1. (a) When \( P = 2,115 \) all qualities are offered for sale. Thus buyers face a lottery with expected value \( \frac{1}{12}1600 + \frac{9}{12}2150 + \frac{2}{12}2300 = 2129.17 \); hence buyers are willing to buy and all cars are traded.

(b) When \( P = 2,020 \) only qualities L and M are offered for sale. Thus buyers face a lottery with expected value \( \frac{1}{10}1600 + \frac{2}{10}2150 = 2095 \); hence buyers are willing to buy and all cars of qualities L and M are traded, while cars of quality H are not traded.

2. (a) \[ p_H \sqrt{w-x} + (1 - p_H) \sqrt{w} = \frac{1}{2} \sqrt{6400 - 2800} + \frac{3}{4} \sqrt{6400} = 75. \]

(b) \[ p_L \sqrt{w-x} + (1 - p_L) \sqrt{w} = \frac{1}{2} \sqrt{6400 - 2800} + \frac{7}{8} \sqrt{6400} = 77.5. \]

(c) Contract A is the full-insurance contract that makes the \( H \) types indifferent between insuring and not insuring. To calculate the premium for contract A solve \( 75 = \sqrt{6400 - h} \). The solution is \( h_H = 775 \). Contract A will be bought only by the \( H \) types. Thus the monopolist’s profits are \( \pi_A = [h_H - p_H x]N_H = [775 - \frac{1}{4}2,800]400 = 30,000 \).

(d) Contract B is the full-insurance contract that makes the \( L \) types indifferent between insuring and not insuring. To calculate the premium for contract B solve \( 77.5 = \sqrt{6400 - h} \). The solution is \( h_L = 393.75 \). Contract B will be bought by both types. Thus the monopolist’s profits are \( \pi_B = [h_L - p_H x]N_H + [h_L - p_L x]N_L = [393.75 - \frac{1}{4}2,800]400 + [393.75 - \frac{7}{8}2,800]3,000 = 8,750 \).

(e) Contract D is a full-insurance contract with premium $500. Let \( h_c \) be the premium of contract C and \( d_c \) the deductible. The first equation reflects the fact that the \( H \) types are indifferent between contract C and contract D:
\[ p_H \sqrt{w-h_c-d_c} + (1 - p_H) \sqrt{w-h_c} = \sqrt{w-500} , \text{that is} \]
\[ \frac{1}{4} \sqrt{6400 - h_c - d_c} + \frac{3}{4} \sqrt{6400 - h_c} = \sqrt{5900} \]

The second equation reflects the fact that the \( L \) types are indifferent between contract C and not insuring:
\[ p_L \sqrt{w-h_c-d_c} + (1 - p_L) \sqrt{w-h_c} = 77.5 , \text{that is} \]
\[ \frac{1}{2} \sqrt{6400 - h_c - d_c} + \frac{7}{8} \sqrt{6400 - h_c} = 77.5 \].

(f) (f.1) Type \( H \) are indifferent between contract \( C \) and contract \( D \) and consider both contracts to be better than no insurance. Given our assumption about how individuals break indifference, type \( H \) will choose contract \( D \), since it has a lower deductible.

(f.2) For type \( L \) contract \( D \) is worse than not insuring and contract \( C \) is just as good as not insuring. Thus the \( L \) type will choose contract \( C \).

3. (a) The slope is given by \(-\frac{p}{1-p} U'(W-x) = -\frac{1}{4} \frac{100}{1,600} - \frac{12}{25} = -0.48 \).

(b) The expected wealth with no insurance is \( \frac{1}{4}(2,304 - 704) + \frac{3}{4}(2,304) = 2,128 \). The expected utility with no insurance is given by
\[ EU_{NI} = \frac{1}{4}100 \ln(2,304 - 704) + \frac{3}{4}100 \ln(2,304) = 765.124 \]. The risk premium is given by
the solution to \(100 \ln(2,128 - r) = 765.124\) which is \(r = 24.748\). [The risk premium is also equal to the difference between the maximum premium that Susan is willing to pay for full insurance, namely 200.748, and the expected loss, namely 176.]

(c) Profits are given by \(\pi(D) = h - p(x - D) = 192 - \frac{1}{4}D - \frac{1}{4}(704 - D) = 16\). Thus profits are independent of \(D\) and equal to 16; in particular, \(\pi(360) = 16\).

(d) \(\frac{1}{4}100\ln\left(2,304 - \left(192 - \frac{1}{4}360\right)\right) - 360\) + \(\frac{1}{4}100\ln\left(2,304 - \left(192 - \frac{1}{4}360\right)\right) = 765.249.

(e) Because of part (c), \(\pi(0) = 16\).

(f) With full insurance she has a guaranteed wealth of \(w - h(0) = 2,304 - 192 = 2,112\) thus her utility is \(100\ln(2,112) = 765.539\).

(g) In \(h = 192 - \frac{1}{4}D\) replace \(h\) with \(W - W_2 = 2,304 - W_2\) and \(D\) with \((W_2 - W_1)\) obtaining \(2,304 - W_2 = 192 - \frac{1}{4}(W_2 - W_1)\). Solving for \(W_2\) we get \(W_2 = 2,816 - \frac{1}{3}W\).

4. (a)

(b.1) The straight lines are the Agent’s indifference curves and the curved lines are the Principal’s indifference curves. The two indifference curves are tangent at contract \(B\), while the Principal’s indifference curve is less steep than the Agent’s indifference curve at contract \(C\):
(b.2) Pareto efficiency requires that the risk-averse party be guaranteed a fixed income. Thus the only Pareto efficient contract is contract $B$ (which is on the $45^\circ$ line for the Principal).

(b.3) Since the Agent is risk neutral, she ranks the contracts according to their expected values. From the Agent’s point of view, contract $A$ is the lottery $\begin{pmatrix} 3,200 \\ 1/3 \end{pmatrix} \begin{pmatrix} 1,600 \\ 4/3 \end{pmatrix}$ whose expected value is 1,920, contract $B$ is the lottery $\begin{pmatrix} 3,500 \\ 1/3 \end{pmatrix} \begin{pmatrix} 1,500 \\ 4/3 \end{pmatrix}$ whose expected value is 1,900 and contract $C$ is the sure lottery $\begin{pmatrix} 2,000 \\ 1 \end{pmatrix}$ whose expected value is 2,000. Thus the Agent ranks the contracts as follows: $C \succ A \succ B$.

(b.4) From the Principal’s point of view, contract $A$ is the lottery $\begin{pmatrix} 2,800 \\ 1/3 \end{pmatrix} \begin{pmatrix} 2,400 \\ 4/3 \end{pmatrix}$ whose expected utility is $\frac{1}{3} \sqrt{2,800} + \frac{4}{3} \sqrt{2,400} = 49.77$, contract $B$ is the sure lottery $\begin{pmatrix} 2,500 \\ 1 \end{pmatrix}$ whose expected utility is $\sqrt{2,500} = 50$ and contract $C$ is the lottery $\begin{pmatrix} 4,000 \\ 1/3 \end{pmatrix} \begin{pmatrix} 2,000 \\ 4/3 \end{pmatrix}$ whose expected utility is $\frac{1}{3} \sqrt{4,000} + \frac{4}{3} \sqrt{2,000} = 48.43$. Thus the Agent ranks the contracts as follows: $B \succ A \succ C$. 

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