

**WINTER 2024 - THIRD MIDTERM EXAM**      Version 2

Answer all questions. **If you don't explain (= show your work for) your answers you will get no credit.**

**NAME:** \_\_\_\_\_ **University ID:** \_\_\_\_\_

- **By writing your name on this exam you certify that you have not violated the University's Code of Academic Contact** (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).
  
- **If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.**
  
- **If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.**

1. [33 points] Consider the market for second-hand iPods. There are many potential sellers and many potential buyers. Second-hand iPods differ in their quality. Each seller knows the quality of her iPod, while the buyer does not. Let  $Q$  denote quality and assume that  $Q$  belongs to the set  $\{5,6,7,8\}$ . **There are 12,000 iPods.** The proportion of iPods of each quality is as follows:

Quality	5	6	7	8
Proportion	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{4}$

Buyers and sellers are **risk-neutral**. Let  $p$  denote the price. The owner of an iPod of quality  $Q$  who sells the iPod at price  $p$  gets a utility of  $(p - Q)$ , while her utility if she doesn't sell is zero. **If indifferent between selling and not selling, the seller chooses to sell.** The buyer's utility, if he buys an iPod of quality  $Q$  at price  $p$  is given by  $(1 + Q - p)$  while his utility if he doesn't buy is zero. **If indifferent between buying and not buying, the buyer chooses to buy.**

- (a) Suppose that  $p = 7.5$ .

[3 points] (a.1) Find the **number** of iPods that are offered for sale at that price,

[4 points] (a.2) Write the lottery (**in terms of quality levels**) that a buyer faces if he buys at that price,

[4 points] (a.3) calculate the buyer's expected **utility** if he buys at that price.

- (b) Suppose that  $p = 8.3$ .

[3 points] (b.1) Find the **number** of iPods that are offered for sale at that price,

[4 points] (b.2) write the lottery (**in terms of quality levels**) that a buyer faces if he buys at that price,

[4 points] (b.3) calculate the buyer's expected **utility** if he buys at that price.

Quality	5	6	7	8
Proportion	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{4}$

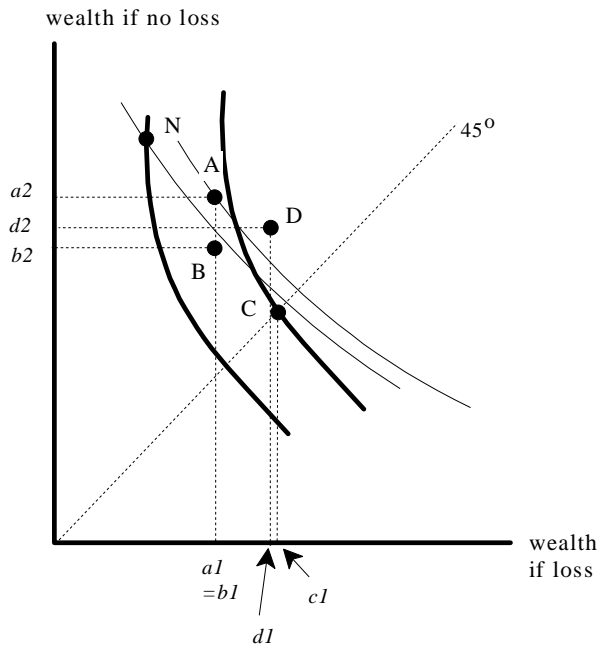
(c) Suppose that  $p = 6.4$ .

[3 points] **(dc.1)** Find the **number** of iPods that are offered for sale at that price,

[4 points] **(c.2)** write the lottery (**in terms of quality levels**) that a buyer faces if he buys at that price,

[4 points] **(c.3)** calculate the buyer's expected **utility** if he buys at that price.

- 2.** [24 points] URC is a monopolist in the market for insurance. There are two types of potential customers, Type  $a$  and Type  $b$ . Both types have the same initial wealth  $W = 42,000$  and face the same potential loss  $\ell = 18,000$ . Furthermore, they have the same utility-of-money function  $U(m)$  such that  $U'(m) > 0$  and  $U''(m) < 0$ . However, they differ in the probability of suffering a loss. Type  $a$  customers will incur a loss with probability  $\frac{1}{120}$ , while Type  $b$  will incur a loss with probability  $\frac{1}{40}$ . While each consumer knows her own type, URC cannot tell whether an applicant is of one type or the other. **There are 1,000 consumers of each type.** Assume that, if a potential customer is indifferent between insuring and not insuring, then she will choose to buy insurance. In the following diagram the thick indifference curves are the indifference curves of one type and the thin indifference curves are those of the other type (I am not telling you which is which, because you have to figure it out yourself).  $N$  is the no insurance point and  $A, B, C$  and  $D$  are possible insurance contracts.

