1. [Note: this was Homework 3!] If she chooses deductible $d$ then she has to pay a premium of $h = 0.4(30,000 - d) = 12,000 - 0.4d$. Thus
   (a) $W_1 = 40,000$, $W_2 = 70,000$. (b) The premium is 4,000. Thus $W_1 = 46,000$, $W_2 = 66,000$.
   (c) The premium is 8,000. Thus $W_1 = 52,000$, $W_2 = 62,000$.
   (d) The premium is 12,000. Thus $W_1 = W_2 = 58,000$.
   (e) If she chooses deductible $d$ then her income will be $W_l = 58,000 - 0.6d$ if sick and $W_2 = 58,000 + 0.4d$ if well. Thus to have $2,000 more when sick, she must reduce the deductible $d$ by $20,000/6 = 3,333.33$, in which case her premium increases by $4,000/3 = 1,333.33$, thereby reducing her wealth in the good state by this amount.
   (f) We need to solve for $W_2$ the following expression: $70,000 - W_2 = 12,000 - 0.4(W_2 - W_1)$.

   Thus $W_2 = \frac{290,000}{3} - \frac{2}{3}W_1 = 96,666.67 - 0.67W_1$.

   (g.1) The slope of the indifference curve at point $(w_1, w_2)$ is $-p u'(w_2) / (1 - p u'(w_2))$. Thus at the no insurance point the slope is $-\frac{7}{6} = -1.1667$.

   (g.2) The slope of the indifference curve at the full insurance point is: $-\frac{2}{3} = -0.67$, the same as the slope of the zero-profit line.

   (h) The premium is $h = \frac{50}{100}(30,000 - d) = 15,000 - \frac{1}{2}d$. Thus solve for $W_2$ the expression $70,000 - W_2 = 15,000 - \frac{1}{2}(W_2 - W_1)$ to get $W_2 = 110,000 - W_1$ (hence the slope is $-1$).

   (i) She chooses a point on the above budget line where the slope of the indifference curve is equal to $-1$. The solution is $w_1 = 44,000$ and $w_2 = 66,000$. In order to get this contract she chooses a deductible of $22,000 with a corresponding premium of $4,000.

2. (a) The Principal is risk averse, since his utility function is concave: $\frac{d^2 \sqrt{x}}{dx^2} = -\frac{1}{4\sqrt{x}^3} < 0$.

   (b) The expected utility of the lottery $\left(\frac{9}{3}, \frac{36}{3}\right)$ is $\frac{1}{3}\sqrt{9} + \frac{2}{3}\sqrt{36} = \frac{1}{3}3 + \frac{2}{3}6 = 5$. The expected value is $\frac{1}{3}9 + \frac{2}{3}36 = 27$. The risk premium is given by the solution to $\sqrt{27 - r} = 5$, namely $r = 2$.

   (c) The Agent is also risk averse since his utility-of-money function is concave: $\frac{\partial^2 V(w, F)}{\partial w^2} = -2 < 0$.

   (d) The Agent’s Arrow-Pratt measure of absolute risk-aversion is $R_a(w) = -\frac{\partial^2 V(w, F)}{\partial w^2} = -\frac{-2}{20 - 2w} = \frac{1}{10 - w}$. When $w = 4$ this is equal to $\frac{1}{6}$.

   (e) (e.1) For the Principal the lottery is $\left(\frac{1}{3}, \frac{5}{3}\right)$. (e.2) For the Agent the lottery is $\left(\frac{1}{3}, \frac{4}{3}\right)$.

   (e.3) Let $w_1$ be what the Agent gets (and $y_1$ what the Principal gets) if the outcome is
x = 2, and w_2 what the Agent gets (y_2 what the Principal gets) if the outcome is x = 9. A necessary condition for Pareto efficiency is that \( \frac{U'(y_1)}{U'(y_2)} = \frac{V'(w_1)}{V'(w_2)} \).

In the first contract F = 1, w_1 = 1, w_2 = 4, y_1 = 1, y_2 = 5. Thus \( \frac{U'(y_1)}{U'(y_2)} = \frac{1}{\sqrt{15}} = \sqrt{5} = 2.236 \) and \( \frac{V'(w_1)}{V'(w_2)} = \frac{3}{2} = 1.5 \). Since 2.236 \( \neq \) 3/2, the contract is not Pareto efficient.

(f.1) For the Principal the lottery is \( \begin{pmatrix} \$1 \\ \frac{2}{3} \end{pmatrix} \). (f.2) For the Agent the lottery is \( \begin{pmatrix} \$1 \\ \frac{2}{3} \end{pmatrix} \).

(f.3) Again, let w_1 be what the Agent gets (and y_1 what the Principal gets) if the outcome is x = 2, and w_2 what the Agent gets (y_2 what the Principal gets) if the outcome is x = 9. Again, a necessary condition for Pareto efficiency is that \( \frac{U'(y_1)}{U'(y_2)} = \frac{V'(w_1)}{V'(w_2)} \).

