0.662	0.202	1.180	-1.429	-1.050
GDP	2.270	2.814	0.376	4.301	7.310	4.465
*GDP**	-2.241	-2.783	-0.366	-3.912	-7.648	-4.499
X^*/GDP	3.264	2.430	0.431	5.881	3.997	3.002
trade balance/GDP	0.032	0.024	0.004	0.058	0.040	0.030
PDV home utility	-3.010	-3.261	-0.817	-7.562	-19.319	-3.324
PDV foreign utility	3.249	3.539	0.915	8.578	28.514	3.623

Unless stated otherwise, all cases assume production using intermediates (with share ζ =0.33) and an elasticity between non-differentiated goods of η =15, with trade costs τ_D = 0.33 and τ_N = 0.

Figure 1. Effect of elasticity (η) on steady states of home variables, without intermediates

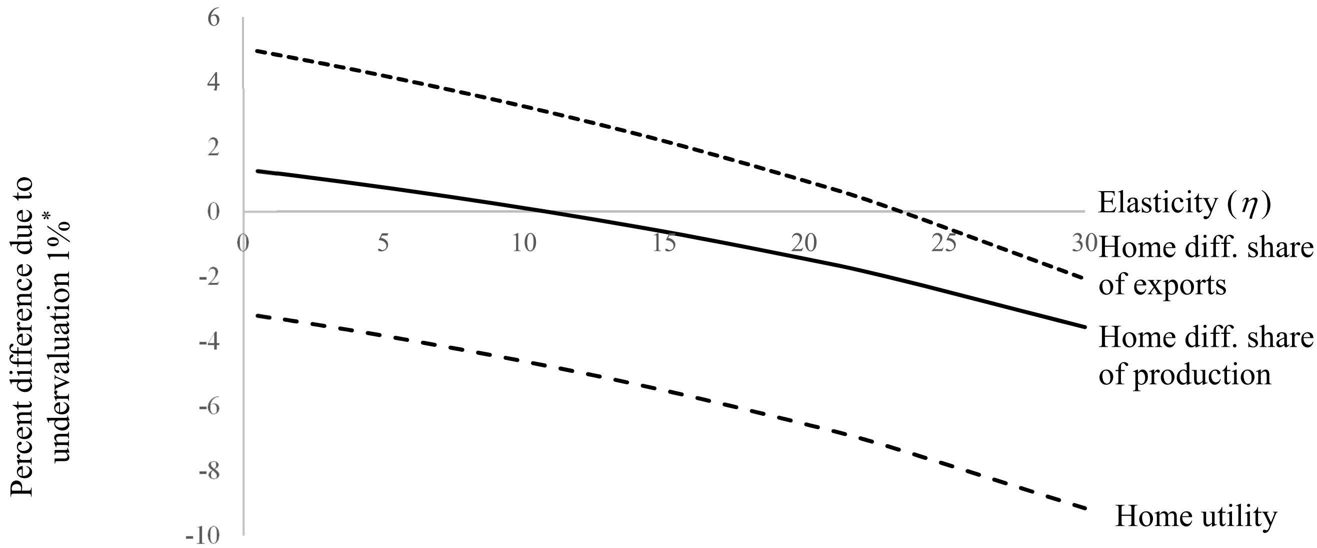
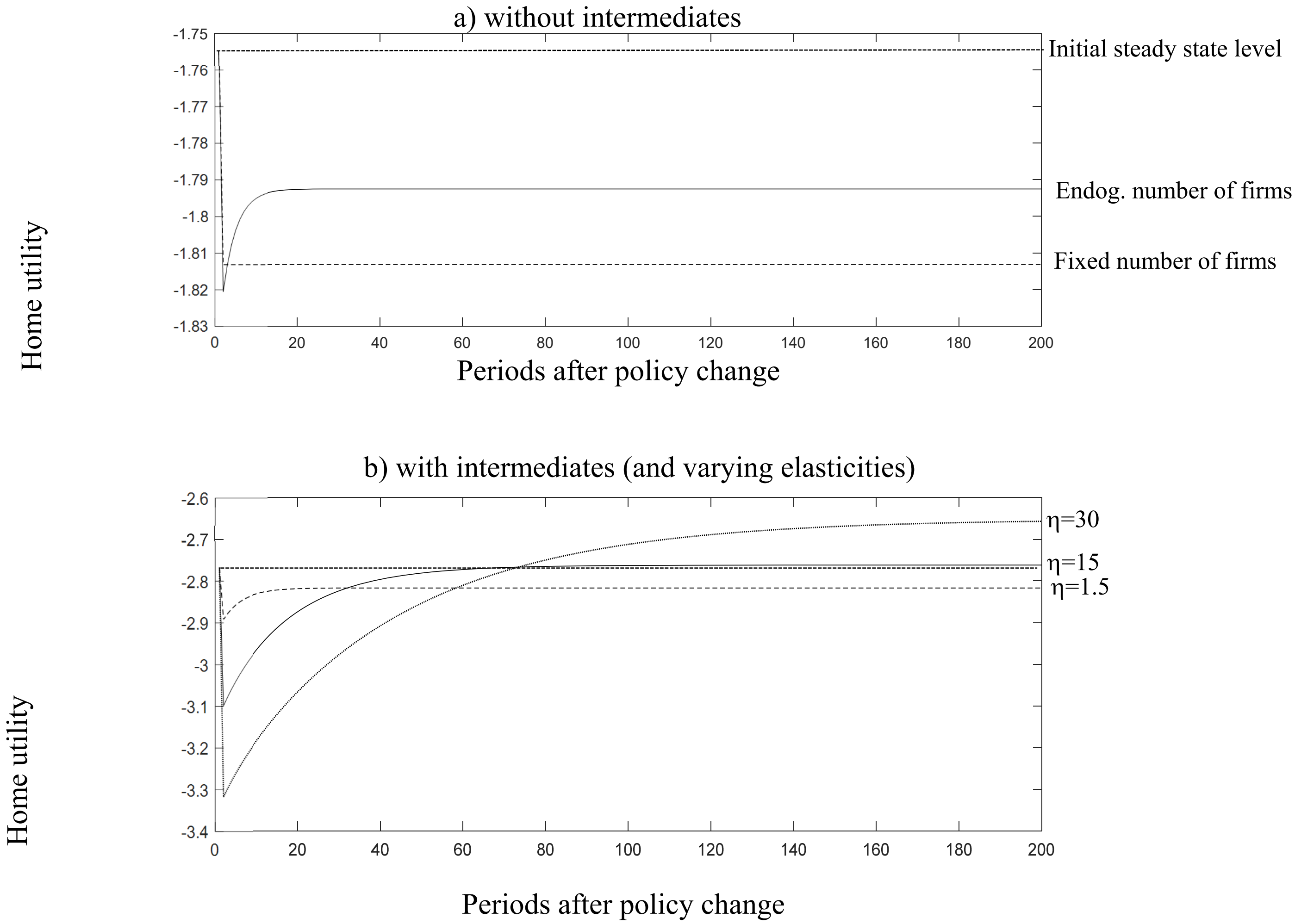


