ANSWERS TO PRACTICE PROBLEMS

PROBLEM # 1: ANSWERS

The expected value of the lottery: \[
\begin{pmatrix}
24 & 12 & 48 & 6 \\
1 & 2 & 1 & 2 \\
6 & 6 & 6 & 6
\end{pmatrix}
\]
is \[
\frac{1}{6} 24 + \frac{2}{6} 12 + \frac{1}{6} 48 + \frac{2}{6} 6 = 18
\]

PROBLEM # 2: ANSWERS

The expected value cannot be computed because the prizes or basic outcomes are not numbers.

PROBLEM # 3: ANSWERS

The decision not to buy insurance is the decision to face the following lottery: with probability 0.9 Shirley’s wealth will be $200,000, with probability 0.1 it will be $125,000. The expected value of this lottery is:

\[
0.9 (200,000) + 0.1 (125,000) = $192,500.
\]
The insurance policy guarantees a wealth of $200,000 – $7,500 = $192,500. Hence Shirley will buy the insurance policy if she is risk-averse, will be indifferent between buying and not buying if she is risk-neutral and will prefer not to buy if she is risk-loving.

PROBLEM # 4: ANSWERS

If Bill refuses to invest his wealth is $12,000 for sure. If Bill gives $10,000 to Bob to invest then he faces the following lottery: \[
L = \begin{pmatrix}
2,000 & \$82,000 \\
88% & 12%
\end{pmatrix}
\]
The expected value of \( L \) is \[
2,000 \frac{88}{100} + 82,000 \frac{12}{100} = $11,600.
\]
If Bill were risk averse he would prefer $11,600 for sure to the investment (lottery \( L \)) and obviously he will prefer $12,000 to $11,600; thus he would prefer $12,000 for sure to the investment; since he decides to go ahead with the investment he is not risk averse. If Bill were risk neutral he would be indifferent between $11,600 for sure and the investment (lottery \( L \)) and obviously he will prefer $12,000 to $11,600; thus he would prefer $12,000 for sure to the investment; since he decides to go ahead with the investment he is not risk averse. Hence Bill is risk-loving.

PROBLEM # 5: ANSWERS

(a) First of all note that the initial wealth is \( W = 1,600 \) and the potential loss is \( x = 1,600 – 1,024 = 576 \). We write each point as a pair \((h, D)\) where \( h = W – W’ \) is the premium and \( D = W’ – W_1 \) is the deductible. Thus \( NI = (0, 576), \ A = (1600 – 1556, 1556 – 1390) = (44, 166), \ B = (1600 – 1480, 1480 – 1390) = (120, 90), \ A = (1600 – 1390 = 210, 0) \).
(b) The expected profit from a contract \((h, D)\) is \(\pi = h - p(x - D)\). Thus \(\pi(NI) = 0\), \(\pi(A) = -38\), \(\pi(B) = 22.8\) and \(\pi(C) = 94.8\).

(c) All the isoprofit lines are straight lines and all have the same slope given by \(-\frac{p}{1 - p} = -\frac{\frac{1}{4}}{\frac{3}{4}} = -\frac{1}{4}\).

Thus starting at a point \((W_1, W_2)\), if you reduce \(W_1\) to 0 then the vertical coordinate changes to \(W_2 + \frac{1}{4}W_1\), yielding the vertical intercept. Applying this to point NI we get that by reducing the horizontal coordinate by 1,024, the vertical coordinate increases by \(\frac{1,024}{4} = 256\) to \(1,600 + 256 = 1,856\). Hence the equation of the isoprofit line that goes through point NI (which is the zero-profit line) is \(W_2 = 1,856 - \frac{1}{4}W_1\). Applying the same procedure we get that

| Equation of isoprofit line through A | \(W_2 = 1,903.5 - \frac{1}{4}W_1\) |
| Equation of isoprofit line through B | \(W_2 = 1,827.5 - \frac{1}{4}W_1\) |
| Equation of isoprofit line through C | \(W_2 = 1,735.5 - \frac{1}{4}W_1\) |

PROBLEM #6: ANSWERS

Since Ben prefers B to A, he must prefer D to C.

Proof: Let \(V\) be a vonNeumann-Morgenstern utility function that represents Ben's preferences. Let \(V(4,000) = a\), \(V(3,000) = b\), \(V(0) = c\).

We can safely assume that \(a > b > c\) (more money is better). Define the function \(U\) as follows:

\[
U(x) = \frac{1}{a - c} V(x) - \frac{c}{a - c}.
\]

Then \(U\) represents the same preferences as \(V\). Hence we can work with \(U\). Now,

\[
U(4000) = \frac{V(4000) - c}{a - c} = \frac{a - c}{a - c} = 1, \quad U(3000) = \frac{V(3000) - c}{a - c} = \frac{b - c}{a - c},
\]

\[
U(0) = \frac{V(0) - c}{a - c} = \frac{c - c}{a - c} = 0. \quad \text{Let } q = \frac{b - c}{a - c}. \quad \text{Then } 0 < q < 1. \quad \text{Thus}
\]

\[
U(4000) = 1, \quad U(3000) = q \quad \text{(with } 0 < q < 1) \quad \text{and} \quad U(0) = 0.
\]

[You could have started directly with this normalized utility function without going through the normalization procedure.]

Now (EU stands for Expected Utility), \(EU(A) = 0.8 \, U(4000) + 0.2 \, U(0) = 0.8\), \(EU(B) = 1 \, U(3000) = q\). Since Ben prefers B to A, it must be \(q > 0.8\). Let us now compare C and D:

\[
EU(C) = 0.2 \, U(4000) + 0.8 \, U(0) = 0.2, \quad EU(D) = 0.25 \, U(3000) + 0.75 \, U(0) = 0.25 \, q.
\]

Since \(q > 0.8\), it follows that \(0.25q > (0.25)(0.8) = 0.2\). Hence \(EU(D) > EU(C)\).
PROBLEM # 7: ANSWERS

Suppose that there is a vonNeumann-Morgenstern utility function $U$ that represents Jennifer’s preferences. We can normalize it so that $U(3000)=1$ and $U(500)=0$.

Since Jennifer is indifferent between $L_1$ and $\$2000$, $U(2000) = \frac{5}{6}$.

Since she is indifferent between $L_2$ and $\$1000$, $U(1000) = \frac{2}{3}$.

Thus $EU(L_3) = 0 \left(1 + \frac{1}{4} \left(\frac{5}{6}\right) + \frac{1}{4} \left(\frac{2}{3}\right) + \frac{1}{4} (0) \right) = \frac{5}{8}$ and

$EU(L_4) = 0 \left(1 + \frac{1}{2} \left(\frac{5}{6}\right) + \frac{1}{2} \left(\frac{2}{3}\right) + 0 (0) \right) = \frac{3}{4}$.

Since $\frac{3}{4} > \frac{5}{8}$, Jennifer should prefer $L_4$ over $L_3$. Hence she is not rational according to the theory of expected utility.

