



## Editorial

# Logic and the foundations of the theory of games and decisions: introduction

This special issue of *Research in Economics* contains a selection of papers presented at the fifth conference on “Logic and the Foundations of the Theory of Games and Decisions” (LOFT5), which took place in Torino at the International Center for Economics Research (ICER) in June 2002.<sup>1</sup>

The LOFT conferences have been a regular biannual event since 1994. With the exception of the first conference, which was hosted by the Centre International de Recherches Mathématiques in Marseille, the LOFT events have taken place at ICER and would not have been possible without ICER’s generous support and hospitality.

The LOFT conferences are interdisciplinary events that bring together researchers from a variety of fields: computer science, economics, game theory, logic, mathematical psychology, philosophy and statistics. There is substantial overlap between the LOFT community and the community of researchers who are active in another regular, biannual event, namely the conferences on Theoretical Aspects of Rationality and Knowledge (TARK), which have a longer history than the LOFT conferences.<sup>2</sup>

In its original conception, LOFT had as its central theme the application of logic, in particular modal epistemic logic, to foundational issues in the theory of games and individual decision-making. Epistemic considerations have been central to game theory for a long time. For example, work has been done on the role of beliefs in refinements of Nash equilibrium since the 1970s and much has been written on common knowledge and common belief since Aumann’s seminal paper during that time. The expression *interactive epistemology* has been used in the game-theory literature to refer to the analysis of what individuals involved in a strategic interaction know about facts concerning the external world as well as facts concerning each other’s knowledge and beliefs. What is relatively new is the realization that the tools and methodology that were used in game theory are

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<sup>1</sup> Collections of papers from previous LOFT conferences can be found in a special issue of *Theory and Decision* (vol. 37, 1994, edited by M. Bacharach, P. Mongin), the volume *Epistemic logic and the theory of games and decisions* (edited by M. Bacharach, L.-A. Gérard-Varet, P. Mongin, H. Shin, and published by Kluwer Academic, 1997), two special issues of *Mathematical Social Sciences* (vols. 36 and 38, 1998, edited by G. Bonanno, M. Kaneko, P. Mongin,) and two special issues of *Bulletin of Economic Research* (vol. 53, October 2001 and vol. 54, January 2002, edited by G. Bonanno, W. van der Hoek).

<sup>2</sup> See [www.tark.org](http://www.tark.org)

closely related to those already used in other fields, notably computer science and philosophy. Modal logic turned out to be the common language that made it possible to bring together different professional communities. The reasons motivating the game theorist's and economist's interest in epistemic logic may differ from those in other disciplines. However, the insights gained and the methodology employed in one field can benefit researchers in a different field. Indeed, new and active areas of research have sprung from the interdisciplinary exposure provided by the LOFT conferences.

Over time the scope of the LOFT conferences has broadened to encompass other tools, besides modal logic, that can be used to shed light on the general issues of rationality and agency. Topics that have fallen within the LOFT umbrella include epistemic and temporal logic, theories of information processing and belief revision, models of bounded rationality, non-monotonic reasoning, theories of learning and evolution, mental models, etc.<sup>3</sup>

The papers collected in this issue of *Research in Economics* reflect the interdisciplinary composition of the participants in the LOFT conferences and the cross-fertilization that has taken place among different fields.

The paper by **Oliver Board** re-examines the paradox of the absent-minded driver, which was introduced by Piccione and Rubinstein and gave rise to an entire issue of *Games and Economic Behavior* (vol. 20, 1997) being focused on the consequences of relaxing the assumption of perfect recall in extensive games and decision problems. Piccione and Rubinstein showed that absent-mindedness can give rise to time inconsistency, whereby a decision-maker's ex ante optimal plan appears sub-optimal at a later stage, despite the fact that no unanticipated information is received by the agent. In the same issue of *Games and Economic Behavior*, Aumann, Hart and Perry argue that the time inconsistency is only apparent and that there is a time-consistent optimal plan for the decision-maker, which involves explicit randomization. Oliver Board observes that the use of mixed strategies to describe choices runs counter to the recent literature on the epistemic foundations of game-theoretic solution concepts, where players are modeled as choosing *pure* strategies and mixed strategies are interpreted as expressing the uncertainty in the mind of the *other* players. The author proposes an epistemic model along these lines, which provides an alternative interpretation of the solution proposed by Aumann, Hart and Perry. In the second part of his paper, Board relaxes the assumption that a player's information sets partition his set of decision nodes. He shows that in this case planning optimality no longer guarantees action-optimality and an absent-minded driver with more information may do worse than a driver with less information.