*Figure plots percent difference in steady state value when real exchange rate pegged at 1.01 compared to when pegged at 1.00.

Figure 2. Dynamic effects on home utility of adopting policy of undervaluation



Figures show levels of home utility for various periods after a change in policy from targeting exchange rate at 1.00 to targeting 1.01. Initial value is steady state utility under old policy, ending value approaches steady state under new policy. Panel (a) assumes no intermediates (ς =0), panel (b) assumes intermediates (ς =0.33).

Figure 3. Effect of elasticity (η) on steady states of home variables, with intermediates

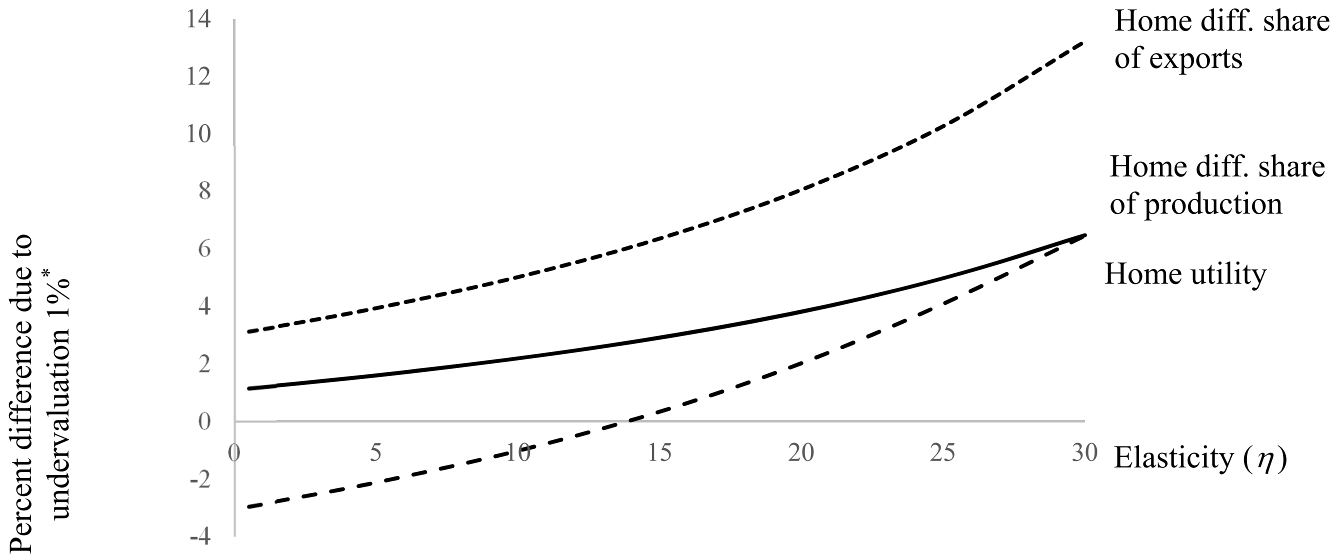


*Figure plots percent difference in steady state value when real exchange rate pegged at 1.01 compared to when pegged at 1.00.

Figure 4. Effect of intermediates share (ς) on steady states of home variables

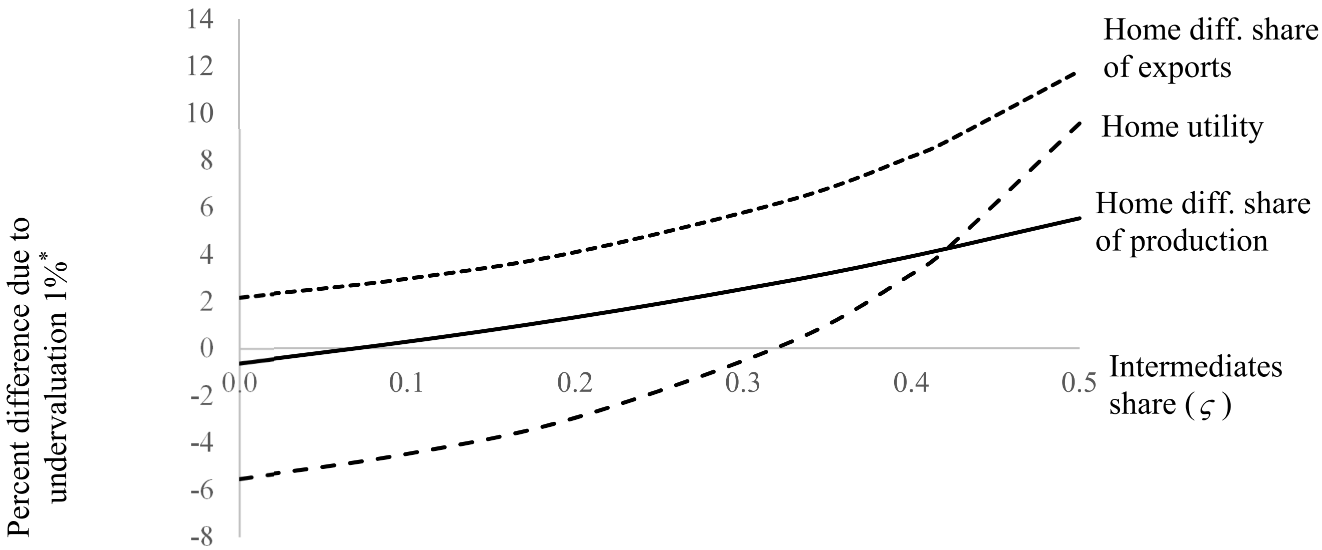


*Figure plots percent difference in steady state value when real exchange rate pegged at 1.01 compared to when pegged at 1.00.