PROBLEM # 8: ANSWERS

Normalize her utility function so that $U(z_1) = 1$ and $U(z_4) = 0$. Then, since Rachel is indifferent

between $\left(\begin{array}{c} z_2 \\ 1 \end{array}\right)$ and $\left(\begin{array}{c} z_3 \\ 8 \\ 2 \\ 10 \end{array}\right)$, we have that $U(z_2) = \frac{8}{10}$. Similarly, since she is indifferent

between $\left(\begin{array}{c} z_3 \\ 1 \end{array}\right)$ and $\left(\begin{array}{c} z_1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \end{array}\right)$, $U(z_3) = \frac{1}{2}$. Then the expected utility of $L_4 = \left(\begin{array}{c} z_1 \\ z_2 \\ z_3 \\ z_4 \end{array}\right)$ is

$\frac{1}{8} \times 1 + \frac{2}{8} \times 8 + \frac{3}{8} \times \frac{1}{2} + \frac{2}{8} \times 0 = \frac{41}{80} = 0.5125$ while the expected utility of $L_2 = \left(\begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}\right)$ is

$\frac{1}{5} \times 1 + \frac{3}{5} \times \frac{8}{10} + \frac{1}{5} \times \frac{1}{2} = \frac{39}{50} = 0.78$. Hence she prefers $L_2$ to $L_1$.

PROBLEM # 9: ANSWERS

(a) The expected value of $L_1$ is $\frac{2}{10} \times 30 + \frac{1}{10} \times 28 + \frac{1}{10} \times 24 + \frac{2}{10} \times 18 + \frac{4}{10} \times 8 = 18$. The expected value of $L_2$ is $\frac{1}{10} \times 30 + \frac{4}{10} \times 28 + \frac{5}{10} \times 8 = 18.2$. Hence a risk-neutral person would prefer $L_2$ to $L_1$.

(b) The expected utility of $L_1$ is $\frac{1}{5} \ln(30) + \frac{1}{10} \ln(28) + \frac{1}{10} \ln(24) + \frac{1}{5} \ln(18) + \frac{2}{5} \ln(8) = 2.741$

while the expected utility of $L_2$ is $\frac{1}{10} \ln(30) + \frac{2}{5} \ln(28) + \frac{1}{2} \ln(8) = 2.713$. Thus Paul prefers $L_1$ to $L_2$.

(c) The graph of $\ln(m)$ is as follows:
PROBLEM # 10:  **ANSWERS**

(i) $u(x) = x^{\frac{1}{2}}$,  $u'(x) = \frac{1}{2} x^{-\frac{1}{2}}$,  $u''(x) = -\frac{1}{4} x^{-\frac{3}{2}} = -\frac{1}{4 \sqrt{x^3}} < 0$ hence concave, thus risk-aversion.

(ii) $u(x) = \ln(x)$,  $u'(x) = \frac{1}{x}$,  $u''(x) = -\frac{1}{x^2} < 0$ hence concave, hence risk-aversion.

(iii) $u(x) = x^2$,  $u'(x) = 2x$,  $u''(x) = 2 > 0$ hence convex, hence risk-loving.

(iv) $u(x) = 5x + 2$,  $u'(x) = 5$,  $u''(x) = 0$ hence straight line, hence risk-neutral.

PROBLEM # 11:  **ANSWERS**

(i) $U'(x) = -2x + 2 \geq 0$,  $U''(x) = -2 < 0$. Risk averse.

(ii) $V'(x) = -6x + 6 \geq 0$,  $V''(x) = -6 < 0$. Risk averse. Thus they have the same attitude to risk.

(iii) Since $V(x) = a U(x) + b$ with $a = 3$ and $b = 12$, $V(x)$ and $U(x)$ represent the same preferences.

(iv) $U(x) = \sqrt{x}$,  $V(x) = \ln(x)$. The second derivative is negative in both cases, hence both display risk-aversion. On the other hand, it is not possible to find two numbers $a > 0$ and $b$ such that $U(x) = a V(x) + b$ (for all $x$) hence they do not represent the same preferences.

(v) We already saw that $U$ and $V$ represent the same preferences. This is confirmed by the Arrow-Pratt measure of absolute risk aversion: it is the same for both. In fact,

$$-\frac{U''(x)}{U'(x)} = -\frac{V''(x)}{V'(x)} = -\frac{1}{x-1}.$$
PROBLEM # 12: ANSWERS

The expected value of the lottery \[ \begin{pmatrix} 24 & 12 & 48 & 6 \\ 2 & 3 & 1 & 0 \\ 6 & 6 & 6 & 0 \end{pmatrix} \] is \[ \frac{24}{6} + \frac{12}{6} + \frac{48}{6} + 0(6) = 22. \]

(a) The risk premium is $22 - 18 = $4.
(b) Amy is risk-averse since she considers the lottery to be equivalent to a sum of money which is less than the expected value of the lottery (hence she prefers the expected value of the lottery for sure to the lottery).

PROBLEM # 13: ANSWERS

(a) The expected value of both lotteries is 18, hence Bill considers them to be equivalent (i.e. he is indifferent between the two). (b) Zero. (c) Zero

PROBLEM # 14: ANSWERS

(a) The expected value of \( L \) is \[ \frac{2}{10} \times 30 + \frac{1}{10} \times 28 + \frac{1}{10} \times 24 + \frac{2}{10} \times 18 + \frac{4}{10} \times 8 = 18. \]

(b) The expected utility of \( L \) is \[ \frac{2}{10} \times \sqrt{30} + \frac{1}{10} \times \sqrt{28} + \frac{1}{10} \times \sqrt{24} + \frac{2}{10} \times \sqrt{18} + \frac{4}{10} \times \sqrt{8} = 4.094. \]

(c) The risk premium is the value of \( r \) that solves the following equation: \( \sqrt{18} - r = 4.094 \). Thus \( r = $1.24. \)

(d) \[ \frac{d \sqrt{m}}{dm} = \frac{1}{2 \sqrt{m}}. \]

(e) \[ \frac{d^2 \sqrt{m}}{dm^2} = -\frac{1}{4m \sqrt{m}} < 0 \text{ for every } m > 0. \]

(f) Jennifer is risk averse.

(g) \[ R_A(m) = \frac{U''(m)}{U'(m)} = -\frac{1}{4 \sqrt{m}} = \frac{1}{2m} > 0. \]

(h) \[ R_A(900) = \frac{1}{2(900)} = \frac{1}{1,800}, \quad R_A(1,600) = \frac{1}{2(1,600)} = \frac{1}{3,200}. \]

PROBLEM # 15: ANSWERS

(a) Obviously, the expected value is still 18. (b) The expected utility of \( L \) is \[ \frac{2}{10} \times (20\sqrt{30} - 4) + \frac{1}{10} \times (20\sqrt{28} - 4) + \frac{1}{10} \times (20\sqrt{24} - 4) + \frac{2}{10} \times (20\sqrt{18} - 4) + \frac{4}{10} \times (20\sqrt{8} - 4) = 77.88. \]

Note that this is equal to 20(4.094) − 4 (recall that 4.094 was the expected utility with the function \( U(m) = \sqrt{m} \)).

(c) The risk premium is the value of \( r \) that solves the following equation: \( 20\sqrt{18} - r - 4 = 77.88 \). Thus \( r = $1.24 \) (same as with the function \( U(m) = \sqrt{m} \)).

