Insurance Part 3

Two types of customers (different risks)

Suppose that there are two types of individuals. They are all identical in terms of the initial wealth (or wealth in the good state), denoted by $W$, and in terms of the potential loss that they face, denoted by $x$. They also have the same utility-of-money function $U$. What they differ in is the probability of loss: it is $p_H$ for type H (high-risk) individuals and $p_L$ for type L (low-risk) individuals with $1 > p_H > p_L > 0$. Then type-H individuals have steeper indifference curves than type-L individuals. In fact, fix an arbitrary point $(W_1, W_2)$. As we saw above, the slope of the indifference curve going through this point is $\frac{dW_2}{dW_1} = -\frac{p}{1 - p} \frac{U'(W_1)}{U'(W_2)}$. Thus for type-H individuals it is $1 - \frac{p_H}{1 - p_H} U'(W_1)$ and for type-L individuals it is $-\frac{p_L}{1 - p_L} U'(W_1)$. From $p_H > p_L$ we get that $1 - p_H < 1 - p_L$. Hence $\frac{p_L}{1 - p_L} < \frac{p_H}{1 - p_H}$, where the latter inequality follows from $p_H > p_L$.

![Figure 1](image-url)

[Note: if we measured wealth in good state on the horizontal axis, then the opposite would be true: the L type indifference curve would be steeper than the H type.]

Suppose that the insurance companies cannot tell who is who. However it is known that the proportion $q_H$ are of high risk and the proportion $(1 - q_H)$ are of low risk, with $0 < q_H < 1$. Let $N$ be the total number of potential customers, so that $q_H N$ are of type $H$ and $(1 - q_H)N$ are of type $L$. 

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Let $h^*_H$ be the maximum premium that the $H$ people would be willing to pay for full insurance and $h^*_L$ be the maximum premium that the $L$ people would be willing to pay for full insurance:

**Case 1: MONOPOLY**

What policy or policies would the monopolist want to offer? There are three options.

**OPTION 1.** Offer only one contract, which is attractive only to the $H$ type. In this case the monopolist will want to offer a contract which is on the indifference curve of the $H$ type that goes through the No Insurance point. Since profits increase along that indifference curve moving towards the $45^\circ$ line, the profit-maximizing contract under Option 1 is the full insurance contract with premium $h^*_H$ and the corresponding profits will be:

$$\pi_1 = q_H N(h^*_H - p_H x)$$

**OPTION 2.** Offer only one contract, which is attractive to both types. In this case the monopolist will want to offer a contract which is on the indifference curve of the $L$ type that goes through the No Insurance point. However, it is not optimal to offer full insurance (at premium $h^*_L$). To see this, note that when both types apply, profits from a contract $(h, D)$ are given by $\pi = N \left[ h - \overline{p} (x - D) \right]$ where $\overline{p} = q_H p_H + (1 - q_H) p_L$ is the average probability of loss. Clearly, $p_L < \overline{p} < p_H$. Thus the isoprofit line that goes through contract $(h, D)$ is the straight line with slope $-\overline{p} / (1 - \overline{p})$ and, since $p_L < \overline{p} < p_H$, $\frac{\overline{p}}{1 - \overline{p}} > \frac{p_L}{1 - p_L}$. Thus, since the slope of the $L$-indifference curve along the $45^\circ$ line is $-\frac{p_L}{1 - p_L}$, the isoprofit line is steeper than the indifference curve at the full insurance contract and, therefore, there is a contract, like $B$ in Figure 2 below, that yields higher profits than the full insurance contract $A$.

**Figure 2**

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The same argument applies to any other point on the indifference curve of the \( L \) type that goes through the No Insurance point at which the slope of the indifference curve is less than \( \frac{\bar{p}}{1-\bar{p}} \). Similarly, a contract on the indifference curve at which the slope of the indifference curve is larger than \( \frac{\bar{p}}{1-\bar{p}} \), cannot be optimal (moving to the right towards the 45° line would increase profits). Thus the best contract under Option 2 is that contract on the indifference curve of the \( L \) type that goes through the No Insurance point at which the slope is equal to \( -\frac{\bar{p}}{1-\bar{p}} \).

There is no need to compute the optimal contract under Option 2, because we will show later that Option 2 is never optimal.

**OPTION 3.** Offer two contracts, one targeted to the \( H \) type and the other targeted to the \( L \) type. Then, by the usual argument, the contract targeted to the \( H \) type must be a full insurance contract. Then the first constraint the monopolist faces is that the premium \( h \) for the full insurance policy targeted to the \( H \) type must be \( h \leq h^*_H \). The second constraint is that the other policy must be less attractive than the full insurance policy for the \( H \) type, that is, it must lie below the indifference curve going through the full insurance policy. The third constraint is that the policy targeted to the \( L \) type must be attractive to them, that is, it cannot lie below their indifference curve that goes through the No Insurance point. Since profits from the \( L \) type increase along this indifference curve moving towards the 45° line, the contract targeted to them must be the contract that lies at the intersection of the two indifference curves (see Figure 3 below).