Figure 5. Effect of increasing the size of undervaluation on steady states of home variables (η=1.5, ς=0.33)

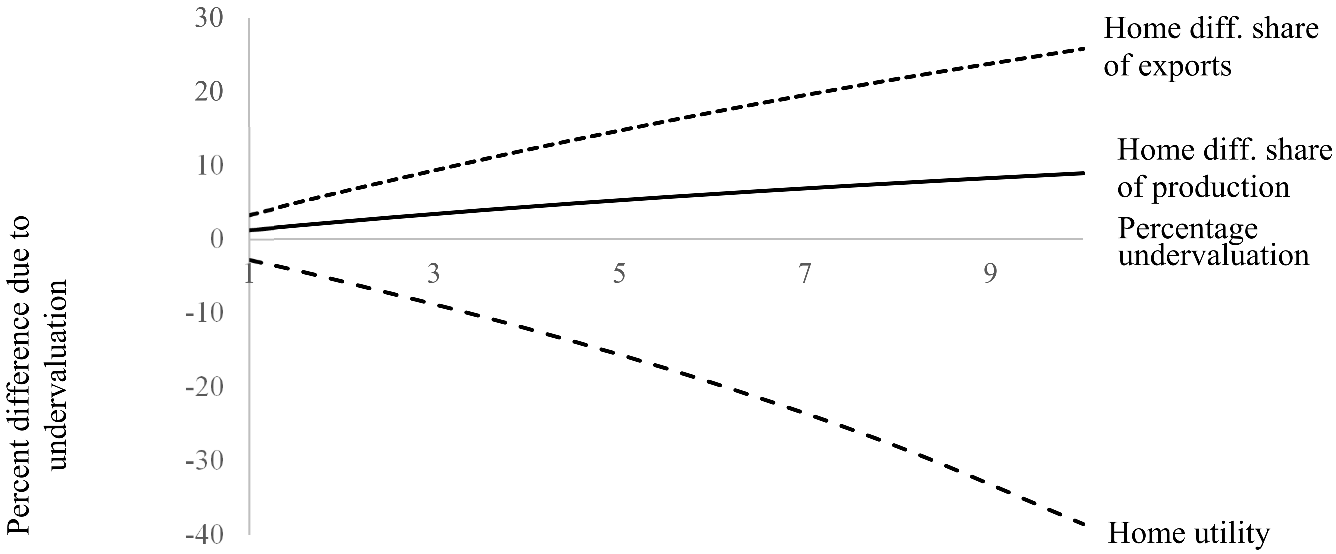


*Figure plots percent difference in steady state value when real exchange rate pegged at varying degrees of undervaluation compared to when pegged at 1.00.

Supplementary Online Appendix for
Currency Undervaluation and Comparative Advantage
by Paul R. Bergin

Appendix A: Model equations

$P_t \equiv \theta^{-\theta}(1-\theta)^{\theta-1} P_{D,t}^{\theta} P_{N,t}^{1-\theta}$ $\quad$ $P_t^* \equiv \theta^{-\theta}(1-\theta)^{\theta-1} P_{D,t}^{*\theta} P_{N,t}^{*1-\theta}$

$P_{D,t} = \left(n_t p_t(h)^{1-\phi} + n_t^* p_t(f)^{1-\phi}\right)^{\frac{1}{1-\phi}}$ $\quad$ $P_{D,t}^* = \left(n_t p_t^*(h)^{1-\phi} + n_t^* p_t^*(f)^{1-\phi}\right)^{\frac{1}{1-\phi}}$

$P_{N,t} = \left(\nu P_{H,t}^{\ 1-\eta} + (1-\nu) P_{F,t}^{\ 1-\eta}\right)^{\frac{1}{1-\eta}}$ $\quad$ $P_{N,t}^* = \left(\nu P_{F,t}^{*\ 1-\eta} + (1-\nu) P_{H,t}^{*\ 1-\eta}\right)^{\frac{1}{1-\eta}}$

$C_{D,t} = \theta P_t C_t \,/\, P_{D,t}$ $\quad$ $C_{D,t}^* = \theta P_t^* C_{tt}^* \,/\, P_{D,t}^*$

$C_{N,t} = (1-\theta) P_t C_t \,/\, P_{N,t}$ $\quad$ $C_{N,t}^*] = (1-\theta) P_t^* C_t^* \,/\, P_{H,t}^*$

$c_t(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} C_{D,t}$ $\quad$ $c_t^*(h) = \left(p_t^*(h)/P_{D,t}^*\right)^{-\phi} C_{D,t}^*$

$c_t(f) = \left(p_t(f)/P_{D,t}\right)^{-\phi} C_{D,t}$ $\quad$ $c_t^*(f) = \left(p_t^*(f)/P_{D,t}^*\right)^{-\phi} C_{D,t}^*$

$d_{gt}(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} n_t G_t(h)$ $\quad$ $d_{g,t}^*(h) = \left(p_t^*(h)/P_{D,t}^*\right)^{-\phi} n_t^* G_t^*(h)$