(d) \[ \frac{d}{dm} (20\sqrt{m} - 4) = 20 \times \frac{1}{2 \sqrt{m}} = \frac{10}{\sqrt{m}}. \]
(e) \( \frac{d^2}{dm^2} (20\sqrt{m} - 4) = -20 \frac{1}{4\sqrt{m^3}} = -\frac{5}{\sqrt{m^3}} < 0 \) for every \( m > 0 \).

(f) John is risk averse.

(g) \( R_a(m) = \frac{V^*(m)}{V'(m)} = \frac{-\frac{5}{\sqrt{m^3}}}{\frac{10}{\sqrt{m}}} = \frac{1}{2m} > 0 \) (same as before).

(h) \( R_a(900) = \frac{1}{2(900)} = \frac{1}{1,800}, \quad R_a(1,600) = \frac{1}{2(1,600)} = \frac{1}{3,200} \) (same as before).

Comments: the utility function \( V(m) = 20\sqrt{m} - 4 \) represents the same preferences as the utility function \( U(m) = \sqrt{m} \) (in fact \( V(m) \) can be obtained from \( U(m) \) by multiplying the latter by 20 and subtracting 4). Thus anything which is true for Jennifer is also true for John (e.g. risk attitude, ranking of any two lotteries, risk premium, measures of risk aversion, etc.). In an exam, if you are given the utility function \( V(m) = 20\sqrt{m} - 4 \) you can simplify your life by transforming it to the equivalent function \( U(m) = \sqrt{m} \) and then use the latter for your calculations.

**PROBLEM #16: ANSWERS**

(a) The expected utility of lottery \( A = \left( \begin{array}{cc} 10 & 40 \\ \frac{1}{3} & \frac{2}{3} \end{array} \right) \) is \( \frac{1}{3} \ln(10) + \frac{2}{3} \ln(40) = 3.227 \).

(b) The expected utility of lottery \( B = \left( \begin{array}{cc} 10 & 10 \\ \frac{1}{3} & \frac{2}{3} \end{array} \right) \) is \( \frac{1}{3} \ln(10) + \frac{2}{3} \ln(10) = 2.303 \).

(c) and (d). The indifference curve that goes through \( A \) is the set of lotteries that yield an expected utility of 3.227. It is a convex curve since the function \( U \) displays risk aversion. The slope of the indifference curve at point \( A \) is equal to \( -\frac{p}{1-p} \frac{U'(10)}{U'(40)} = -\frac{\frac{1}{3}}{\frac{1}{10}} = -\frac{10}{3} \).

Similarly, the indifference curve that goes through \( B \) is the set of lotteries that yield an expected utility of 2.303. It is a convex curve. The slope of the indifference curve at point \( B \) is equal to \( -\frac{p}{1-p} \frac{U'(10)}{U'(100)} = -\frac{\frac{1}{3}}{\frac{1}{10}} = -\frac{10}{3} \).

**PROBLEM #17: ANSWERS**

Since the individual is risk neutral, we can take his utility-of-money function to be the identity function \( U(m) = m \). Thus the expected utility of a lottery coincides with the expected value.
(a) The expected utility of lottery $A = \begin{pmatrix} 10 & 40 \\ 1/3 & 2/3 \end{pmatrix}$ is $\frac{1}{3} \cdot 10 + \frac{2}{3} \cdot 40 = \frac{90}{3} = 30$

(b) The expected utility of lottery $B = \begin{pmatrix} 10 & 10 \\ 1/3 & 2/3 \end{pmatrix}$ is 10.

(c) and (d). The indifference curve that goes through $A$ is the set of lotteries whose expected value is 30. It is a straight line with slope $-\frac{1}{2} = -\frac{1}{2}$. Similarly, the indifference curve that goes through $B$ is the set of lotteries whose expected value is 10. It is a straight line with slope $-\frac{1}{2}$.

PROBLEM # 18: ANSWERS

If he does not buy insurance he faces the lottery $\begin{pmatrix} W & W-x \\ 1-p & p \end{pmatrix}$ whose expected value is $(W - px)$. If $h^*$ is the maximum premium he is willing to pay for insurance [that is, $h^*$ is the solution to the equation $U(W - h^*) = (1 - p)U(W) + pU(W - x)]$ and $r$ is the risk premium associated with the no insurance lottery [that is, $r$ is the solution to the equation]
\[ U(W - px - r) = (1 - p)U(W) + pU(W - x) \], then \( h^* = px + r \). Thus we have the following equation \( 800 = \frac{1}{10} x + 500 \), which gives \( x = 3,000 \).

**PROBLEM # 19: ANSWERS**

(i) The probability of the loss is 0.05, the size of the loss is $110,000. Thus the expected loss is \( 110,000 \times 0.05 = $5,500 \).

(ii) Bob’s expected wealth if he does not buy insurance is:

\[
(0.05) \times 85,000 + (0.95) \times 195,000 = 189,500.
\]

(iii) Bob’s expected utility if he does not buy insurance is:

\[
(0.05) \left[800 - (20 - 8.5)^2 \right] + (0.95) \left[800 - (20 - 19.5)^2 \right] = 793.15.
\]

(iv) If Bob buys full insurance for $5,500, he is guaranteed a wealth of $189,500. Thus his utility is

\[
\left[800 - (20 - 18.95)^2 \right] = 798.8975.
\]

(v) Since 798.8975 > 793.15, Bob would indeed by insurance if the premium were $5,500.

(vi) To find the maximum premium Bob is willing to pay, we need to solve the following equation with respect to \( h \) (the LHS is Bob’s expected utility if he does not buy insurance):

\[
793.15 = 800 - [20 - (19.5 - h)]^2.
\]

The solution is \( h = $21,173 \): almost four times the expected loss!

An alternative way of obtaining the same answer is the following. Solve the following equation with respect to \( w \):

\[
793.15 = 800 - (20 - w)^2
\]

The solution is \( w = 17.3827 \). Thus if Bob were guaranteed a wealth of $173,827, his utility would be the same as his expected utility without insurance. Thus Bob is willing to pay up to $(195,000 - 173,827) = $21,173 for full insurance.

**PROBLEM # 20: ANSWERS**

(a) \( 0.9\sqrt{80,000} + 0.1\sqrt{60,000} = 279.053 \)

(b) Expected utility from a policy with premium \( h \) and deductible \( D \) is

\[
0.9\sqrt{80,000 - h} + 0.1\sqrt{80,000 - h - D}
\]

Substituting the values from the possible policies we get:

<table>
<thead>
<tr>
<th>premium</th>
<th>deductible</th>
<th>expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,340</td>
<td>500</td>
<td>278.586</td>
</tr>
<tr>
<td>2,280</td>
<td>1,000</td>
<td>278.603</td>
</tr>
<tr>
<td>2,220</td>
<td>1,500</td>
<td>278.62</td>
</tr>
<tr>
<td>2,160</td>
<td>2,000</td>
<td>278.637</td>
</tr>
</tbody>
</table>

None of the policies gives her a higher expected utility than no insurance. Hence she will not buy insurance.
PROBLEM # 21: **ANSWERS**

\[ u(z) = a - b \, e^{-z} \quad (a>0, \ b>0), \quad u'(z) = b \, e^{-z} > 0, \quad u''(z) = -b \, e^{-z} < 0 \] thus risk-averse. Absolute risk aversion: \(-\frac{u''}{u'} = 1\), hence constant. The maximum premium \(h\) is determined by the solution to: 

\[ u(w - h) = p \, u(w - x) + (1 - p) \, u(w), \] i.e.

\[ a - b \, e^{-w+h} = p [a - b \, e^{-w+x}] + (1 - p) \left[ a - b \, e^{-w} \right]. \]

Thus: 

\[ a - b \, e^{-w+h} = a - b \left[ p \, e^{-w+x} + (1-p) \, e^{-w} \right]. \]

Subtract a from both sides and then multiply by \(-\frac{1}{b} \): 

\[ e^{-w} \, e^{h} = p \, e^{-w} \, e^{x} + (1-p) \, e^{-w}. \]

Multiply both sides by \(e^{w}\) to obtain 

\[ e^{h} = p \, e^{x} + (1-p) \]. Thus:

\[ h = \ln (p \, e^{x} + 1 - p) \] hence independent of \(w\).