**Giacomo Bonanno's** paper deals with the notion of perfect recall in extensive games. The property of perfect recall was introduced by Kuhn<sup>4</sup> who interpreted it as "equivalent to the assertion that each player is allowed by the rules of the game to remember everything he knew at previous moves and all of his choices at those moves". The recent debate (referred to above) on the paradoxes of decision-making when perfect recall is

<sup>3</sup> The programs of the last four LOFT conferences can be found at the following web site: <http://www.econ.ucdavis.edu/faculty/bonanno/loft.html>

<sup>4</sup> Kuhn, H.W., 1953. Extensive games and the problem of information. In: Kuhn, H.W., Tucker, W.W. (Eds.), *Contributions to the theory of games*, vol. II, Princeton University Press, pp. 193–216.

lacking points to the need for a deeper understanding of the different aspects and components of perfect recall (or memory in general) and their role in rational decision-making. Bonanno's paper offers a syntactic characterization of perfect recall in an extension of basic temporal logic obtained by adding a knowledge operator for every player. He also studies an implication of perfect recall, namely the property of remembering what one knew in the past, and discusses its relationship to a similar property investigated in the computer science literature called "no forgetting". In the second part of the paper the author discusses the relationship between the axiom he proposes for perfect recall and a simpler axiom suggested by van Benthem. He shows that van Benthem's axiom is only appropriate in von Neumann games, where the players can be thought of as having a "common clock".

**Barbara Fasolo, Raffaella Misuraca and Gary McClelland** investigate the relationship between decision strategies (they distinguish between compensatory strategies—in which information is processed in an exhaustive, option-based fashion—and non-compensatory strategies) and person characteristics (such as self-reported choice style, open-mindedness and reasoning ability). In an experiment in which 123 students are presented with a Web-based choice task involving digital cameras, they give a systematic analysis of self-reported and demonstrated choice style, and perceptions of confidence, difficulty and satisfaction. One of their conclusions is that certain person characteristics are associated with *switches* in strategy: open minded persons with good reasoning capabilities seem to be also more *adaptive* decision makers.

The paper by **Wiebe van der Hoek and Michael Wooldridge** deals with the notions of co-operation, knowledge and time. Their starting point is an extension of temporal logic, called Alternating-time Temporal Logic (ATL), which allows one to express properties concerning what certain coalitions of players can or cannot achieve. Thus ATL is especially suitable for strategic reasoning in games. In previous work, van der Hoek and Wooldridge enriched the ATL language by adding a knowledge operator for each coalition, thus making it possible to express complex epistemic statements such as "common knowledge about  $\phi$  in the group of agents  $G$  is sufficient to enable the members of  $G$  to cooperate and ensure  $\psi$ ". The aim of their paper in this issue is to show how model checking, a technique developed by computer scientists for showing that computer programs are correct, can be applied to the problem of proving that game-like multi-agent encounters have certain properties. They do so by means of a case study: the alternating bit protocol. This is a communication protocol that is designed to allow two agents to reliably communicate a string of bits between each other over a potentially faulty communications channel. In this protocol, the knowledge that one agent has about the knowledge of the other party is essential.

**Rohit Parikh's** article also deals with interactive knowledge. The notion of common knowledge was introduced by the philosopher David Lewis.<sup>5</sup> A formal treatment of common knowledge in game theory was first provided by Robert Aumann.<sup>6</sup> Intuitively a fact is common knowledge among a group of agents if everybody knows it, everybody knows that everybody knows it, and so on ad infinitum. While not denying that common

<sup>5</sup> Lewis, D., 1969. *Convention, a philosophical study*. Harvard University Press.

<sup>6</sup> Aumann, R., 1976. Agreeing to disagree. *Annals of Statistics* 4, 1236–1239.

knowledge is an important concept, Rohit Parikh argues that often only lower levels of knowledge arise in practical situations, e.g. with e-mail or “snail mail”. The author observes that in some situations high levels of knowledge may actually be undesirable. For example, Ann might want Bob to know something but not want him to know that she knows. The purpose of Parikh’s paper is to study levels of knowledge other than common knowledge and how they affect the actions of groups. After developing a formal framework for the analysis of different levels of knowledge, the author explores the connection between game theoretic strategies and levels of knowledge.

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