$d_{gt}(f) = \left(p_t(f)/P_{D,t}\right)^{-\phi} n_t G_t(h)$ $\quad$ $d_{g,t}^*(f) = \left(p_t^*(f)/P_{D,t}^*\right)^{-\phi} n_t^* G_t^*(h)$

$d_{kt}(h) = \left(p_t(h)/P_{D,t}\right)^{-\phi} n_t \overline{K}$ $\quad$ $d_{k,t}^*(h) = \left(p_t^*(h)/P_{D,t}^*\right)^{-\phi} n_t^* \overline{K}$

$d_{k,t}(f) = \left(p_t(f)/P_{D,t}\right)^{-\phi} n_t \overline{K}$ $\quad$ $d_{k,t}^*(f) = \left(p_t^*(f)/P_{D,t}^*\right)^{-\phi} n_t^* \overline{K}$

$C_{H,t} = \nu\left(P_{H,t}/P_{N,t}\right)^{-\eta} C_{N,t}$ $\quad$ $C_{H,t}^* = \nu\left(P_{H,t}^*/P_{N,t}^*\right)^{-\eta} C_{N,t}^*$

$C_{F,t} = (1-\nu)\left(P_{F,t}/P_{N,t}\right)^{-\eta} C_{N,t}$ $\quad$ $C_{F,t}^* = (1-\nu)\left(P_{F,t}^*/P_{N,t}^*\right)^{-\eta} C_{N,t}^*$

$\beta(1+i_t)E_t\left[P_t C_t^{\sigma} \big/ \left(P_{t+1} C_{t+1}^{\sigma}\right)\right] = 1$ $\quad$ $\beta(1+i_t^*)E_t\left[P_t^* C_t^{*\sigma} \big/ \left(P_{t+1}^* C_{t+1}^{*\ \sigma}\right)\right] = 1$

$W_t / P_t = l_t^{\psi} C_t^{\sigma}$ $\quad$ $W_t^* / P_t^* = l_t^{*\psi} C_t^{*\sigma}$

$M_t / P_t = C_t^{\sigma}(1+i_t)/i_t$ $\quad$ $M_t^* / P_t^* = C_t^{*\sigma}(1+i_t^*)/i_t^*$

$y_t(h) = \alpha_D\left[G_t(h)\right]^{\zeta}\left[l_t(h)\right]^{1-\zeta}$ $\quad$ $y_t^*(f) = \alpha_D\left[G_t^*(f)\right]^{\zeta}\left[l_t^*(f)\right]^{1-\zeta}$

$n_{t+1} = (1-\delta)(n_t + ne_t)$ $\quad$ $n_{t+1}^* = (1-\delta)(n_t^* + ne_t^*)$

$d_t(h) = c_t(h) + d_{G,t}(h) + d_{K,t}(h)$ $\quad$ $d_t^*(h) = c_t^*(h) + d_{g,t}^*(h) + d_{k,t}^*(h)$

$d_t(f) = c_t(f) + d_{G,t}(f) + d_{K,t}(f)$ $\quad$ $d_t^*(f) = c_t^*(f) + d_{g,t}^*(f) + d_{k,t}^*(f)$

$y_t(h) = d_t(h) + (1+\tau_D) d_t^*(h)$ $\quad$ $y_t^*(f) = d_t^*(f) + (1+\tau_D) d_t(f)$

$\pi_t(h) = p_t(h) d_t(h) + e_t p_t^*(h) d_t^*(h) - mc_t y_t(h)$ $\quad$ $\pi_t^*(f) = p_t(f) d_t(f)/e_t + p_t^*(f) d_t^*(f) - mc_t^* y_t^*(f)$

$v_t(h) = E_t\left\{\sum_{s=0}^{\infty}\left(\beta(1-\delta)\right)^s \frac{\mu_{t+s}}{\mu_t}\pi_{t+s}(h)\right\}$ $\quad$ $v_t^*(f) = E_t\left\{\sum_{s=0}^{\infty}\left(\beta(1-\delta)\right)^s \frac{\mu_{t+s}^*}{\mu_t^*}\pi_{t+s}^*(f)\right\}$

