1. If Jane does not buy insurance, with probability 0.8 her wealth will be $10,000 (no theft), while with probability 0.2 it will be $2000 (the car is stolen). If she buys insurance at premium $h$ her wealth will be $(10,000-h)$ with probability 0.8 (no theft), and $(10,000-1,000-h)$ with probability 0.2 (the car is stolen). The maximum premium she is willing to pay is the value of $h$ that solves the following equation, where the LHS is Jane's expected utility if she doesn't buy insurance and the RHS is her expected utility if she does:

\[
0.8 \left[ 40(10) - (10)^2 \right] + 0.2 \left[ 40(2) - 2^2 \right] = \frac{0.8}{1,000} - \frac{h}{1,000} + \frac{0.2}{1,000} - \frac{h}{1,000}
\]

The solution is $h = 1,826.64$. Thus Jane is willing to pay up to $1826.64 for the insurance policy.

2. (a) If she doesn't buy insurance then her wealth will be 200,000 + 120,000 = 320,000 if there is no fire (and this happens with probability 0.99) and 200,000 if there is a fire (and this happens with probability 0.01). Thus her expected wealth is 320,000 * 0.99 + 200,000 * 0.01 = 318,800.

(b) If they sign the contract then Carla faces the following lottery:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>Both houses burn down</th>
<th>Only Carla's house burns down</th>
<th>Only Natasha's house burns down</th>
<th>Neither house burns down</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROBABILITY</td>
<td>(0.01)(0.01) = 0.0001</td>
<td>(0.01)(0.99) = 0.0099</td>
<td>(0.99)(0.01) = 0.0099</td>
<td>(0.99)(0.99) = 0.9801</td>
</tr>
<tr>
<td>Carla's wealth</td>
<td>200,000</td>
<td>260,000</td>
<td>260,000</td>
<td>320,000</td>
</tr>
</tbody>
</table>

Thus her expected wealth is:

200,000 * 0.0001 + 260,000 * 0.0198 + 320,000 * 0.9801 = 318,800,

the same as without the contract.

(c) Since her expected wealth is the same with or without the contract, if she is risk-neutral she does not gain by signing the contract (nor does she lose: she is indifferent).

(d) Normalize Carla's utility function so that $U(200,000) = 0$, $U(260,000) = a$ and $U(320,000) = 1$ (with $0 < a < 1$). Then her expected utility without the contract is:

$0.01(0) + 0.99(1) = 0.99$.

(e) Her expected utility with the contract is:

$0.0001(0) + 0.0198(a) + 0.9801(1) = 0.0198a + 0.9801$. 


From (d) and (e) we deduce that she is better off with the contract if and only if
\[0.0198 \times a + 0.9801 > 0.99,\]
that is, if and only if
\[a > 0.5.\]
Thus, since \(a = 0.6\), she is better off with the contract.

For a risk-averse person the utility of the expected value of a lottery is greater than the expected utility of the lottery. Let us construct a lottery with prizes 200,000 [with probability \((1-p)\)] and 320,000 [with probability \(p\)] whose expected value is 260,000:
\[(1-p) \times 200,000 + p \times 320,000 = 260,000.\]
Solving for \(p\) we get: \(p = 0.5.\) Now, \(U(260,000) = 0.6,\) while denoting by \(A\) is the lottery \(\begin{pmatrix} 320,000 & 200,000 \\ 0.5 & 0.5 \end{pmatrix}\), \(EU(A) = 0.5 \times U(200,000) + 0.5 \times U(320,000) = 0.5 \times 0 + 0.5 \times 1 = 0.5.\)
Thus we have that the utility of the expected value of \(A\) is greater than the expected utility of \(A,\) hence Carla is risk averse.

### 3. (a)

<table>
<thead>
<tr>
<th>PRIZE</th>
<th>$2,000</th>
<th>$4,000</th>
<th>$8,000</th>
<th>$16,000</th>
<th>$32,000</th>
<th>$64,000</th>
<th>$0</th>
</tr>
</thead>
<tbody>
<tr>
<td>COIN SEQUENCE</td>
<td>H</td>
<td>TH</td>
<td>TTH</td>
<td>TTTH</td>
<td>TTTTH</td>
<td>TTTTTTH</td>
<td>TTTTTTT</td>
</tr>
<tr>
<td>PROBABILITY</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{8})</td>
<td>(\frac{1}{16})</td>
<td>(\frac{1}{32})</td>
<td>(\frac{1}{64})</td>
<td>(\frac{1}{64})</td>
</tr>
</tbody>
</table>

(b) The expected prize is:
\[
\frac{1}{2} \times 2,000 + \frac{1}{4} \times 4,000 + \frac{1}{8} \times 8,000 + \frac{1}{16} \times 16,000 + \frac{1}{32} \times 32,000 + \frac{1}{64} \times 64,000 = \$6,000.
\]
Since it costs $5,000 to play the game, the expected net gain is $1,000.

(c) If you don’t enter the casino, your utility is \(\sqrt{15,000} = 122.474\)

If you enter (by paying $5,000) and play the game, your expected utility is:
\[
\frac{1}{2} \times \sqrt{12,000} + \frac{1}{4} \times \sqrt{14,000} + \frac{1}{8} \times \sqrt{18,000} + \frac{1}{16} \times \sqrt{26,000} + \frac{1}{32} \times \sqrt{42,000} + \frac{1}{64} \times \sqrt{74,000} + \frac{1}{64} \times \sqrt{10,000} = 123.494
\]
(note: the last term in the sum is for the case where the outcome is TTTTTTT and you are left with the initial $15,000 minus the price of $5,000 you paid to play).

