Department of Economics, University of California, Davis

Ecn 103 – Uncertainty and Information – Professor Giacomo Bonanno

**HOMEWORK # 3 (for due date see the web page)**

Consider the following risk-sharing problem. A Principal (owner of a firm) wants to hire an Agent to manage the firm. The Principal's utility-of-money function is

$$U(m) = \sqrt{m}$$

while the Agent's utility-of-money function is:

$$V(m) = \ln(m)$$

where ln denotes the natural logarithm. The profit of the firm will be $\$x_1$ with probability $p$ and $\$x_2$ with probability $(1-p)$. Consider the following contract: the Agent is paid $\$w_1$ if the outcome is $x_1$ and is paid $\$w_2$ if the outcome is $x_2$, where

$$x_1 = \$2{,}500, \quad x_2 = 1{,}300, \quad p = \frac{1}{3}, \quad w_1 = \$900 \quad \text{and} \quad w_2 = \$400.$$

**(a)** Calculate the Principal's and Agent's expected utility from this contract.

**(b)** Prove that this contract is not Pareto efficient.

**(c)** **(c.1)** Draw an Edgeworth box to represent all the possible contracts and identify the point that corresponds to the above contract.

**(c.2)** Draw the indifference curve of the Principal that goes through that point and the indifference curve of the Agent that goes through that point.

**(c.3)** State in words which of the two (if any) is a straight line.

**(d)** If you were to recommend a contract that is Pareto superior to the one given above, which of the following four suggestions would you make? (1) Increase both $w_1$ and $w_2$; (2) decrease both $w_1$ and $w_2$; (3) increase $w_1$ and decrease $w_2$; (4) decrease $w_1$ and increase $w_2$. Justify your suggestion using the Edgeworth box that you drew for part (c).