$v_t(h) = P_{D,t}\overline{K}$ $\quad$ $v_t^*(f) = P_{D,t}^*\overline{K}$

$$\left(1-\zeta\right)P_{D,t}G_t(h)=\zeta W_t l_t(h) \qquad \left(1-\zeta\right)P^*_{D,t}G^*_t(f)=\zeta W_t^{*} l^*_t(f)$$

$$p_t\left(h\right)=\left(\phi/\left(\phi-1\right)\right)mc_t \qquad p^*_t\left(f\right)=\left(\phi/\left(\phi-1\right)\right)mc^*_t$$

$$p^*_t\left(h\right)=\left(1+\tau_D\right)p_t\left(h\right)/e_t \qquad p_t\left(f\right)=\left(1+\tau_D\right)p^*_t\left(f\right)e_t$$

$$y_{H,t}=\alpha_N l_{H,t} \qquad y^*_{F,t}=\alpha^*_N l^*_{F,t}$$

$$P_{H,t}=W_t/\alpha_N \qquad P^*_{F,t}=W^*_t/\alpha^*_N$$

$$P^*_{H,t}=P_{H,t}\left(1+\tau_N\right)/e_t \qquad P_{F,t}=P^*_{F,t}\left(1+\tau_N\right)e_t$$

$$T_t+\left(M_t-M_{t-1}\right)+\left(B^s_{H,t}-\left(1+i_{t-1}\right)B^s_{H,t-1}\right)=e_t\left(R_{F,t}-\left(1+i^*_{t-1}\right)R_{F,t-1}\right)+e_tX_t$$

$$T^*_t+\left(M^*_t-M^*_{t-1}\right)+\left(B^{s*}_{F,t}-\left(1+i^*_{t-1}\right)B^{s*}_{F,t-1}\right)+X_t=0$$

$$M_t=\overline{M} \qquad M^*_t=\overline{M^*}$$

$$B^s_{H,t}=\overline{B^s_H} \qquad B^{s*}_{Ft}=\overline{B^{s*}_F}$$

$$y_{H,t}=C_{H,t}+\left(1+\tau_N\right)\left(C^*_{H,t}\right) \qquad y^*_{F,t}=C^*_{F,t}+\left(1+\tau_N\right)\left(C_{F,t}\right)$$

$$l_t=n_t l_t\left(h\right) \qquad l^*_t=n^*_t l^*_t\left(f\right)$$

$$B_{Ht}=B^s_{Ht} \qquad B^*_{Ft}+R_{F,t}=B^{s*}_{Ft}$$

$$X_t=\psi R_{Ft-1}$$

$$e_tP^*_t/P_t=1,1.01$$

$$n_t e_t p^*_t\left(h\right)d^*_t\left(h\right)+e_tP^*_{Ht}C^*_{H,t}-n^*_t p_t\left(f\right)d_t\left(f\right)-P_{F,t}C_{F,t}=e_t\left(R_{Ft}-\left(1+i^*_{t-1}\right)R_{Ft-1}\right)+e_tX_t$$

Appendix B: Steady state

The steady state is defined by the equations above when time subscripts are dropped. In the case of the consumption Euler equations, this defines the steady state interest rate: $i=1/\beta-1$ and $i^*=1/\beta-1$. The firm entry dynamics equation defines steady state investment in replacement for firm deaths: $ne=\delta n$ and $ne^*=\delta n^*$; firm value becomes $v(h)=\left(1-\beta\delta\right)^{-1}\pi\left(h\right)$ and $v^*\left(f\right)=\left(1-\beta\delta\right)^{-1}\pi^*\left(f\right)$. Government budget constraints define steady state taxes (transfers) to households; $T=ei^*R_F-iB^s_H+eX$ and $T^*=-i^*B^{s*}_F-X$. The balance of payments condition becomes
$nep^*\left(h\right)d^*\left(h\right)+eP^*_HC^*_H-n^*p\left(f\right)d\left(f\right)-P_FC_F=ei^*R_F+eX$.

Appendix C: Model solution

The numerical experiment assumes the economy starts at a symmetric steady state in which the real exchange rate is *rer* = 1. The experiment specifies that the target exchange rate is permanently reset starting in period 1, and we solve for the transition dynamics from the initial steady state to the new steady state assuming no further surprises.

Steady state values are found by numerically solving the system of nonlinear steady-state equations using a Newton-Raphson based algorithm. Solution for the dynamic model is found by solving the model as a nonlinear forward looking deterministic system using a Newton-Raphson method as described in Laffargue (1990). This method solves simultaneously all equations for each period over the simulation horizon.