PROBLEM # 22: **ANSWERS**

Expected utility from policy \((h, D)\) with \(h = 0.8 - 0.2D\) is:

\[ f(D) = \frac{1}{6} \ln[10 - D - (0.8 - 0.2D)] + \frac{5}{6} \ln[10 - (0.8 - 0.2D)]. \]

Optimal deductible given by the solution to \(f'(D) = 0\), i.e.

\[ \frac{1}{6} \frac{1}{10 - D - (0.8 - 0.2D)} (-1 + 0.2) + \frac{5}{6} \frac{1}{10 - (0.8 - 0.2D)} (0.2) = 0 \]

which gives \(D = 1.92\) and \(h = 0.8 - 0.2(1.92) = 0.416\).

PROBLEM # 23: **ANSWERS**

The expected utility from choosing a contract with deductible \(D\) is

\[ f(D) = (1-p) \ln(w-h) + p \ln(w-h-D) \]

where \(h = p(1+k)(x-D)\). The optimal deductible is given by the solution to \(f'(D) = 0\).

Recall that \(\frac{d}{dx} \ln(x) = \frac{1}{x}\).

(a) if \(w = 10, \ x = 4, \ p = \frac{1}{6}, \ k = \frac{1}{5}\), then \(h = p(1+k)(x-D) = \frac{1}{6} \left( 1 + \frac{1}{5} \right) (4-D) = \frac{4-D}{5} \).

Thus

\[ f(D) = \frac{5}{6} \ln \left( \frac{10-4-D}{5} \right) + \frac{1}{6} \ln \left( \frac{10-4-D}{5} - D \right) = \frac{5}{6} \ln \left( \frac{46+D}{5} \right) + \frac{1}{6} \ln \left( \frac{46-4D}{5} \right) \].

\[ f'(D) = \frac{5}{6} \left( \frac{1}{46+D} \right) \left( \frac{1}{5} \right) + \frac{1}{6} \left( \frac{1}{46-4D} \right) \left( -\frac{4}{5} \right) = \frac{5}{6} \left( \frac{1}{46+D} \right) - \frac{4}{6} \left( \frac{1}{46-4D} \right) \]

This is equal to zero when \(\frac{5}{46+D} = \frac{4}{46-4D}\), that is, when \(5(46-4D) = 4(46+D)\), that is, when \(D = \frac{46}{24} = 1.92\) and \(h = \frac{5}{12} = 0.416\) (same as in Problem 18)
The solution to \( f'(D) = 0 \) is \( D = \frac{62}{21} = 2.952 \) which gives a premium of \( h = \frac{8}{21} = 0.381 \).

**PROBLEM # 24: ANSWERS**

Contract \( A \) gives rise to the following lottery for the Agent: \( \left( \begin{array}{c} 700 \\ 1 \end{array} \right) \) and therefore an expected utility of \( V(700) = 2(700) + 2 = 1402 \). The lottery for the Principal is
\[
\left( \begin{array}{ccc} 2,400 - 700 & 1,600 - 700 & 900 - 700 \\ \frac{1}{5} & 2 & 2 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{array} \right) = \left( \begin{array}{ccc} 1,700 & 900 & 200 \\ \frac{1}{5} & 2 & 2 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{array} \right)
\]
with corresponding expected utility of \( EU = \frac{1}{5} \ln(1,700) + \frac{2}{5} \ln(900) + \frac{2}{5} \ln(200) = 6.328 \).

Contract \( B \) gives rise to the following lottery for the Agent: \( \left( \begin{array}{ccc} 1,200 & 1,000 & 200 \\ \frac{1}{5} & 2 & 2 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{array} \right) \) and therefore an expected utility of \( EV = \frac{1}{5} 2(1,200) + \frac{2}{5} 2(1,000) + \frac{2}{5} 2(200) + 2 = 1442 \). The lottery for the Principal is
\[
\left( \begin{array}{ccc} 2,400 - 1,200 & 1,600 - 1,000 & 900 - 200 \\ \frac{1}{5} & 2 & 2 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{array} \right) = \left( \begin{array}{ccc} 1,200 & 600 & 700 \\ \frac{1}{5} & 2 & 2 \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \end{array} \right)
\]
with corresponding expected utility of \( EU = \frac{1}{5} \ln(1,200) + \frac{2}{5} \ln(600) + \frac{2}{5} \ln(700) = 6.597 \).
Thus both Principal and Agent prefer contract \( B \) to contract \( A \), that is, contract \( A \) is Pareto dominated by contract \( B \).

**PROBLEM # 25: ANSWERS**

(a) Since \( U^*(m) = -\frac{1}{4\sqrt{m^3}} < 0 \), John is risk averse.

(b) Since \( V^*(m) = 0 \), Joanna is risk neutral.

(c) Contract \( A \) gives rise to the following lottery for John:
\[
\left( \begin{array}{ccc} 4,000 - 1,000 = 3,000 & 1,600 - 1,000 = 600 \\ \frac{2}{3} & \frac{1}{3} \end{array} \right)
\]
with corresponding expected utility of
\[
\frac{2}{3} \sqrt{3,000} + \frac{1}{3} \sqrt{600} = 44.68 \]. Joanna’s utility from contract \( A \) is \( V(1,000) = 2(1,000) = 2,000 \).

Contract \( B \) gives rise to the following lottery for John:
\[
\left( \begin{array}{ccc} 4,000 - 1,500 = 2,500 & 1,600 \\ \frac{2}{3} & \frac{1}{3} \end{array} \right)
\]
with corresponding expected utility of \( \frac{2}{3} \sqrt{2,500} + \frac{1}{3} \sqrt{1,600} = 46.67 \). Thus John prefers contract \( B \).
For Joanna contract B gives rise to the following lottery \( \begin{pmatrix} 1,500 & 0 \\ 2 & 1 \\ 3 & 3 \end{pmatrix} \) with corresponding expected utility of \( \frac{2}{3} V(2,500) + \frac{1}{3} V(0) = \frac{2}{3} \cdot 2(1,500) + \frac{1}{3} \cdot 2(0) = 2000 \). Thus Joanna is indifferent between the two contracts.