Thus you should play the game.
EXAM 2

1. (a) $U'(x) = \frac{3}{125} - \frac{x}{500,000}$ and $U''(x) = -\frac{1}{500,000} < 0$. Thus Jennifer is risk-averse.

(b) $A(x) = \frac{U''(x)}{U'(x)} = \frac{1}{12,000 - x}$. Thus $A(4,000) = \frac{1}{8,000} = 0.000125$ and $A(6,000) = \frac{1}{6,000} = 0.000167$

(c) Jennifer's expected utility if she bets $2000 is:

$$\frac{3}{4} \left\{ 200 - \left[ 12 - \frac{6000 + 2000}{1000} \right]^2 \right\} + \frac{1}{4} \left\{ 200 - \left[ 12 - \frac{6000 - 2000}{1000} \right]^2 \right\} = 172.$$

(d) If Jennifer does not bet, her utility is:

$$\left\{ 200 - \left[ 12 - \frac{6000}{1000} \right]^2 \right\} = 164$$

(d) If Jennifer bets $y$, her expected utility is:

$$f(y) = \frac{3}{4} \left\{ 200 - \left[ 12 - \frac{6000 + y}{1000} \right]^2 \right\} + \frac{1}{4} \left\{ 200 - \left[ 12 - \frac{6000 - y}{1000} \right]^2 \right\}$$

Jennifer will choose $y$ to maximize $f(y)$. A necessary condition for this is that $f'(y) = 0$, i.e.

$$\frac{3}{4} \left( -2 \right) \left[ 12 - \frac{6000 + y}{1000} \right] \left( -\frac{1}{1000} \right) + \frac{1}{4} \left( -2 \right) \left[ 12 - \frac{6000 - y}{1000} \right] \left( \frac{1}{1000} \right) = 0$$

which gives $y = 3000$, that is, Jennifer will choose to bet $3000$.

(e) Her expected utility if she bets $3000 is:

$$\frac{3}{4} \left\{ 200 - \left[ 12 - \frac{6000 + 3000}{1000} \right]^2 \right\} + \frac{1}{4} \left\{ 200 - \left[ 12 - \frac{6000 - 3000}{1000} \right]^2 \right\} = 173.$$

(f) If she bets the optimal amount of $3000, her utility goes up, compared to not betting, by $(173 - 164) = 9$.

(g) If the probability is 50%, then her expected wealth (whatever the stake) is her initial wealth. Since Jennifer is risk-averse, she will not want to bet, that is, she will choose $y = 0$. 

Page 3 of 4
2. Suppose Peter does satisfy the axioms of expected utility and let $U$ be his utility-of-money function, normalized so that $U(5000) = 1$ and $U(0) = 0$. Let $U(1000) = p$. Then $0 < p < 1$. 

Now, $EU(A) = p$, $EU(B) = 0.1 \times (1 + 0.89 \times p) + 0.01 \times 0 = 0.1 + 0.89 \times p$, $EU(C) = 0.11 \times p$ and $EU(D) = 0.1$. 

Then $EU(A) > EU(B)$ if and only if $p > 0.1 + 0.89 \times p$, i.e. if and only if $p > \frac{10}{11}$. But if $p > \frac{10}{11}$ then $EU(C) = 0.11 \times p > \frac{10}{11} \times \frac{10}{11} = 0.1 = EU(D)$. Thus if Peter satisfies the axioms of expected utility and prefers $A$ to $B$ then he must also prefer $C$ to $D$. Hence he does not satisfy the axioms of expected utility.

3. (a) Let $W_0$ be the initial wealth. Then $h = W_0 - W_2$ and $d = W_2 - W_1$. Replacing in the formula $h = p(\ell - d) + c$ we get $W_0 - W_2 = p \ell - p(W_2 - W_1) + c$. Rearranging we get 

$$W_2 = \frac{W_0 - p \ell - c}{1 - p} - \frac{p}{1 - p} W_1.$$ 

(b) The above is the equation of a straight line with slope $-\frac{p}{1 - p}$, hence an isoprofit line. Any two contracts on this isoprofit line, expressed as wealth lotteries, have the same expected value. Let $F$ be the full-insurance contract on this line and $C$ any partial insurance contract on this line. Then $F$ guarantees the expected value of $C$ for sure and thus, by risk aversion, the individual will strictly prefer $F$ to $C$.

[A mathematically sophisticated student might suggest the following alternative proof, which – however – is less general because it assumes that the individual has vNM preferences. Let $U$ be a vNM utility-of-money function that represents the individual’s preferences. If the individual chooses deductible $d \geq 0$, her utility is ($W$ denotes initial wealth):

$$U(d) = p \ U(W - d - h) + (1-p) \ U(W - h) = p \ U(W - d - p \ell + pd - c) + (1-p) \ U(W - p \ell + pd - c).$$

The individual will then choose $d$ to maximize $U(d)$. Necessary condition is that $U'(d) = 0$. 

$$U'(d) = p \ U'(W - d - p \ell + pd - c) (1 + p) + (1 - p) \ U'(W - p \ell + pd - c) \ p =$$

$$= p \ (1-p) \left[ U'(W - p \ell + pd - c) - U'(W - d - p \ell + pd - c) \right].$$

This is equal to zero if and only if $U'(W - p \ell + pd - c) = U'(W - d - p \ell + pd - c)$. Since the individual is risk-averse, $U'' < 0$, hence $U'$ is decreasing. It follows that the equality is satisfied if and only if $W - p \ell + pd - c = W - d - p \ell + pd - c$ i.e. if and only if $d = 0$. Hence the individual will choose full insurance (= zero deductible).]