ECN 103 : ECONOMICS of UNCERTAINTY Professor Giacomo Bonanno

FINAL EXAM: ANSWERS for VERSION 1

1. **(a)** y =12 is worse than y = 6 (same wage but higher cost) and, similarly, y =21 is worse than y = 18, so each person only needs to consider $y \in \{6,16,18\}$.

Decision problem for person of productivity 20,000

y	gross wage	cost	net income
6	20,000	0	20,000
16	35,000	20000	15,000
18	37,700	24000	13,700

Hence the best choice is y = 6.

Decision problem for person of productivity 35,000

y	gross wage	cost	net income
6	20,000	0	20,000
16	35,000	14000	21,000
18	37,700	16800	20,900

Hence the best choice is y = 16.

Decision problem for person of productivity 37,700

y	gross wage	cost	net income
6	20,000	0	20,000
16	35,000	10000	25,000
18	37,700	12000	25,700

Hence the best choice is y = 18.

Thus there is a signaling equilibrium, which is as follows. All people with productivity 20,000 will choose y = 6 and be paid $20,000, which is their true productivity. All people with productivity 35,000 will choose y = 16 and be paid $35,000, which is their true productivity. Finally, all people with productivity 37,700 will choose y = 18 and be paid $37,700, which is their true productivity. The employers' beliefs that more education correlates with higher productivity are confirmed.

(b) If schools beyond 6th grade were eliminated, the employers would no longer have education as a signal of productivity. Hiring an employee would then be the same as playing the lottery

$$\begin{pmatrix} \text{productivity} & 20,000 & 35,000 & 37,700 \\ \hline \text{probability} & 0.4 & 0.5 & 0.1 \end{pmatrix}$$

which has an expected value of 29,270. Thus employers (being risk-neutral) would pay everybody $29,270. Every employee would be better off (while the employers would be as well off). Hence closing down schools beyond 6th grade would lead to a Pareto superior situation.

2. **[Note: this is a small variation on Exercise 10.3 in the Textbook]**

(a) Bob's initial total wealth is 980,000 + 300,000 = $1,280,000. Thus

$$EU(x) = \left(0.6 - \frac{x}{500,000}\right)\sqrt{1,280,000 - 700,000 - x} + \left(0.4 + \frac{x}{500,000}\right)\sqrt{1,280,000 - x}$$

(b) EU(0) = 909.4947, EU(100,000) = 928.8949 and EU(200,000) = 954.6727. Thus **Bob would choose x = 200,000**.

(c) Since, for every x, a full-insurance contract at premium h guarantees a wealth of 1,280,000 $-h-x$, **Bob would choose x = 0**.

(d) In this case his expected utility, as a function of x, is

$$EU(x)=\left(0.6-\frac{x}{500,000}\right)\sqrt{1,280,000-80,000-100,000-x}+\left(0.4+\frac{x}{500,000}\right)\sqrt{1,280,000-80,000-x}$$

EU(0) = 1,067.4633, EU(100,000) = 1,029.2853 and EU(200,000) = 989.7367. Thus **Bob would choose x = 0.**

(e) From the above calculations we get that under option (1) he would choose x = 200,000 and his expected utility would be 954.6727, under option (2) he would choose x = 0 and his utility would be $\sqrt{1,280,000-150,000}=1,063.0146$ and under option (3) he would choose x = 0 and his expected utility would be 1,067.4634. Thus **Bob will choose the partial insurance contract and x = 0.**

3. **(a.1)** First we need to figure out what level of effort the Agent will choose. Her expected utility is:

- with $e = L$: $\frac{2}{3}\sqrt{20}+\frac{1}{4}\sqrt{29}+\frac{1}{12}\sqrt{34}-2=2.8136$
- with $e = H$: $\frac{1}{12}\sqrt{20}+\frac{1}{2}\sqrt{29}+\frac{5}{12}\sqrt{34}-3=2.4948$

Thus the Agent will choose $e = L$ and her expected utility will be 2.8136.

(a.2) Hence the Principal's expected utility is $\frac{2}{3}\ln(80)+\frac{1}{4}\ln(116)+\frac{1}{12}\ln(136)=4.5191$

(b.1) First we need to figure out what level of effort the Agent will choose. Her expected utility is:

- with $e = L$: $\frac{2}{3}\sqrt{0}+\frac{1}{4}\sqrt{36.25}+\frac{1}{12}\sqrt{42.5}-2=0.0485$
- with $e = H$: $\frac{1}{12}\sqrt{0}+\frac{1}{2}\sqrt{36.25}+\frac{5}{12}\sqrt{42.5}-3=2.7267$

Thus the Agent will choose $e = H$ and her expected utility will be 2.7267.

(b.2) Hence the Principal's expected utility is $\frac{1}{12}\ln(100)+\frac{1}{2}\ln(108.75)+\frac{5}{12}\ln(127.5)=4.7483$

(c) No, because the Principal prefers B to A while the Agent prefers A to B.

4. The consumers with the thicker and steeper indifference curve choose contract B while the others choose A. Thus the expected profits are:

$$50\left[\underbrace{(800-760)}_{h_A}-\frac{1}{40}\left(650-\left(\underbrace{(760-200)}_{d_A}\right)\right)\right]+50\left[\underbrace{(800-520)}_{h_B}-\frac{1}{25}\left(650-\left(\underbrace{(520-280)}_{d_B}\right)\right)\right]$$

$$=1,887.5+13,180=15,067.5$$