\( \textbf{(d)} \) The theory of optimal risk sharing says that if one party is risk neutral and the other risk averse, then Pareto efficiency requires that the risk-averse party be guaranteed a fixed income. This principle, however, applies only if the contract is in the interior of the Edgeworth box. There are also Pareto efficient contracts that lie on the long side of the Edgeworth box and off the 45\( ^{\circ} \) line for the risk-averse person. In the case under consideration, the set of Pareto-efficient contracts is shown as the thick line in the following figure:

Contract B has coordinates \((w_1 = 1,500, w_2 = 0)\) (from the point of view of the Agent) or \((x_1 - w_1 = 2,500, x_2 - w_2 = 1,600)\) (from the point of view of the Principal) and is thus on the horizontal part of the thick line. Hence contract B is Pareto efficient.

(Contract B would not be Pareto efficient if the Agent had some savings she could draw from to pay the Principal if the profits turned out to be $1,600: in that case there is a contract outside the box that Pareto dominates contract B.)

**PROBLEM # 26: ANSWERS**

\( \textbf{(a)} \) The Principal is risk-averse, since \( U''(m) = -\frac{1}{m^2} < 0 \).

\( \textbf{(b)} \) The Agent is also risk-averse, since \( V''(m) = -\frac{1}{100} e^{-\frac{x}{100}} < 0 \) (recall that \( \frac{d}{dx} e^x = e^x \)).

\( \textbf{(c)} \) Pareto efficiency requires \( \frac{U'(x_1 - w_1)}{U'(x_2 - w_2)} = \frac{V'(w_1)}{V'(w_2)} \). Now, \( U'(m) = \frac{1}{m} \) and \( V'(m) = e^{\frac{x}{100}} \). Thus \( \frac{U'(x_1 - w_1)}{U'(x_2 - w_2)} = \frac{U'(600)}{U'(400)} = \frac{\frac{1}{600}}{\frac{1}{400}} = \frac{400}{600} = \frac{2}{3} = 0.67 \) while \( \frac{V'(w_1)}{V'(w_2)} = \frac{V'(400)}{V'(200)} = \frac{e^{4/100}}{e^{2/100}} = e^{2/100} = 1.135 \). Thus the contract is not Pareto efficient.
(d) From the above calculations we see that the Principal’s indifference curve through the proposed contract is steeper than the Agent’s indifference curve. Thus any contract in the shaded region is Pareto superior to the given contract. Hence \( w_1 \) should be decreased and \( w_2 \) should be increased.

![Graph showing indifference curves and contracts](image)

PROBLEM # 27: **ANSWERS**

(i) P’s expected utility is:
\[
\frac{23}{40} \sqrt{520 - 250} + \frac{17}{40} \sqrt{200 - 80} = 14.10
\]

(ii) A’s expected utility is:
\[
\left( 3 \left( \frac{250}{16} \right) + \frac{27}{40} \right) \frac{23}{40} + \left( 3 \left( \frac{80}{16} \right) + \frac{27}{40} \right) \frac{17}{40} = 534.9375
\]

(iii) A necessary condition for there not to exist a Pareto superior contract is that
\[
\frac{U'(270)}{U'(120)} = \frac{V'(250)}{V'(80)}.
\]

Now, \( U'(y) = \frac{1}{2\sqrt{y}} \). Thus
\[
\frac{U'(270)}{U'(120)} = \frac{\sqrt{120}}{\sqrt{270}} = 0.67.
\]

\( V'(w) = 3 \) for every \( w \). Thus
\[
\frac{V'(250)}{V'(80)} = \frac{3}{3} = 1.
\]

Hence the above condition is not satisfied and there is a Pareto superior contract.
Notice that P is risk-averse and is facing a lottery that gives him $270 with probability \(\frac{23}{40}\) and $120 with probability \(\frac{17}{40}\). The expected value of the lottery is $206.25. Thus if given $206.25 for sure, P will be better off. On the other hand, since A is risk-neutral, she will be indifferent between the original contract (that gives her $250 with probability \(\frac{23}{40}\) and 80 with probability \(\frac{17}{40}\), with expected value 177.75) and a contract that gives P $206.25 for sure (so that A would get $313.75 with probability \(\frac{23}{40}\) and would pay $6.25 with probability \(\frac{17}{40}\), with expected value 177.75). Thus a Pareto superior contract is the following:

A will get $313.75 if the firm’s profit is $520 and will pay $6.25 if the firm’s profit is $200.

This contract leaves Ms. A as well off as before, but makes P better off.

The following contract is an example of a contract that is preferred by both:

A gets $320 if the outcome is $520, otherwise she gets nothing. Then P’s expected utility will be (he will be getting $200 in every case):

\[\sqrt{200} = 14.14 > 14.10\]

while A’s expected utility will be

\[
\left(3 \left(\frac{320}{16}\right) \frac{23}{40} + \left(3 \left(0\right) + \frac{27}{16}\right) \frac{17}{40}\right) = 553.69 > 534.9375.
\]

Let R, S and T denote the following contracts:

R = Ms. A gets $250 if the profit is $520, otherwise she gets $80.
S = Ms. A gets $313.75 if the profit is $520, otherwise she pays $6.25.
T = Ms. A gets $320 if the profit is $520, otherwise she gets nothing.

These contracts are represented in the following Edgeworth box.
How on earth did I think of these two points, S and T? Well, the original contract gives Mr. P $270 with probability \((23/40)\) and $120 with probability \((17/40)\). The expected value of this lottery is: \((23/40) \times 270 + (17/40) \times 120 = 206.25\). Being risk-averse, Mr. P would prefer to get $206.25 for sure, that is, would prefer to be at point S (on the 45° line through \(0_p\)). Since S is not in the box, the point in the box which is closest to S and also on the 45° line through \(0_p\) is T. Simple, isn’t it?

**PROBLEM # 28: ANSWERS**

(i) The Agent’s utility is zero, while the Principal’s expected utility is:

\[
\frac{1}{16} \sqrt{\frac{5}{4}} - 1 + \frac{3}{16} \sqrt{\frac{3}{2}} - 1 + \frac{12}{16} \sqrt{2} - 1 = 0.91.
\]

[Note: what sum of money for sure would the Principal consider equivalent to this lottery? That sum of money \(x\) such that \(x = 0.91\). Thus \(x = (0.91)^2 = 0.8281\).

(ii) Since the Principal is risk-averse and the Agent is risk-neutral, Pareto efficiency requires that the Agent bear all the risk (i.e. the Principal must be guaranteed a fixed income). An optimal contract would be, for example: \(w = x - 0.8281\) (so that the Principal gets $0.8281 in every state with a utility of \(\sqrt{0.8281} = 0.91\)). The Agent therefore would get:

\[
\frac{1}{16} \left(\frac{5}{4} - 0.8281 - 1\right) + \frac{3}{16} \left(\frac{3}{2} - 0.8281 - 1\right) + \frac{12}{16} \left(2 - 0.8281 - 1\right) = 0.031 > 0.
\]
PROBLEM # 29: **ANSWERS**

(i) Under contract $C$ the Agent’s utility is $V(\$20) = 20$, while the Principal’s utility is

\[
\frac{1}{5}\sqrt[5]{141 - 20} + \frac{1}{5}\sqrt[5]{164 - 20} + \frac{1}{5}\sqrt[5]{461 - 20} + \frac{1}{5}\sqrt[5]{645 - 20} + \frac{1}{5}\sqrt[5]{749 - 20} = \frac{1}{5}(11 + 12 + 21 + 25 + 27) = 19.2
\]

(ii) Under contract $D$ the Agent’s utility is $\frac{1}{5}0 + \frac{2}{5}60 = 24$ and thus the Agent prefers contract $D$ to contract $C$. Under contract $D$ the Principal’s utility is:

\[
\frac{1}{5}\left(\sqrt[5]{141} + \sqrt[5]{164} + \sqrt[5]{461} + \sqrt[5]{645} - 60 + \sqrt[5]{749} - 60\right) = 19.3174.
\]

Thus also the Principal prefers contract $D$ to contract $C$. Hence contract $D$ is Pareto superior to contract $C$.

