NAME:_________________________________ University ID:__________________

- By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

- If you don’t explain (= show your work for) your answers you will get no credit.

- If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [40 points] Consider the following situation. The Principal’s utility-of-money function is given by $U(m) = 81 - (10 - m)^2$ while the Agent's utility function is given by $V(m, F) = \sqrt{m} - F$ where $F$ denotes effort and $m$ money. There are two possible levels of effort: $F = 1$ and $F = 2$ and two possible levels of income that the Agent can generate: $x = 2$ and $x = 9$. The probabilities are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Prob (x = 2)</th>
<th>Prob (x = 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>if $F = 1$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>if $F = 2$</td>
<td>$\frac{2}{5}$</td>
<td>$\frac{3}{5}$</td>
</tr>
</tbody>
</table>

For questions (a)-(i) assume that effort is observable and verifiable so that it can be specified in the contract.

(a) [1 point] What is the Principal’s attitude to risk? [No credit if you don’t explain your answer.]

(b) [3 points] What is the Principal’s Arrow-Pratt measure of absolute risk-aversion when $m = 4$?

(c) [1 point] What is the Agent’s attitude to risk? [No credit if you don’t explain your answer.]

(d) [3 points] For the Agent, calculate the risk premium associated with the lottery \[
\left( \begin{array}{c}
\frac{1}{3} \\
\frac{2}{3}
\end{array} \right)
\left( \begin{array}{c}
$9 \\
$36
\end{array} \right)
\] when $F = 1$. 

(e) Consider the following contract, call it Contract A: The Agent performs at $F = 1$, gets $w = 1$ if the outcome is $x = 2$ and $w = 4$ if the outcome is $x = 9$; the Principal gets the difference between $x$ and $w$.

(e.1) [2 points] Represent this contract as a lottery, taking the point of view of the Principal.

(e.2) [2 points] Represent this contract as a lottery, taking the point of view of the Agent.

(e.3) [2 points] Is this contract Pareto efficient? [No credit if you don’t explain your answer.]

(f) Consider now the following contract, call it Contract B: The Agent performs at $F = 2$, gets $w = 1$ if the outcome is $x = 2$ and $w = 6$ if the outcome is $x = 9$; the Principal gets the difference between $x$ and $w$.

(f.1) [2 points] Represent this contract as a lottery, taking the point of view of the Principal.

(f.2) [2 points] Represent this contract as a lottery, taking the point of view of the Agent.

(f.3) [3 points] Is this contract Pareto efficient? [No credit if you don’t explain your answer.]

(g) [2 points] Which of the above two contracts does the Principal prefer?
(h) [2 points] Which of the above two contracts does the Agent prefer?

(i) [3 points] Is one of the two contracts Pareto superior to the other? [Explain]

For the following questions, assume that effort is not observable, but the value of $x$ is and thus the contract can only specify the Agent’s payment as a function of $x$. Consider the following two contracts. Contract C: the Agent is paid a fixed salary of $2 (that is, he gets $2 irrespective of the value of $x$). Contract D: the Agent is paid $1 if $x = 2$ and $4 if $x = 9$.

(j) [3 points] What level of effort would the Agent choose under contract C?

(k) [3 points] What level of effort would the Agent choose under contract D?

(l) [3 points] Of the two contracts C and D, which does the Agent prefer?

(m) [3 points] Of the two contracts C and D, which does the Principal prefer?
2. [30 points] There are two types of individuals. They have identical initial wealth of $3,600, they face a potential loss of $1,100 and they have a utility-of-money function \( U(m) = \sqrt{m} \). For individuals of type \( H \) the probability of loss is \( p_H = \frac{1}{10} \) while for individuals of type \( L \) the probability of loss is \( p_L = \frac{1}{20} \). Let \( N_H \geq 1 \) be the number of \( H \) types and \( N_L \geq 1 \) the number of \( L \) types. The insurance market is a monopoly. The monopolist knows all of the above data but cannot tell whether any particular customer is of type \( H \) or type \( L \). The monopolist is considering several options (refer to the following figure). Assume that (1) if indifferent between insuring and not insuring, a consumer would choose to insure and (2) if indifferent between two contracts, then the consumer would choose the one with lower deductible. For (a) and (b), since you have not been given the values of \( N_H \) and \( N_L \) write an expression in terms of these parameters.

(a) [8 points] Option 1: offer only contract \( A \). Calculate the monopolist’s profits in this case.

(b) [8 points] Option 2: offer only contract \( B \). Calculate the monopolist’s profits in this case.
(c) [2 points] If $N_H = 100$ and $N_L = 1,000$, which of the above two options would the monopolist choose?

(d) [2 points] If $N_H = 50$ and $N_L = 2,000$, which of the above two options would the monopolist choose?

(e) Option 3: Offer contracts $C$ and $D$ and let consumer choose. The premium for contract $C$ is $25.15$ and the premium for contract $D$ is $94.18$.

(e.1) [5 points] Write an equation whose solution gives the deductible of contract $C$. [No need to solve it]

(e.2) [5 points] If the deductible of contract $C$ is $660$, calculate the monopolist’s profits when $N_H = 50$ and $N_L = 2,000$. 

3. [30 points] Mary knows that, given her family history, she is threatened by a rather debilitating disease. Her normal disposable income when she works is $W_0 = $80,000. With probability 0.3 she may catch the disease, in which case she will be able to work only half the year, and will have to incur medical expenses, so that her disposable income will be reduced to $30,000 (hence her loss of disposable income is $\ell = $50,000). Suppose that Mary can buy insurance from a not-for-profit company which is willing to offer any fair (i.e. zero profit) contract and lets her choose the deductible $d$. Let $W_1$ denote her wealth if she has the disease (the “bad state”) and $W_2$ her wealth if she does not have the disease (the “good state”). Find the values of $W_1$ and $W_2$ in the following cases:

(a) [1 point] She takes no insurance.

(b) [2 points] $d = $40,000.

(c) [2 points] $d = $30,000.

(d) [2 points] She takes full insurance.

(e) [4 points] If she wants to increase her wealth by $700 in the case where she gets the disease, how much does she need to reduce her wealth in the case where she does not get the disease?

(f) [5 points] Write the equation of the zero-profit line in the wealth diagram.
(g) (g.1) [2 point] Suppose that Mary has the von Neumann-Morgenstern utility-of-money function

\[ U(m) = \ln(m) \]. What is the slope of her indifference curve that goes through the no insurance point at the no insurance point?

(g.2) [2 points] What is the slope of the indifference curve that goes through the full-insurance point on the line of part (f.1) (that is, the zero-profit line) at the full-insurance point?

Suppose now that, unfortunately, Mary cannot find any insurance company willing to insure her at the fair price. She finds, however, an insurance company which accepts to give her insurance if she pays 40 cents of insurance premium for every dollar of insurance (for example, if she insures for $10,000 then her premium is \( \frac{40}{100} \times 10,000 = $4,000 \); note that insuring for $x means having a deductible equal to \( \ell - x : loss - x \)).

(h) [4 points] Find the equation of the insurance budget line in the wealth diagram.

(i) [6 points] Find the optimal insurance contract. Express it both in terms of wealth levels and in terms of premium and deductible.