Appendix D: Discussion of equilibrium

Determination of the equilibrium nominal exchange rate in this model follows Persson (1984), which in turn is based on Helpman (1981) (except that we introduce money using the utility function rather than a cash in advance constraint, and we augment the model with financial market segmentation in the form of capital controls). The balance of payments condition can be viewed as summarizing excess demand for foreign currency. In the dynamic equilibrium, the home policy can use reserve accumulation to alter this excess demand and target a nominal exchange rate. Given that capital controls allow the home government to sterilize these foreign exchange operations from affecting the domestic money market, the policy maker can also choose the level of its CPI or other domestic nominal variable. Given the ability to choose nominal exchange rate as well as CPI, we view the policy maker as targeting a desired CPI-based real exchange rate in the benchmark simulation.

In the steady state version of the balance of payments (listed in the preceding section), since home foreign exchange reserves do not change, the policy maker can choose the level of foreign transfers, *X*, to determine the equilibrium nominal exchange rate. This equilibrium directly corresponds to the "Exchange Rate Union" equilibrium defined in section 5 of Persson (1984).

Appendix E: Additional sensitivity analysis

Sensitivity analysis was conducted to several alternative parameterizations or extensions to the model. Simulation results are reported in Appendix Table 1.

1. Elasticity between sectors

The Cobb-Douglas consumption aggregator of the benchmark model can be replaced by the following CES aggregator, where χ is the elasticity of substitution between differentiated and non-differentiated sectors:

$$C_t \equiv \left(\theta^{\frac{1}{\chi}} C_{Dt}^{\frac{\chi-1}{\chi}} + (1-\theta)^{\frac{1}{\chi}} C_{N,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} .$$

This implies the following demands and price indexes replacing equations (1) and (4a-b):

$$P_t = \left(\theta P_{D,t}{}^{1-\chi} + (1-\theta) P_{N,t}{}^{1-\chi} \right)^{\frac{1}{1-\chi}}$$

$$C_{D,t} = \theta \left(P_{D,t} / P_t \right)^{-\chi} C_t$$

$$C_{N,t} = (1-\theta)\left(P_{N,t} / P_t \right)^{-\chi} C_t .$$

Columns (1) and (2) of Appendix Table 1 consider two alternative values for the elasticity, $\chi = 0.5$ and $\chi = 1.4$, where the latter is the highest value for which I can achieve a numerical steady state solution under the case of home currency undervaluation. Simulation results indicate that a higher degree of substitutability between the two sectors allows for greater home specialization in the differentiated goods sector, so it amplifies the firm delocation effect. The home differentiated goods export share rises by 14.7% due to undervaluation in column (2) of Appendix Table 1 with the high elasticity, rather than the 6.4% in the comparable Cobb-Douglas case reported in Column (3) of Table 3. Conversely, a lower degree of substitutability in column (1) limits the ability to specialize and dampens the home market effect (the home differentiated goods export share rises just 3.0% rather than 6.4%).

2. <u>Asset markets</u>

The assumption of complete capital market segmentation preventing private asset trade between private agents across countries can be relaxed, replaced by a capital control tax in the form of a quadratic cost of holding foreign assets. The household budget constraint below:

$$P_t C_t + \left(M_t - M_{t-1}\right) + \left(B_{Ht} - B_{Ht-1}\right) + e_t\left(B_{Ft} - B_{Ft-1}\right) = W_t l_t + \Pi_t + i_{t-1} B_{Ht-1} + i^*_{t-1} B_{Ft-1} - P_t AC_{Bt} - T_t,$$

where $$AC_{Bt} = \frac{\psi_B\left(e_t B_{Ft}\right)^2}{2P_t p_{Ht} y_{Ht}}$$

is a cost of holding foreign assets, the magnitude of which can modulated by the parameter ψ_B. The foreign budget constraint is modified analogously. It is assumed that this cost is a resource loss, rather than generating revenue for the government. Simulations parameterize the cost parameter at a low value, $\psi_B = 0.002$, implying significant freedom for households to engage in asset trade to promote consumption smoothing.

This constraint implies a nonlinear home-country uncovered interest rate parity condition and foreign counterpart which pin down bond allocations:

$$\mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}}\frac{e_{t+1}}{e_t}\left(1+i^*_t\right)\left(1+\psi_B\left(\frac{e_t B_{Ft}}{p_{Ht} y_{Ht}}\right)\right)\right] = \mathrm{E}_t\left[\frac{\mu_t}{\mu_{t+1}}\left(1+i_t\right)\right].$$

When this augmented model is simulated for the case of a home currency undervaluation, the steady states of variables are nearly the same as the unmodified benchmark model: compare results in column (3) in Appendix Table 1 to column (3) of Table 3 in the main text. However, the discounted value of home welfare is slightly worse in the modified model. Plotting the time paths of variables indicates that this result comes from the fact that with partial capital controls that incompletely prevent private bond holding from undoing the government reserve accumulation, a given exchange rate undervaluation requires a larger amount of reserve accumulation and transfers in the initial period.

