ECN 103 Professor Giacomo Bonanno

SECOND MIDTERM EXAM: ANSWERS for VERSION 2

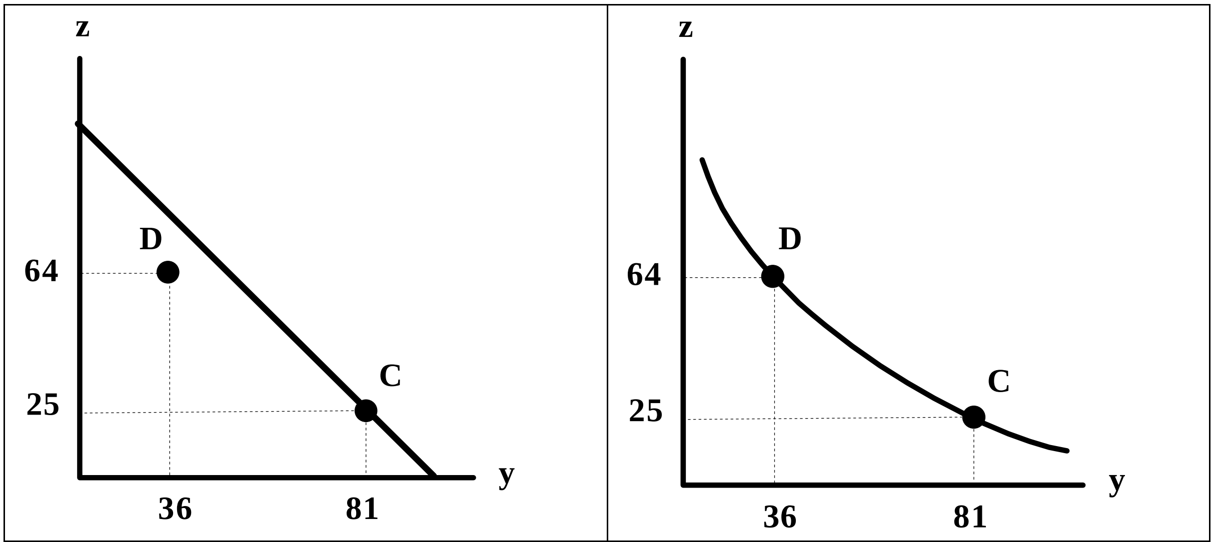
1. First of all, we need the probabilities in lottery B to add up to 1: $\frac{1}{24}+3p+\frac{1}{12}=1$. Solving this equation we get that $\boxed{p=\frac{7}{24}}$ so that $B=\begin{pmatrix}\$30 & \$36 & \$44 & \$48 \\ \frac{1}{24} & \frac{14}{24} & \frac{2}{24} & \frac{7}{24}\end{pmatrix}$. Two equations need to be satisfied in order for A to be a mean-preserving spread of B; the first ensures that the probabilities in lottery B add up to 1 and the second ensures that the expected value of A is equal to the expected value of B: $\boxed{r+s=\frac{14}{24}=\frac{7}{12}}$ and $\boxed{32r+40s=\frac{7}{12}36=21}$. [The second equation can also be written as follows, since $E[B]=\frac{479}{12}$: $\frac{2}{48}30+32r+40s+\frac{4}{48}44+\frac{14}{48}48=\frac{479}{12}$.] [The solution is $r=s=\frac{7}{24}$.]

2. **(a)** Since $\mathbb{E}[A]=\frac{2}{3}36+\frac{1}{3}81=51$ and $\mathbb{E}[B]=\frac{3}{5}16+\frac{2}{5}121=58$, a risk-neutral person prefers B to A.

(b) Since Amy is risk averse she prefers \$51 for sure (that is, $\mathbb{E}[A]$ for sure) to A.

(c) Since $\mathbb{E}[U(A)]=\frac{2}{3}6+\frac{1}{3}9=7$ and $\mathbb{E}[U(B)]=\frac{3}{5}4+\frac{2}{5}11=6.8$ Amy prefers A and B.

(d) and (e) See the following figures. Note that $\mathbb{E}[C]=53$, $\mathbb{E}[D]=50$ and $\mathbb{E}[U(C)]=\mathbb{E}[U(D)]=7$



(d)	€

(f) The slope if the same at every point and equal to $-\frac{\frac{1}{2}}{1-\frac{1}{2}}=-1$.

(g) Recall that $U'(m)=\frac{1}{2\sqrt{m}}$. The slope at C is $-\frac{U'(81)}{U'(25)}\left(\frac{p}{1-p}\right)=-\frac{\frac{1}{2\sqrt{81}}}{\frac{1}{2\sqrt{25}}}(1)=-\frac{5}{9}$ and the slope at D is $-\frac{U'(36)}{U'(64)}\left(\frac{p}{1-p}\right)=-\frac{\frac{1}{2\sqrt{36}}}{\frac{1}{2\sqrt{64}}}(1)=-\frac{4}{3}$.

(h) Since point F is on the 45° line, the two slopes are the same, namely –1.

3. It must be that A dominates B in the sense of First-Order stochastic dominance. There are only two values of x and y that yield that: $\boxed{x = \frac{1}{2} \text{ and } y = 0}$. Explanation: first of all, in order for B to be a lottery we need the probabilities to add up to 1, which requires $x + y = \frac{1}{2}$ or $y = \frac{1}{2} - x$. Thus we have:

$$\begin{matrix} \\ P_A: \\ cdf_A: \end{matrix} \begin{pmatrix} \$15 & \$16 & \$20 & \$28 & \$35 & \$45 \\ \frac{2}{10} & 0 & \frac{6}{10} & 0 & \frac{1}{10} & \frac{1}{10} \\ \frac{2}{10} & \frac{2}{10} & \frac{8}{10} & \frac{8}{10} & \frac{9}{10} & 1 \end{pmatrix} = A$$

$$\begin{matrix} \\ P_B: \\ cdf_B: \end{matrix} \begin{pmatrix} \$15 & \$16 & \$20 & \$28 & \$35 & \$45 \\ \frac{2}{10} & x & \frac{1}{10} & y = \frac{1}{2} - x & \frac{1}{10} & \frac{1}{10} \\ \frac{2}{10} & \frac{2}{10} + x & \frac{3}{10} + x & \frac{8}{10} & \frac{9}{10} & 1 \end{pmatrix} = B$$

In order for A to dominate B in the sense of First-Order stochastic dominance, it cannot be that $x = 0$, because then $cdf_A(\$20) = \frac{6}{10} > cdf_B(\$20) = \frac{3}{10}$. So it must be that $x > 0$. Then we have that $cdf_A(\$16) < cdf_B(\$16)$, which is fine, but we also need $cdf_A(\$20) = \frac{8}{10} \le cdf_B(\$20) = \frac{3}{10} + x$ that is, $x \ge \frac{1}{2}$ and this, together with $x + y = \frac{1}{2}$ gives: $x = \frac{1}{2}$ and $y = 0$.