

HOMEWORK # 4 (for due date see the web page)

Let the quality of a second-hand car be denoted by $\theta \in \{1,2,3\}$, where θ is the number of tune-ups that the car received in the past. The value of a car of quality θ to the seller is $\$800\theta$. Each potential buyer has an initial wealth of $\$9,025$ and the utility of purchasing a car of quality θ at price P is $\sqrt{9,025 - P + 1,000\theta}$ (while the utility of not buying is $\sqrt{9,025} = 95$). Let the proportion of cars of each quality be as follows (where q is a number strictly between 0 and $\frac{1}{3}$): $\left(\begin{array}{c|ccc} \theta & 1 & 2 & 3 \\ \text{proportion} & q & \frac{2}{3} - q & \frac{1}{3} \end{array} \right)$. Suppose that the price of a second-hand car is $P = \$1,700$. [In the following assume that, if indifferent between selling and not selling, the owner of a car would sell and, if indifferent between buying and not buying, a potential buyer would buy.]

- (a) Are there values of q such that ALL cars are traded?
- (b) Are there values of q such that all cars of quality $\theta=1$ and $\theta=2$ are traded?
- (c) Are there values of q such that only cars of quality $\theta=1$ are traded?