1. (a) Barbara is facing the lottery \[ \begin{bmatrix} 2,400 & 3,600 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} \] whose expected value is \( \frac{1}{4} 2400 + \frac{3}{4} 3600 = 3,300 \). Being risk-averse she strictly prefers $3,300 for sure to facing the lottery. Thus if \( x = 300 \) and \( y = 1,200 \) then the contract guarantees her a wealth of $3,300 and she is better off. If Anton does not sign the contract his wealth is $4,000. If he signs the mentioned contract then he faces the lottery \[ \begin{bmatrix} 3,100 & 4,300 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} \] whose expected value is \( \frac{1}{4} 3,100 + \frac{3}{4} 4,300 = 4,000 \). Being risk neutral he is indifferent between signing and not signing the contract.

(b) The lottery she is facing is \( \begin{bmatrix} 3,600 & 2,400 \\ 0.75 & 0.25 \end{bmatrix} \) whose expected value is 3,300. The expected utility of the lottery is \( 0.75 \sqrt{3700} + 0.25 \sqrt{2500} = 58.12 \). The risk premium is given by the solution to \( \sqrt{3300 + 100 - r} = 58.12 \) which is $21.98.

2. (a)

(b) \( \frac{99}{100} (220,000) + \frac{1}{100} (60,000) = 218,400 \).

(c) \( \frac{99}{100} (218,500) + \frac{1}{100} (198,500) = 218,300 \).

(d) \( 1,500 - \frac{1}{100} (160,000 - 20,000) = 100 \).

(e) The deductible of contract B is the solution to \( 100 = 800 - \frac{1}{100} (160,000 - D_B) \) which is \( D_B = 90,000 \). See the above diagram.
(f) The slope of any isoprofit line is \( \frac{-1}{\frac{100}{99}} = -\frac{1}{99} \).

(g) It is the straight line with slope \(-\frac{1}{99}\) that goes through the NI point. Thus it is of the form \( W_2 = a - \frac{1}{99}W_1 \). Replacing the coordinates of the NI point we get \( 220,000 = a - \frac{1}{99}60,000 \) and solving for \( a \) we get \( W_2 = 220,606.61 - \frac{1}{99}W_1 \).

(g) Contract B involves the same expected wealth as contract A, namely $218,300, which is less than the expected wealth from no insurance. Thus a risk neutral person would prefer not to insure. Hence Sam will not buy contract B.

3. (a) Suppose that an individual satisfies the axioms of expected utility. Let \( U \) be her normalized von Neumann-Morgenstern utility function. Then, assuming that she prefers more money to less, \( U(4,000) = 1 \), \( U(3,000) = a \) and \( U(0) = 0 \), with \( 0 < a < 1 \). Thus expected utility of the four lotteries is:

\[
EU(A) = \frac{20}{100}, \quad EU(B) = \frac{25}{100}a, \quad EU(C) = \frac{80}{100} \quad \text{and} \quad EU(D) = a.
\]

If she chooses \( A \) over \( B \) then \( EU(A) = \frac{20}{100} > EU(B) = \frac{25}{100}a \), that is, \( a < \frac{20}{25} = \frac{80}{100} \), so that she must prefer \( C \) to \( D \). Thus choosing \( A \) over \( B \) and \( D \) over \( C \) or choosing \( B \) over \( A \) and \( C \) over \( D \) reveals a violation of expected utility. Thus 44+5=49 people for sure did not satisfy the axioms of expected utility.

(b) A risk neutral person ranks lotteries based on their expected values. Now,

\[
E_A = \frac{20}{100} \times 4,000 = 800, \quad E_B = \frac{25}{100} \times 3,000 = 750, \quad E_C = \frac{80}{100} \times 4,000 = 3,200 \quad \text{and} \quad E_D = 3,000.
\]

Thus he will rank \( A \) above \( B \) and \( C \) above \( D \).

(c) If \( U(m) = \ln(m) \) and your initial wealth is $21,000, then

\[
EU(A) = \frac{20}{100}\ln(25,000) + \frac{80}{100}\ln(21,000) = 9.987,
\]

\[
EU(B) = \frac{25}{100}\ln(24,000) + \frac{75}{100}\ln(21,000) = 9.985,
\]

\[
EU(C) = \frac{80}{100}\ln(25,000) + \frac{20}{100}\ln(21,000) = 10.092 \quad \text{and}
\]

\[
EU(D) = \ln(24,000) = 10.085.
\]

Thus you will choose \( A \) over \( B \) and \( C \) over \( D \).