![Figure 3](image-url)
Let \( L = (h_L, D_L) \) be the policy targeted to the \( L \) types and \( H = (h_H, 0) \) the policy targeted to the \( H \) types and suppose that all these constraints are satisfied. Then profits will be
\[
\pi_3 = (h_H - p_H x)q_H N + (h_L - p_L x + p_L D_L)(1-q_H)N.
\]

Now, **Option 3 yields higher profits than Option 2**. To see this, start with the pooling contract of Option 2 (point \( B \) in Figure 4 below) and draw the indifference curve for the \( H \) type that goes through that contract. Let \( C \) be the contract at the intersection of this indifference curve and the \( 45^\circ \) line. Then profits from the \( H \) type will be higher at \( C \) than at \( B \) (profits increase along an indifference curve when moving towards the \( 45^\circ \) line). If the firm offers a full-insurance contract with a premium slightly lower than the premium associated with \( C \), then the \( H \) people will switch from \( B \) to \( C \), while the \( L \) people will stay at \( B \). Thus profits from the \( L \) people won’t change, but profits from the \( H \) people will increase. Hence the original pooling contract \( B \) is not optimal.

![Figure 4](image)

**In conclusion,**

- when \( q_H \) is close to 1, the monopolist will offer only the full-insurance contract with premium \( h_H' \)
- when \( q_H \) is not close to 1, the monopolist will offer two contracts as explained under Option 3.
Case 2: COMPETITIVE INDUSTRY

Consider now a competitive industry where free entry leads to zero profits. Define an equilibrium as a set of contracts such that (1) every firm makes zero profits and (2) no (existing or new) firm could make positive profits by introducing a new contract.

Now, could there be a pooling equilibrium where only one contract is offered (with non-negative deductible, so that \( W_1 \leq W_2 \)), everybody buys it and the firms make zero profits? The answer is No. Let \( A \) be such a contract. By the crossing property of the indifference curves there is a contract \( B \) which is between the two indifference curves, so that \( B \) would be preferred by the \( L \) type but not by the \( H \) type and therefore would attract only and all the \( L \) types.

![Figure 5](image)

Contract \( A \) consists of a premium \( h_A > 0 \) and deductible \( D_A \geq 0 \). Since it is on the average fair odds line, \( h_A - \overline{p}(x-D_A) = 0 \). Since \( p_L < \overline{p} \), it follows that \( h_A - p_L(x-D_A) > 0 \). Choose a contract \( B = (h_B, D_B) \) between the two indifference curves (as shown in the picture above) with \( h_B < h_A \) and \( D_B > D_A \) but small enough so that \( h_B - p_L(x-D_B) > 0 \) (such a contract exists because the function \( f(h, D) = h - p_L(x-D) \) is continuous and \( f(h_A, D_A) = h_A - p_L(x-D_A) > 0 \)). Then a firm offering such a contract would attract all and only the \( L \) types and make positive profits.

Thus if there is a zero-profit equilibrium it must be an equilibrium with at least two contracts. Such an equilibrium is called a separating equilibrium if all the \( L \) types buy one contract and all the \( H \) types buy a different contract. What would such an equilibrium look like with exactly two contracts? The zero-profit equilibrium requires that the \( H \)-contract be on the fair
odds line for the $H$ type, that is, on the line with slope $-\frac{p_H}{1-p_H}$, and that the $L$-contract be on the fair odds line for the $L$ type, that is, on the line with slope $-\frac{p_L}{1-p_L}$.

![Diagram](image)

Figure 6

By the argument used above, if the $H$ type is not offered full insurance, then somebody could step in and offer a full-insurance contract attractive to the $H$ type and make positive profits (profits from the $H$ type increase when traveling along an indifference curve towards the $45^\circ$ line). Thus the contract designed for the $H$ type must be on the $45^\circ$ line. It is shown as point $H$ in the following diagram.

An analogous full insurance contract for the $L$ type (given by the intersection of the $45^\circ$ line and the fair odds line for the $L$ type) cannot be offered, because such a contract would be more attractive than contract $H$ for the $H$ type, everybody would buy it and it would yield negative profits (because, when everybody buys the same contract the relevant fair odds line is the average one, which is steeper than the $L$ one). The incentive compatibility constraint for the $H$ type requires the contract designed for the $L$ type to be below or on the indifference curve of the $H$ type that goes through the full insurance contract $H$. The zero profit condition requires it to be on the fair odds line of the $L$ type as close as possible to the $45^\circ$ line (because traveling along the $L$ indifference curve towards the $45^\circ$ line increases profits from the $L$ type). Such a point is point $L$ in the above figure. The $L$ types prefer contract $L$ (because of the way the $L$ and $H$ indifference curves cross at the $H$ contract: see Figure 28 below).
Is this an equilibrium? It depends on the position of the fair odds line. Consider contract $P$ in Figure 9 below. It is more attractive than $L$ and $H$ for both groups, thus a firm offering it would attract both types. Since point $P$ lies below the average fair odds line, a firm offering it would make positive profits. Hence the pair $L$ and $H$ would not be an equilibrium (on the other hand we know from the previous analysis that $P$ cannot be an equilibrium either, because there cannot be a pooling equilibrium).
Thus for a separating equilibrium it must be the case that the average fair odds line be below (or at most tangent to) the indifference curve of the $L$ type through contract $L$. This amounts to saying that the fraction of $H$ type in the population is sufficiently high.