Economics 103

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PRACTICE EXAM FOR THE FINAL

Note: a typical exam would consist of four questions. The extra questions are for additional practice.

1. Consider all the wealth lotteries of the form $\begin{pmatrix} \$x & \$y \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}$ with $x \geq 0, y \geq 0$. We will denote such a lottery as the pair (x, y). Consider an individual with von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$.

(a) Calculate the expected utility of lottery $A = (25,100)$.

(b) Calculate the expected utility of lottery $B = (100, 25)$.

(c) Calculate the slope of the indifference curve through point *A* at point *A*.

(d) Calculate the slope of the indifference curve through point *B* at point *B*.

(e) Write the equation of the indifference curve through point *A*.

(f) Write the equation of the indifference curve through point *B*.

2. Anna's vNM utility-of-money function is $U(m) = \sqrt{m}$. Her initial wealth is \$3,600 and she faces a potential loss of \$2,700 with 15% probability. An insurance company is offering Anna any contract such that premium *h* and deductible *d* satisfy the following equation: $h = 1,200 - \frac{2}{5}d$.

(a) Translate the equation $h = 1,200 - \frac{2}{5}d$ into an equation in terms of wealth levels.

(b) Does the equation found in Part (a) correspond to an isoprofit line?

(c) Does the insurance budget line of Part (a) go through the no-insurance point?

(d) Are there any contracts on the insurance budget line that Anna prefers to no insurance? To answer this question it is sufficient to write the relevant equation(s): you don't need to compute the solution (if any).

Suppose now that the insurance company is offering Anna any contract such that premium *h* and deductible *d* satisfy the following equation: $h = 1,080 - \frac{2}{5}d$.

(e) Translate the equation $h = 1,080 - \frac{2}{5}d$ into an equation in terms of wealth levels.

(f) Does the insurance budget line of Part (a) go through the no-insurance point?

(g) Are there any contracts on the insurance budget line that Anna prefers to no insurance? [Hint: try the full-insurance contract.]

3. There are two types of potential workers: those (Group I) with marginal productivity $\left(6 + \frac{2}{3}\,\mathrm{y}\right)$ and those (Group II) with marginal productivity $\left(10 + \frac{1}{2}\,\mathrm{y}\right)$, where y denotes the amount of education. Each worker knows whether she belongs to Group I or Group II, while the potential employer does not. The cost of acquiring y units of education is $4\mathrm{y}$ for Group I and $2\mathrm{y}$ for Group II. The potential employer believes that those applicants with education below a belong to Group I and those with education of at least a belong to Group II and offers each applicant a wage equal to the applicant's estimated productivity. Find all the values of a that give rise to a signaling equilibrium.

4. There are two types of individuals. They all have the same initial wealth of \$15,000, they all face the same potential loss of \$9,000 and the all have the same vNM utility-of-money function $U(\$m) = \ln\left(\frac{m}{1,000}\right)$. What they differ in is the probability of loss: for the *H* type it is $p_H = \frac{1}{3}$ and for the *L* type it is $p_L = \frac{1}{12}$. There are $N_H = 1,800$ individuals of type *H* and $N_L = 7,200$ individuals of type *L*. The insurance industry is a monopoly. The monopolist is considering various options. Assume throughout that when indifferent between insuring and not insuring an individual will choose to insure.

OPTION 1. Offer only one insurance contract designed in such a way that it attracts only the *H* types.

(a) Find the profit maximizing contract for the monopolist under Option 1 and calculate the monopolist's expected profit per contract and expected total profits.

OPTION 2. Offer only one insurance contract designed in such a way that it attracts both types.

(b) Can the monopolist make positive profits under Option 2?

(c) Write two equations whose solution gives the profit maximizing contract for the monopolist under Option 2.

OPTION 3. Offer only a menu of two contracts: one, call it C_H, targeted to the *H* types and the other, call it C_L, targeted to the *L* types.

(d) Consider the following pair of contracts, expressed in terms of premium *h* and deductible *d*: $C_H = (h = 3,600, d = 600)$, targeted to the H type, and $C_L = (h = 200, d = 2000)$, targeted to the L type. Do these two contracts satisfy the four constraints (Individual Rationality for each type and Incentive Compatibility for each type)? State which are satisfied and which are not.

(e) Suppose that the monopolist offers the two contracts of Part (d). What will its total expected profits be?

(f) Suppose that the monopolist intends to offer a pair of contracts. The contract targeted to the H type is the same as in Part (d), namely $C_H = (h = 3,600, d = 600)$. What should the other contract, C_L, be if the monopolist wants to maximize profits (conditional on offering C_H)? Write two equations whose solution gives the answer to this question.

5. Albert's initial wealth is \$900 and he faces a potential loss of \$275 with a probability that depends on whether or not he spends some money on preventive measures, as follows:

$\frac{2}{5}$	if he does not spend money on prevention
$\frac{1}{10}$	if he spends \$78 on preventive measures

Albert's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$.

(a) If Albert does not buy insurance and does not spend money on prevention, what is his expected utility?

(b) If Albert does not buy insurance and spends \$78 on prevention, what is his expected utility?

(c) If Albert gets full insurance with premium \$90, what is his expected utility?

(d) If the insurance company offers Albert a partial-insurance contract with premium \$50 and deductible \$150, what is the insurance company's expected profit?

6. Andrea wants to hire Bill to run her shop. Andrea's von Neumann-Morgenstern utility-of-money function is $U_A(m) = m$ while Bill's von Neumann-Morgenstern utility-of-money function is as follows:

$U_B(m) = \sqrt{m}$	if he puts in no effort
$U_B(m) = \sqrt{m} - 2$	if he works hard

The profit of the shop can take on two values: X_L = \$1,300 or X_H = \$1,900. The probability of X_L depends on Bill's effort as follows (and the remaining probability is the probability of X_H):

probability of $X_L = \frac{1}{2}$	if he puts in no effort
probability of $X_L = \frac{1}{6}$	if he works hard

Suppose first that Andrea and Bill agree on the following contract, call it *C*: Bill will get \$324 if the profit is X_L and will get \$900 if the profit is X_H.

(a). What is Bill's expected utility from contract *C*?

(b) What is Andrea's expected utility from contract *C*?

Suppose now that Andrea and Bill agree on the following contract, call it *D*: Bill will get \$400 if the profit is X_L and will get \$484 if the profit is X_H.

(c) What is Bill's expected utility from contract *D*?

(d) What is Andrea's expected utility from contract *D*?