PROBLEM # 30: **ANSWERS**

CONTRACT A. If the Agent works hard, he will get a utility of $V(10, H) = \sqrt{90 + 10} - 10 = 10 - 10 = 0$. If he does not work hard, his utility will be: $V(10, L) = \sqrt{90 + 10} - 9 = 10 - 9 = 1$. Since $1 > 0$, he will choose $e = L$, that is, he will not work hard. The Principal can figure this out and will realize that his expected utility from this contract is: $\frac{1}{2} (-100 - 10) + \frac{1}{4} (100 - 10) + \frac{1}{4} (500 - 10) = 90$.

CONTRACT B. If the Agent works hard, his expected utility will be: $\frac{1}{4} (\sqrt{90} - 10) + \frac{1}{4} (\sqrt{90} - 10) + \frac{1}{2} (\sqrt{90 + 100} - 10) = 1.635$. If he does not work hard, his expected utility will be: $\frac{1}{2} (\sqrt{90} - 9) + \frac{1}{4} (\sqrt{90} - 9) + \frac{1}{4} (\sqrt{90 + 100} - 9) = 1.561$. Since $1.635 > 1.561$, he will choose $e = H$, that is, he will work hard. The Principal can figure this out and will realize that his expected utility from this contract is: $\frac{1}{4} (-100) + \frac{1}{4} (100) + \frac{1}{2} (500 - 100) = 200$.

Thus contract B is Pareto superior to contract A, despite the fact that it does not allocate risk optimally.
PROBLEM # 31:  \textbf{ANSWERS}

\textbf{CONTRACT A}

If Ms. Managy is lazy she will get a utility of $10 - 8 = 2$, if she works hard she will get a utility of $10 - 10 = 0$. Hence she will be lazy.

Mr. Owny’s expected utility will be:

$$
\frac{1}{2} (-10) + \frac{1}{4} (100 - 10) + \frac{1}{4} (800 - 10) = 215.
$$

\textbf{CONTRACT B}

If Ms. Managy is lazy she will get an expected utility of:

$$
(-8) \frac{1}{2} + (-8) \frac{1}{4} + (24 - 8) \frac{1}{4} = -2.
$$

If she works hard she will get an expected utility of:

$$
(-10) \frac{1}{4} + (-10) \frac{1}{4} + (24 - 10) \frac{1}{2} = 2.
$$

Hence she will work hard.

Given that she will work hard, Mr. Owny’s expected utility will be:

$$
0 \frac{1}{4} + 100 \frac{1}{4} + (800 - 24) \frac{1}{2} = 413.
$$

Hence Ms. Managy would be indifferent between the two contracts (and accept either of them), while Mr. Owny prefers contract B. Unlike Contract A, Contract B provides Ms. Managy with the incentive to work hard.

PROBLEM # 32:  \textbf{ANSWERS}

(a) From column E in the following table we get that she should plan to work for at least 5 years after obtaining the Master’s degree (thus 7 years into the future).

(b) From column G in the following table we get that she should plan to work for at least 11 years after obtaining the Master’s degree (thus 13 years into the future).
<table>
<thead>
<tr>
<th>Year</th>
<th>Master degree cost/benefit</th>
<th>current job (opportunity cost)</th>
<th>Net benefit of MBA</th>
<th>Cumulative net benefit</th>
<th>Discounted benefit</th>
<th>Cumulative discounted benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-20,000</td>
<td>40,000</td>
<td>-60,000</td>
<td>-60,000</td>
<td>-52,174</td>
<td>-52,174</td>
</tr>
<tr>
<td>2</td>
<td>-20,000</td>
<td>40,000</td>
<td>-60,000</td>
<td>-120,000</td>
<td>-45,369</td>
<td>-97,543</td>
</tr>
<tr>
<td>3</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>-95,000</td>
<td>16,438</td>
<td>-81,105</td>
</tr>
<tr>
<td>4</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>-70,000</td>
<td>14,294</td>
<td>-66,811</td>
</tr>
<tr>
<td>5</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>-45,000</td>
<td>12,429</td>
<td>-54,381</td>
</tr>
<tr>
<td>6</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>-20,000</td>
<td>10,808</td>
<td>-43,573</td>
</tr>
<tr>
<td>7</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>5,000</td>
<td>9,398</td>
<td>-34,175</td>
</tr>
<tr>
<td>8</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>30,000</td>
<td>8,173</td>
<td>-26,002</td>
</tr>
<tr>
<td>9</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>55,000</td>
<td>7,107</td>
<td>-18,896</td>
</tr>
<tr>
<td>10</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>80,000</td>
<td>6,180</td>
<td>-12,716</td>
</tr>
<tr>
<td>11</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>105,000</td>
<td>5,374</td>
<td>-7,342</td>
</tr>
<tr>
<td>12</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>130,000</td>
<td>4,673</td>
<td>-2,670</td>
</tr>
<tr>
<td>13</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>155,000</td>
<td>4,063</td>
<td>1,393</td>
</tr>
<tr>
<td>14</td>
<td>65,000</td>
<td>40,000</td>
<td>25,000</td>
<td>180,000</td>
<td>3,533</td>
<td>4,927</td>
</tr>
</tbody>
</table>

**PROBLEM # 33: ANSWERS**

Nobody will want to acquire a level of education $y > y^*$ or a level $0 < y < y^*$. Every individual will limit himself to choosing between $y = 0$ and $y = y^*$.

| For a GROUP L individual | If choose $y = 0$ | get $w = 1$  
|                         |                  | pay $C = 0$   
|                         |                  | net wage = 1 |
| If choose $y = y^*$    | get $w = 2$      
|                         | pay $C = y^*$    
|                         | net wage = $2 - y^*$ |

| For a GROUP H individual | If choose $y = 0$ | get $w = 1$  
|                         |                  | pay $C = 0$   
|                         |                  | net wage = 1 |
| If choose $y = y^*$    | get $w = 2$      
|                         | pay $C = y^*$    
|                         | net wage = $2 - \frac{y^*}{2}$ |
If $y^*$ is such that $2 - y^* < 1$ and $2 - \frac{y^*}{2} > 1$, that is, if $1 < y^* < 2$ then group L individuals will choose $y = 0$ and group H individuals will choose $y = y^*$ and therefore the employers’ beliefs will be confirmed: people with low education will turn out to be of low productivity and people with high education will turn out to have high productivity. This is called a separating signaling equilibrium.

**PROBLEM # 34: ANSWERS**

The wage offered by the employer is represented by the thick lines in the following diagram:

For example, if $y^0 = 13$, anybody with $y = 12$ will be offered a salary of $1 + 12/4 = 4$ (even if he belongs to Group II, because the employer does not know to which group the applicant belongs) and anybody with $y = 16$ will be offered a salary of $2 + 16/4 = 6$ (even if he belongs to Group I).

