Economics 103

PRACTICE EXAMS for the FIRST MIDTERM

Note: a typical exam would consist of three questions. The extra question is for additional practice.

1. You are an insurance agent. You know Jane well enough to be sure that her preferences can be represented by the vonNeumann-Morgenstern utility-of-money function:

$$U(\$x) = 40\left(\frac{x}{1,000}\right) - \left(\frac{x}{1,000}\right)^2$$

where x denotes wealth. She told you that her entire wealth consists of \$10,000 and that she is planning to spend \$8,000 on a new car and she wants to insure it against theft. She lives in an area where the probability that a new car is stolen is 20%. You want to offer her a policy with a deductible of \$1,000 (so that if her car is stolen she is paid \$7,000 rather than \$8,000). Write an equation whose solution gives the maximum premium that you can charge her. [Assume that, if she is indifferent between buying insurance and not buying insurance, she will decide to buy insurance.]

2. Carla has just bought a house in California for \$120,000. She does not trust insurance companies and is not going to buy fire insurance. She learned that in her area there is a 1% probability that a fire will completely destroy her house. Her best friend, Natasha, has just bought a house in Florida for the same price of \$120,000. Natasha feels the same about insurance companies and is not going to buy fire insurance. Also in the area where Natasha lives there is a 1% probability of fire. Carla's wealth is equal to the value of her house plus her bank account balance which is \$200,000.

(a) What is Carla's expected wealth if she doesn't buy fire insurance?

A student of Economics 103, who is a friend of both Carla and Natasha, suggests to them that they should write the following contract:

> «If both Carla's house and Natasha's house burn down, or if neither house burns down, then each person will owe nothing to the other. If Carla's house burns down while Natasha's house does not, then Natasha will give \$60,000 to Carla. Finally, if Natasha's house burns down while Carla's house does not, then Carla will give \$60,000 to Natasha.»

Assume independence, that is, that the probabilities are as follows:

both houses destroyed	only Carla's house destroyed	only Natasha's house destroyed	neither house destroyed
0.01×0.01	0.01×0.99	0.01×0.99	0.99×0.99

(b) What is Carla's expected wealth if they sign this contract?

(c) Suppose that Carla is risk-neutral. Would she gain by signing this contract?

(d) Let U be Carla's vonNeumann-Morgenstern *normalized* utility-of-wealth function. What is Carla's expected utility if she **doesn't** sign the contract?

(e) What is Carla's expected utility if she **does** sign the contract? [Hint: some value(s) of the utility function has/have to be treated as an unknown at this stage.]

(f) As before, denote by U Carla's vonNeumann-Morgenstern *normalized* utility-of-wealth function. Suppose that $U(260{,}000) = 0.6$.

(1) Prove that if they sign the contract Carla is better off than if they don't sign the contract [this is an application of the general principle that if risk averse individuals pool their independent risks then they end up better off].
(2) Prove that Carla is risk-averse, by constructing a lottery relative to which she is risk averse.

3. A new casino in Reno offers the following deal. You pay $5,000 and a fair coin is tossed repeatedly – up to six times – until it shows heads for the first time. If it shows heads for the first time on the k^{th} trial, you win $\$2^k(1{,}000)$ (thus if it shows heads on the first trial you get $2,000, if it shows tails on the first trial and heads on the second you get $4,000, etc.). If it never shows heads, you get nothing.

(a) Fill in the third row of the following table.

PRIZE	$2,000	$4,000	$8,000	$16,000	$32,000	$64,000
COIN SEQUENCE	H	TH	TTH	TTTH	TTTTH	TTTTTH
PROBABILITY						

(b) If you are risk neutral, what is your expected **net** gain if you play this game?

(c) Would you accept to play this game if your von Neumann-Morgenstern utility-of-money function were $U(\$x) = \sqrt{x}$ and your initial wealth was $15,000?

4. Consider the following lotteries (the numbers in the top row are sums of money, measured in dollars, and the numbers in the bottom row are the respective probabilities):

$$A = \begin{pmatrix} 1000 \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} 5000 & 1000 & 0 \\ 0.1 & 0.89 & 0.01 \end{pmatrix} \quad C = \begin{pmatrix} 1000 & 0 \\ 0.11 & 0.89 \end{pmatrix} \quad D = \begin{pmatrix} 5000 & 0 \\ 0.1 & 0.9 \end{pmatrix}$$

(thus, for example, B says that with probability 0.1 you win $5000, with probability 0.89 you win $1000 and with probability 0.01 you win nothing). Peter's preferences are such that, between A and B he strictly prefers A, and between C and D he strictly prefers D. Does Peter satisfy the axioms of expected utility theory? (Assume that Peter prefers more money to less). [Note: a "Yes" or "No" answer is not enough: you need to justify your answer.]