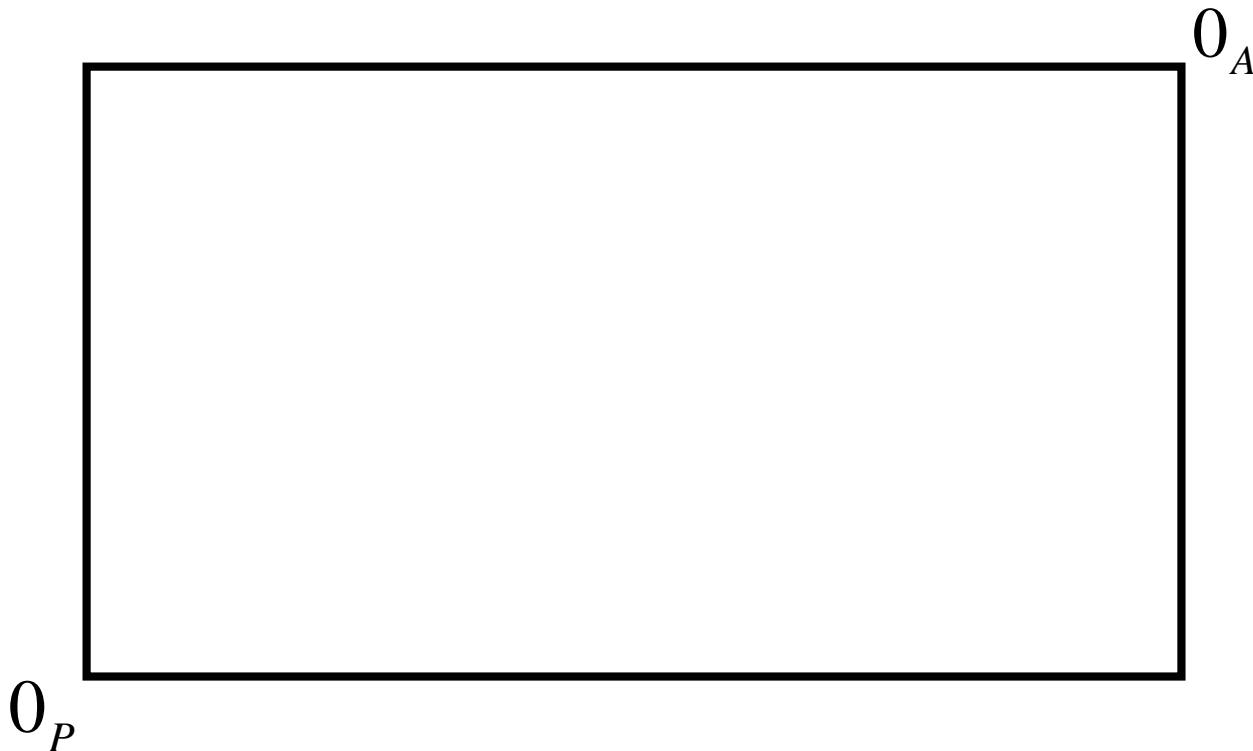


$$a1 = b1 = 39,900; a2 = 41,700; b2 = 41,400; c1 = 41,100; d1 = 40,650; d2 = 41,550.$$

- (a) [6 points] What are URC's expected profits if it decides to offer only contract *C*?
- (b) [6 points] What are URC's expected profits if it decides to offer contracts *B* and *C*?
- (c) [12 points] What are URC's expected profits if it decides to offer contracts *A*, *B* and *C*?

**3.** [43 points] The owner of a firm (the Principal) wants to hire a manager (the Agent) to run the firm. There are two possible outcomes, measured in terms of the profits of the firm:  $\pi_1 = \$6,000$  and  $\pi_2 = \$4,000$ . The probability of outcome  $\pi_1$  is 20% and the probability of outcome  $\pi_2$  is 80%. A contract is a pair  $(w_1, w_2)$ , where  $w_1$  is the payment to the Agent if the profit turns out to be  $\pi_1$  and  $w_2$  is the payment to the Agent if the profit turns out to be  $\pi_2$ . Consider the following contracts:  $A = (3,200, 1,600)$ ,  $B = (3,500, 1,500)$  and  $C = (2,000, 2,000)$ . **The Principal is risk averse and the Agent is risk neutral.** In the Edgeworth boxes below,  $0_p$  is the origin for the Principal and  $0_A$  is the origin for the Agent.

(a) (a.1) [9 points] In the Edgeworth box below draw the 45° lines and show where the three contracts  $A$ ,  $B$  and  $C$ , lie.



(a.2) [12 points] In the same Edgeworth box above, draw the indifference curves through contracts  $B$  and  $C$ , for both the Principal and the Agent and **clearly indicate the person to whom each indifference curve refers and the shape of each curve.**

(b) [10 points] Of the three contracts,  $A$ ,  $B$  and  $C$ , which (if any) are Pareto efficient? Explain your answer.

(c) [12 points] If the Principal's utility-of-money function is  $U(m) = \sqrt{m}$ , how does she rank the three contracts?