First of all, note that the only sensible choices are $y = 0$ and $y = y^0$. In fact, if you increase $y$ by 1 unit, you get an extra $$(1/4) but you pay more than this (you pay $1 if you belong to Group I and $ 0.5 if you belong to Group II).

Decision for a Group I person:

<table>
<thead>
<tr>
<th>wage</th>
<th>cost</th>
<th>net income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 0$</td>
<td>$1 + 0/4 = 1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$y = y^0$</td>
<td>$2 + y^0/4$</td>
<td>$y^0$</td>
</tr>
</tbody>
</table>

In equilibrium a Group I person chooses what the employer expects him to choose, namely $y = 0$. However, it must be in his interest to make this choice, i.e. it must be the case that

$$1 > 2 + \frac{y^0}{4} - y^0$$

Decision for a Group II person:

<table>
<thead>
<tr>
<th>wage</th>
<th>cost</th>
<th>net income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ y = 0 \quad 1 + 0/4 = 1 \quad 0 \quad 1 \]

\[ y = y^0 \quad 2 + y^0/4 \quad y^0/2 \quad 2 + y^0/4 - y^0/2 \]

In equilibrium a Group II person chooses what the employer expects him to choose, namely \( y = y^0 \). However, it must be in his interest to make this choice, i.e. it must be the case that

\[ 2 + \frac{y^0}{4} - \frac{y^0}{2} > 1. \]

These two inequalities are simultaneously satisfied if and only if \( \frac{4}{3} < y^0 < 4 \).

### PROBLEM #35: ANSWERS

(a) \[
\begin{pmatrix}
6,000 & 5,000 & 4,000 & 3,000 \\
\frac{1}{8} & \frac{2}{8} & \frac{1}{8} & \frac{4}{8}
\end{pmatrix}
\]
The expected value is $4,000.

(b) No, because if the price is $3,800, only owners of cars of quality C and D will offer their cars for sale and thus buying a car means facing the lottery \[
\begin{pmatrix}
4,000 & 3,000 \\
\frac{1}{5} & \frac{4}{5}
\end{pmatrix}
\]
which has an expected value of $3,200 (less than the price of $3,800).

(c) Because of the adverse selection problem, only the lowest quality cars (those of quality D) will be traded. Any price \( P \) such that $2,700 < P < $3,000 would lead to all and only cars of quality D being traded. Thus the total number of cars traded would be 400.

### PROBLEM #36: ANSWERS

(a) If the firm offers $20,100 then everybody will apply and the firm’s expected profit will be

\[
\frac{1}{4}(25,000 - 20,100) + \frac{3}{4}(18,000 - 20,100) = \frac{1}{4}(4,900) + \frac{3}{4}(-2,100) = -$350
\]

Thus the firm would lose money.

(b) Clearly, requirement (1) amounts to \( w > 20,000 \). Since the firm cannot tell applicants apart, everybody will apply and for each applicant the firm will face a probability \( \frac{1}{4} \) that the applicant is H and probability \( \frac{3}{4} \) that the applicant is L. Thus the expected profit from each hired worker is

\[
\frac{1}{4}(25,000 - w) + \frac{3}{4}(18,000 - w) = \frac{1}{4}(25,000) + \frac{3}{4}(18,00) - w = 19,750 - w.
\]

Thus requirement (1) is \( w > 20,000 \) and requirement (2) is \( 19,750 - w > 0 \), i.e. \( w < 19,750 \). Obviously they are incompatible.
(c) Clearly, profits are zero (for each unit of output the firm collects $1,000 from consumers and pays $1,000 to the worker who produced it). [By the way, only H-workers would apply: see below.]

(d) An H-worker will apply if and only if $25b > 20,000$, that is, if and only if $b > 800$. Similarly, an L-worker will apply if and only if $18b > 20,000$, that is, if and only if $b > 1,111.11$. Thus if $b = $900, only H-workers apply.

(e) Profits would be zero, because nobody would apply for a job at this firm!

**PROBLEM # 37: ANSWERS**

<table>
<thead>
<tr>
<th>If price is</th>
<th>Number of cars offered for sale</th>
<th>Average quality of cars offered for sale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2500</td>
<td>(1/8)200 = 25</td>
<td>2000</td>
</tr>
<tr>
<td>$3100</td>
<td>[(1/8)+(3/8)]200=100</td>
<td>(1/4)2000+(3/4) 3000 = 2750</td>
</tr>
<tr>
<td>$4600</td>
<td>(6/8)200 = 150</td>
<td>(1/6)2000+(3/6)3000+(2/6)4000 = 3166</td>
</tr>
<tr>
<td>$5250</td>
<td>200 (all the cars)</td>
<td>(1/8)2000+(3/8)(3000)+(2/8)4000+(2/8)5000=3625</td>
</tr>
<tr>
<td>$6100</td>
<td>200 (all the cars)</td>
<td>(1/8)2000+(3/8)(3000)+(2/8)4000+(2/8)5000=3625</td>
</tr>
</tbody>
</table>

**PROBLEM # 38: ANSWERS**

(a) Since the owner of a car of quality $\theta=3$ values the car at $2,400$, when $P = $1,700 she will not be willing to sell. Hence the answer is No.

(b) The owner of a car of quality $\theta=1$ values it at $800$ and the owner of a car of quality $\theta=2$ values it at $1,600$. Hence both qualities will be offered for sale. Thus the buyer faces the following lottery: $\begin{pmatrix} \theta = 1 & \theta = 2 \\ \frac{1}{2} q & 1 - \frac{1}{2} q \end{pmatrix}$ whose expected utility is:

$$\frac{1}{2} q \sqrt{9,025 - 1,700 + 1,000 + (1 - \frac{1}{2} q) \sqrt{9,025 - 1,700 + 2,000}} = 136.86q + 96.57 - 144.85q = 96.57 - 7.99q$$

The buyer will be willing to buy if $96.57 - 7.99q \geq \sqrt{9025} = 95$, that is, if $q \leq 0.1965$. So the answer is: Yes all the values of $q$ less than or equal to 0.1965.

(c) No. If $q \leq 0.1965$ then both qualities $\theta=1$ and $\theta=2$ are traded and if $q > 0.1965$ then both qualities $\theta=1$ and $\theta=2$ are offered for sale, but buyers are not willing to buy.
PROBLEM # 39: ANSWERS

(a) For a U type the expected utility of not insuring is
\[
9 \frac{1}{10} \sqrt{193,600} + \frac{1}{10} \frac{1}{10} \sqrt{193,600 - 17,200} = 43,800.
\]
Thus the maximum premium that she is willing to pay for full insurance is given by the solution to
\[
100 \sqrt{193,600 - h} = 43,800\text{ which is } h = 1,756.
\]

(b) If the premium is $3,450, it follows from part (a) that the U types will not apply for insurance. For a V type the expected utility of not insuring is
\[
\frac{4}{5} 100 \ln(193,600) + \frac{1}{5} 100 \ln(193,600 - 17,200) = 1,215.494145.
\]
The utility from the contract is
\[
100 \ln(193,600 - 3,450) = 1,215.556851.
\]
Thus the V types will buy insurance and the monopolist’s expected profits are
\[
\pi_v(n_v) = \left[3,450 - \frac{1}{5}(17,200)\right] n_v = 10n_v.
\]

