1. (a) With a fixed wage of $200 Barb will choose low effort. Hence her utility will be $V(200,L) = 153.95$. Since Al is risk neutral, we can take his utility-of-money function to be $U(m) = m$ so that his expected utility will be $EU = \left( \frac{1}{10} \times 630 + \frac{4}{10} \times 360 + \frac{5}{10} \times 210 \right) - 200 = 112$.

(b) With contract B we first need to find out what level of effort Barb would choose. Her expected utility is as follows:

- If she chooses $e = L$: $\frac{1}{10} V(420,L) + \frac{4}{10} V(240,L) + \frac{5}{10} V(140,L) = 153.013$
- If she chooses $e = M$: $\frac{1}{8} V(420,M) + \frac{4}{8} V(240,M) + \frac{3}{8} V(140,M) = 154.454$
- If she chooses $e = H$: $\frac{2}{10} V(420,H) + \frac{3}{10} V(240,H) + \frac{5}{10} V(140,H) = 152.692$

Thus she would choose $e = M$ and have an expected utility of 154.454. Al’s expected utility would be $EU = \left( \frac{1}{8} \times 210 + \frac{4}{8} \times 120 + \frac{3}{8} \times 70 \right) = 112.5$.

(c) With contract C again we first need to find out what level of effort Barb would choose. Her expected utility is as follows:

- If she chooses $e = L$: $\frac{1}{10} V(225,L) + \frac{4}{10} V(105,L) + \frac{5}{10} V(105,L) = 136.905$
- If she chooses $e = M$: $\frac{1}{8} V(225,M) + \frac{4}{8} V(105,M) + \frac{3}{8} V(105,M) = 136.477$
- If she chooses $e = H$: $\frac{2}{10} V(225,H) + \frac{3}{10} V(105,H) + \frac{5}{10} V(105,H) = 137.192$

Thus she would choose $e = H$ and have an expected utility of 137.192. Al’s expected utility would be $EU = \left( \frac{2}{10} \times 405 + \frac{3}{10} \times 255 + \frac{5}{10} \times 105 \right) = 210$.

(d) For Al the ranking is $\begin{cases} \text{best} & C \\ \text{worst} & A \end{cases}$, while for Barb it is $\begin{cases} \text{best} & B \\ \text{worst} & A \end{cases}$.

(e) Yes: $A$ is Pareto dominated by $B$ (both Al and Barb prefer $B$ to $A$).

(f) Since the highest expected profit is obtained when $e = H$ (with relatively low utility loss for the extra effort for Barb) and Barb is risk-averse, a Pareto efficient contract would require $e = H$ and guarantee a fixed wage to Barb (i.e. let the risk-neutral Al bear all the risk).

2. [Note: this question was essentially the same as Q. 1 in the practice exam on the web page.] At a signaling equilibrium the employer's beliefs must be confirmed. Thus Group B workers must choose $y < y^*$ (in which case they would choose $y = 0$) and Group A workers must choose $y \geq y^*$ (in which case they would choose $y = y^*$). For Group B this requires: $8 > 10 + y^* - 3y^*$, that is, $y^* > 1$, while for Group A this requires: $10 + y^* - 2y^* \geq 8$, that is, $y^* \leq 2$. Thus $y^*$ gives rise to a signaling equilibrium if and only if $1 < y^* \leq 2$. 

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3. (a) Consider first \( p \geq 3,400 \). Then all surgeons are willing to perform surgery. Thus the expected value to a buyer is

\[
\frac{3}{10} \times 6,000 + \frac{3}{10} \times 3,000 + \frac{3}{10} \times 2,000 - \frac{1}{10} \times 2,000 = 3,100 < 3,400
\]

Thus no such \( p \) gives rise to an equilibrium.

Consider now \( 2,400 \leq p < 3,400 \). At any such price, Type D doctors drop out. Thus the expected value to a patient (updating probabilities according to Bayes’ rule) is:

\[
\frac{3}{7} \times 3,000 + \frac{3}{7} \times 2,000 - \frac{1}{7} \times 2,000 = \frac{13,000}{7} < 2,400
\]

Thus no such \( p \) gives rise to an equilibrium.

Consider now \( 1,200 \leq p < 2,400 \). At any such price, Types C and D drop out. Thus the expected value to a patient (updating probabilities according to Bayes’ rule) is:

\[
\frac{3}{4} \times 2,000 - \frac{1}{4} \times 2,000 = 1,000 < 1,200
\]

Thus no such \( p \) gives rise to an equilibrium.

Clearly \( p < 1,200 \) cannot yield an equilibrium because buyers know that, if at all, they would be dealing with a Type A doctor. Thus there is no value of \( p \) that gives rise to an equilibrium.

(b) Because of the new law, the situation is now as follows:

<table>
<thead>
<tr>
<th>Type of surgeon</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of surgery to surgeon</td>
<td>$1,400</td>
<td>$2,600</td>
<td>$3,600</td>
</tr>
<tr>
<td>Value of surgery to patient</td>
<td>$2,000</td>
<td>$3,000</td>
<td>$6,000</td>
</tr>
<tr>
<td>Proportion</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

Consider first \( p \geq 3,600 \). The expected value to a buyer is

\[
\frac{1}{3} \times 6,000 + \frac{1}{3} \times 3,000 + \frac{1}{3} \times 2,000 = \frac{11,000}{3} = 3,666.67
\]

Thus, for every \( p \) such that \( 3,600 \leq p \leq 3,666.67 \) we have an equilibrium where all (licensed) types of surgeons are active in the market.

Consider now \( 2,600 \leq p < 3,600 \). At any such price, Type D doctors drop out. Thus the expected value to a patient (updating probabilities according to Bayes’ rule) is:

\[
\frac{1}{2} \times 3,000 + \frac{1}{2} \times 2,000 = 2,500 < 2,600
\]

Thus no such \( p \) gives rise to an equilibrium.

Consider now \( 1,400 \leq p < 2,600 \). In this case only Type B doctors participate. Since surgery performed by a type B doctor is worth $2,000 to the patient, for an equilibrium we need \( p \leq 2,000 \). **Hence any \( p \) with \( 1,400 \leq p \leq 2,000 \) gives rise to an equilibrium.**

Clearly, when \( p < 1,200 \) no doctor is willing to offer surgery and therefore there is no market.