University of California, Davis -- Department of Economics
ECN 103 : ECONOMICS of UNCERTAINTY  Professor Giacomo Bonanno
SPRING 2019 - SECOND MIDTERM EXAM  Version 2

Answer all questions.

NAME: ___________________________ University ID: ________________

- By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

- If you don’t explain (= show your work for) your answers you will get no credit.

- If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [65 points] Suppose that there are two groups of individuals:

<table>
<thead>
<tr>
<th>Group L</th>
<th>Group H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity = 8,000</td>
<td>Productivity = 15,000</td>
</tr>
<tr>
<td>Proportion in population: $q$</td>
<td>Proportion in population: $1 - q$</td>
</tr>
</tbody>
</table>

Education does not affect productivity. Workers of both types are able to buy any level of education $y$, at a cost. The amount of education $y \in [0, \infty)$ is a continuous variable and it is fully verifiable (e.g. through a certificate). Type-L workers face a higher cost of acquiring education than type-H workers:

- Cost of education for Group L individuals: $C_L(y) = 800y$
- Cost of education for Group H individuals: $C_H(y) = 400y$

Employers believe that anybody with a level of education less than $y^*$ has a productivity of 8,000 (and thus is offered a wage of 8,000) while everybody with a level of education greater than or equal to $y^*$ has a productivity of 15,000 (and thus is offered a wage of 15,000).

(a) [20 points] What values of $y^*$ give rise to a separating signaling equilibrium where H-types choose a level of education not less than $y^*$ while L-types choose a level of education less than $y^*$?
(b) [15 points] Suppose that \( y^* = 10 \) and the government steps in and forces everybody to choose \( y = 0 \). Employers react to this by paying everybody the average productivity of the population. For what values of \( q \) (with \( 0 < q < 1 \)) does the government’s intervention give rise to a Pareto improvement?

2. [65 points] Dan wants to sue his doctor for malpractice. He is now talking to his lawyer, Mr Getem. Mr Getem explains to Dan that there are four possible outcomes, expressed in terms of the sum of money that the hospital will have to pay Dan for damages. Mr Getem thinks that the amounts and probabilities are as follows:

\[
\begin{pmatrix}
\$15,000 & \$35,000 & \$45,000 & \$75,000 \\
\frac{2}{15} & \frac{1}{3} & \frac{4}{15} & \frac{4}{15}
\end{pmatrix}
\]

Dan’s current wealth is $10,000 and his utility-of-money function is \( U(m) = \sqrt{m} \). On the other hand, Mr Getem is risk-neutral and his current wealth is \( W \). He suggests one of two contracts. Contract A involves a payment to the lawyer equal to 20% of the awarded amount. Contract B involves a payment of $5,000 from Dan to the lawyer, if the judgment is less than 40,000, and a payment of $15,000 from Dan to the lawyer, if the judgment is more than 40,000.

(a) [4 points] What is Mr Getem’s expected utility from contract A?

(b) [4 points] What is Mr Getem’s expected utility from contract B?

(c) [4 points] Calculate Dan’s expected utility from contract A. [Don’t just give a number: write down the expression that you are evaluating.]
(d) [4 points] Calculate Dan’s expected utility from contract B. [Don’t just give a number: write down the expression that you are evaluating.]

(e) [4 points] Does one contract Pareto dominate the other? Explain.

The following day, Mr. Getem calls Dan and tells him that he did some more research and he now thinks that the judgment will not be $35,000, nor will it be $75,000. He thinks that it is going to be one of the other two amounts and that the probabilities should be updated according to Bayes’ rule. From now on use the updated probabilities.

(f) [5 points] What are the updated probabilities?

(g) [2 points] What is Mr Getem’s expected utility from contract A?

(h) [2 points] What is Mr Getem’s expected utility from contract B?

(i) [2 points] What is Dan’s expected utility from contract A?

(j) [2 points] What is Dan’s expected utility from contract B?
(k) [2 points] Does one of the two contracts Pareto dominate the other? Explain why.

(l) [4 points] Is contract A Pareto efficient? Explain why.

(m) [4 points] Is contract B Pareto efficient? Explain why.

(n) (n.1a) [4 points] Assume that Mr. Getem’s initial wealth is \( W = 20,000 \), so that he can use some of this money to pay Dan, if he so chooses. Find a contract which is just as good as contract B from the point of view of Dan and gives income certainty to Dan. [Mr Getem’s bank account allows the contract to be outside the Edgeworth box.]

(n.1b) [3 points] Show that the contract of part (n.1a) is better for Mr Getem than contract B.
(o) (n.2) [7 points] Assume now that Mr. Getem’s initial wealth is $W = 0$, so that he cannot give any
extra money to Dan, that is, the contract must be within the Edgeworth box. Find a Pareto
efficient contract which is just as good as contract B from the point of view of Mr Getem,
but better for Dan. You do need to compute the contract.

(p) [8 points] Suppose now that Mr Getem’s utility-of-money function is $V(m) = \ln(m)$ (the natural
logarithm function) and his initial wealth is $W = 0$. Show that contract A is not Pareto efficient.