3. <u>Production function</u>

The specification of roundabout production, involving the differentiated goods sector but not the non-differentiated sector, goes back to the specification of Krugman and Venables (1995), which was designed to create agglomeration specific to the manufacturing industry. One can easily consider a generalization that specifies the roundabout material input that combines both sectors analogously to how they are combined in consumption. Assume that the input in the production function follows the same aggregator over differentiated goods and non-differentiated that consumption does, with the same expenditure share parameter θ.

$$G_t \equiv G_{D,t}^{\theta} G_{N,t}^{1-\theta}$$

This implies materials demand equations for non-differentiated goods that are the same as the consumption demand equations

$$G_{D,t} = \theta P_t G_t / P_{D,t}$$

$$G_{N,t} = (1-\theta) P_t G_t / P_{N,t}$$

$$G_{H,t} = \nu \left(P_{H,t} / P_{N,t}\right)^{-\eta} G_{N,t}$$

$$G_{F,t} = (1-\nu)\left(P_{F,t} / P_{N,t}\right)^{-\eta} G_{N,t}$$

Market clearing conditions for non-differentiated goods are adjusted accordingly to account of this additional component of demand.

Results in column (4) of Appendix Table 1 show that reducing the share of differentiated goods in the roundabout input weakens the firm delocation effect arising from the given currency undervaluation: the home differentiated export share rises by 5.6% rather than 6.4% in the model with just differentiated inputs (from column (3) of Table 3).

References:

Helpman, Elhanan, 1981. “An exploration in the theory of exchange rate regims,” *Journal of Political Economy* 89, 865 -890.

Persson, Thorsten, 1984. “Real transfers in fixed exchange rate systems and international adjustment mechanism,” *Journal of Monetary Economics* 13, 349-369.

Appendix Table 1. Additional Sensitivity Analysis
Effect of undervaluation policy
(%Δ in steady state values for *e=1.01* versus *e=1.00*)

	(1)	(2)	(3)	(4)
	Elasticity between sectors		Private bond trade	Roundabout with non-differentiateds
	χ =0.5	χ =1.4		
n	5.105	9.788	6.636	5.990
n^*	-5.026	-9.701	-6.557	-5.876
diff. export share, ω_H	2.956	14.681	6.356	5.581
diff. export share, ω_F	-3.094	-13.455	-6.397	-5.685
y_D	4.436	8.351	5.725	5.237
y_D^*	-4.421	-8.434	-5.737	-5.215
y_H	0.029	-4.304	-1.191	0.051
y_F^*	-0.052	4.563	1.227	-0.083
diff. prod. share	-4.361	-13.344	-7.098	-1.271
diff. prod. share*	4.391	13.652	7.166	1.280
p_D	0.685	-3.337	-0.409	0.878
p_D^*	-0.712	3.282	0.377	-0.887
$p^*(h)$	1.212	7.481	2.910	1.780
$p(f)$	-1.284	-7.567	-3.053	-1.901
p^*_H	0.029	-4.304	-1.191	0.051
p_F	-0.052	4.563	1.227	-0.083
$(p^*(h)/ p^*_H)/(p(f) /p_F)$	-0.728	3.605	0.447	-0.996
home utility	0.712	-3.492	-0.439	0.987
foreign utility	-0.416	4.280	0.872	-0.802
C	0.398	-4.152	-0.866	0.793
C^*	-0.625	-1.326	-0.849	-0.387
P	-1.816	6.584	0.298	-1.907
P^*	1.881	-6.012	-0.296	1.974
L	-0.094	1.939	0.446	-0.186
L^*	0.107	-1.871	-0.444	0.199
W/P	0.189	-3.769	-0.887	0.373
W^*/P^*	-0.213	3.849	0.895	-0.397
GDP	1.881	-4.517	0.156	1.840
GDP*	-1.883	4.631	-0.159	-1.836
X^**/GDP*	0.795	1.419	0.977	0.589
trade balance/GDP	-0.783	-1.385	-0.970	-0.576
PDV home utility	-4.703	-11.689	-7.201	-10.406
PDV foreign utility	5.130	13.721	7.376	12.285