In the second contract F = 2, w_1 = 1, w_2 = 6, y_1 = 1, y_2 = 3. Thus \( \frac{U'(y_1)}{U'(y_2)} = \sqrt{3} = 1.732 \) and \( \frac{V'(w_1)}{V'(w_2)} = \frac{9}{4} = 2.25 \). Since they are not equal, also this contract is not Pareto efficient.

(g) For the Principal Contract A is the lottery \( \begin{pmatrix} \$1 \\ \frac{2}{5} \\ \frac{4}{5} \end{pmatrix} \) with an expected utility of \( \frac{4}{5} \sqrt{1} + \frac{1}{5} \sqrt{5} = 1.247 \), while Contract B is the lottery \( \begin{pmatrix} \$1 \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \) with an expected utility of \( \frac{3}{5} \sqrt{1} + \frac{2}{5} \sqrt{3} = 1.439 \). Thus the Principal prefers contract B.

(h) For the Agent Contract A is the lottery \( \begin{pmatrix} \$1 \\ \frac{2}{5} \\ \frac{4}{5} \end{pmatrix} \) with an expected utility of \( \frac{4}{5} V(1,1) + \frac{1}{5} V(4,1) = \frac{4}{5} 0 + \frac{1}{5} 45 = 9 \), while Contract B is the lottery \( \begin{pmatrix} \$1 \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \) with an expected utility of \( \frac{3}{5} V(1,2) + \frac{2}{5} V(6,2) = \frac{3}{5} (-1) + \frac{2}{5} 64 = 38 \). Thus also the Agent prefers contract B.

(i) Contract B is Pareto superior to contract A, since both Principal and Agent prefer B to A.

(j) With a fixed salary the Agent will choose the lowest level of effort, namely F = 1, and his EU is \( 82 - (10 - 2)^2 \). If the Agent chooses F = 2, his EU is \( \frac{2}{5} \left(82 - (10 - 1)^2\right) + \frac{3}{5} \left(82 - (10 - 4)^2\right) = \frac{2}{5} (-1) + \frac{3}{5} 44 = 26 \). Thus he will choose F = 2.

(l) Contract D.

(m) With contract C the Principal’s EU is \( \frac{4}{5} \sqrt{0} + \frac{1}{5} \sqrt{7} = 0.52915 \). With contract D the Principal’s EU is (recall that the Agent chooses F = 2) \( \frac{4}{5} \sqrt{1} + \frac{3}{5} \sqrt{5} = 1.74164 \). Thus he prefers contract D.
3.

(a) Contract \( A \) is a full-insurance contract that will be bought only by the \( H \) types. To calculate the premium for contract \( A \) solve \( \frac{4}{5}\sqrt{2500} + \frac{1}{5}\sqrt{1600} = \sqrt{2500 - h} \). The solution is \( h = 196 \).

Thus the monopolist’s profits are \( \pi_1 = \left[196 - \frac{1}{5} \times 900\right]N_H = 16N_H \).

(b) Contract \( B \) is a full-insurance contract that will be bought by all types. To calculate the premium for contract \( B \) solve \( \frac{1}{10}\sqrt{2500} + \frac{1}{10}\sqrt{1600} = \sqrt{2500 - h} \). The solution is \( h = 99 \).

Thus the monopolist’s profits are \( \pi_2 = \left[99 - \frac{1}{10} \times 900\right]N_H + \left[99 - \frac{1}{10} \times 900\right]N_L = 9N_L - 81N_H \).

(c) When \( N_H = 100 \) and \( N_L = 1,000 \), \( \pi_1 = 16 \times 100 = 1,600 \) and \( \pi_2 = 9 \times 1,000 - 81 \times 100 = 900 \) thus Option 1 is better.

(d) When \( N_H = 50 \) and \( N_L = 2,000 \), \( \pi_1 = 16 \times 50 = 800 \) and \( \pi_2 = 9 \times 2,000 - 81 \times 50 = 13,950 \) thus Option 2 is better.

(e) (e.1) The \( H \) types would choose the full-insurance contract \( D \) while the \( L \) types would choose the partial-insurance contract \( C \). To compute the deductible for contract \( C \) solve either

\[
\frac{9}{10} \sqrt{2,500 - 6.24} + \frac{1}{10} \sqrt{2,500 - 6.24 - d} = \frac{9}{10} \sqrt{2500} + \frac{1}{10} \sqrt{1600}
\]

or

\[
\frac{4}{5} \sqrt{2,500 - 6.242} + \frac{1}{5} \sqrt{2,500 - 6.242 - d} = \sqrt{2,500 - 190}.
\]

(e.2) The solution is \( d = 848.48 \). Thus the monopolist’s profits are

\[
\pi_3 = \left[190 - \frac{1}{5} \times 900\right]N_H + \left[6.24 - \frac{1}{10} \left(900 - 848.48\right)\right]N_L = 10N_H + 1.088N_L = 1,000 + 1,088 = 2,088.
\]