(c) We already know that the U types will not consider the full insurance contract. The partial insurance contract gives them an expected utility of
\[
9 \frac{1}{10} \sqrt{193,600 - 225} + \frac{1}{10} \frac{1}{10} \sqrt{193,600 - 225 - 15000} = 43,800.428407,
\]
higher than no insurance. Thus they will buy the partial insurance contract. For the V type the partial insurance contract gives an expected utility of
\[
\frac{4}{5} 100 \ln(193,600 - 225) + \frac{1}{5} 100 \ln(193,600 - 225 - 15,000) = 1,215.623794,
\]
higher than the expected utility from the full-insurance contract. Thus everybody will buy the partial insurance contract and the monopolist’s expected profits are
\[
\pi_2(n_u, n_v) = \left[225 - \frac{1}{5}(17,200 - 15,000)\right] n_u + \left[225 - \frac{1}{10}(17,200 - 15,000)\right] n_v = 5n_u - 215n_v.
\]

(d) \( \pi_1(100) = 1,000 \) and \( \pi_2(4,400) = 500 \). Thus option (b) is better.

(e) \( \pi_1(100) = 1,000 \) and \( \pi_2(4,700) = 2,000 \). Thus option (c) is better.
PROBLEM # 40: **ANSWERS**

(a) If she chooses low effort, her utility is 
\[ 0.15 \sqrt{80,000 - 36,000} + 0.85 \sqrt{80,000} = 271.881 \]
and if she chooses high effort, her utility is
\[ 0.05 \left( \sqrt{80,000 - 36,000} - 1 \right) + 0.95 \left( \sqrt{80,000} - 1 \right) = 278.189 \] . Thus she will choose high effort.

(b) Low effort, because her utility will be \( \sqrt{80,000 - 2,250} \) while with high effort it would be \( \sqrt{80,000 - 2,250} - 1 \).

(c) Yes, because her utility if she accepts it is \( \sqrt{80,000 - 2,250} = 278.837 \). The best alternative would be to remain uninsured and choose high effort with an expected utility of 278.189.

(d) Since Emily will indeed buy insurance (and exert low effort), expected profits will be \( 2,250 - 0.15 \times 36,000 = -3,150 \), that is, a loss. Thus it would not be a good idea for the insurance company to offer this contract.

PROBLEM # 41: **ANSWERS**

(a) If he chooses to spend $x on preventive measures his expected utility – when not insured – is
\[ NI(x) = \left( 1 - \frac{x}{15,000} \right) \left( 10 \ln(950,000 - 40,000 - x) \right) + \left( 1 - \left( \frac{x}{15,000} \right) \right) \left( 10 \ln(950,000 - x) \right) \]
Since \( NI(0) = 137.096 \), \( NI(400) = 137.237 \), \( NI(750) = 137.361 \) and \( NI(1,000) = 137.449 \), of the four options he will choose \( x = 1,000 \).

(b) If he is fully insured at a premium of \( h \) then his utility if he does not spend any money on preventive measures is \( \ln(950,000 - h) \), while if he spends $x (with \( x > 0 \)) then his utility is \( \ln(950,000 - h - x) \). Thus he will choose \( x = 0 \).

(c) As determined in part (b), if he buys the full insurance contract with premium $40,000 then he will choose \( x = 0 \), so that his utility will be
\[ 10 \ln(950,000 - 40,000) = 10 \ln(910,000) = 137.21 \] . This is less than the expected utility if he remains uninsured and spends $1,000 on preventive measures (which is 137.449 as calculated in part a). Thus he will not accept the contract.
(a) In the following figure the loss is denoted by $\ell$ rather than $x$:

(b) The slope of the $e$-indifference curve at NI is

$$-\frac{p_e}{1-p_e}\left(\frac{U'(W-x)}{U'(W)}\right) = -\frac{1}{19} \left(\frac{1}{2\sqrt{900}}\right) = -\frac{5}{57} = -0.0877$$

and the slope of the $n$-indifference curve is

$$-\frac{p_n}{1-p_n}\left(\frac{U'(W-x)}{U'(W)}\right) = -\frac{1}{9} \left(\frac{1}{2\sqrt{900}}\right) = -\frac{5}{27} = -0.1852.$$ 

(c) $EU_n(NI) = \frac{1}{10}\sqrt{900} + \frac{9}{10}\sqrt{2,500} = 48$ and $EU_e(NI) = \frac{1}{20}\sqrt{900} + \frac{19}{20}\sqrt{2,500} - \frac{15}{16} = 48.0625$. Thus she would exert effort.

(d) $EU_n(NI) = 48$ and $EU_e(NI) = \frac{1}{20}\sqrt{900} + \frac{19}{20}\sqrt{2,500} - \frac{3}{2} = 47.5$. Thus she would not exert effort.

(e) Using the calculations of part (c) contracts $E$ and $F$ are shown in the following figure:
(e.1) The premium of contract $E$ is given by the solution to $\sqrt{2,500 - h} \frac{15}{16} = 48.0625$ which is $h = 99$ (and zero deductible).

(e.2) The premium of contract $F$ is given by the solution to $\sqrt{2,500 - h} = 48$ which is $h = 196$ (and zero deductible).

(e.3) The customer will be indifferent between choosing NI with effort and contract $E$ with effort and thus, given the assumption made, she will choose $E$ and the monopolist’s profits will be $99 - \frac{1}{20}(1,600) = 19$.

(e.4) The customer has the following options: (1) choose NI and no effort, with a corresponding utility of 48, (2) choose NI and effort, with a corresponding utility of 48.0625, (3) choose contract $F$ and no effort, with a corresponding utility of 48 and (4) choose contract $F$ and effort, with a corresponding utility of $48 - \frac{15}{16}$. Thus she will choose option (2), that is, no insurance and the monopolist’s profits will be zero.

(e.5) Contract $N$ is shown in the figure below:

The deductible is zero and the premium is given by the solution to $\sqrt{2,500 - h} = 48.0625$ which is $h = 189.99$. 

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PROBLEM # 43: ANSWERS

Recall that if the highest offer so far is a wage of x, then one more search would give rise to one of the following two events: the event that the new offer, call it y, is less than or equal to x and the event that the new offer y is greater than x. If the first event occurs, you go back to the highest offer obtained so far, namely x. If the second event occurs you have to compute the expected wage. Now,

CASE 1: the highest offer so far is w=10. Searching once more means playing the lottery
\[
\left( \begin{align*}
10 & 20 & 30 & 40 & 50 & 60 \\
\frac{1}{12} & \frac{2}{12} & \frac{2}{12} & \frac{4}{12} & \frac{1}{12} & \frac{1}{12}
\end{align*} \right)
\]
which has an expected value of 33.33. Thus it pays to search once more (you expect to increase the wage by 23.33 at a cost of 0.25). An alternative way of thinking about it is as follows. \( \Pr\{y \leq 10\} = \frac{1}{12} \) and \( \Pr\{y > 10\} = \frac{11}{12} \). Thus with probability \( \frac{1}{12} \) the highest offer will remain 10, while with probability \( \frac{11}{12} \) the highest offer will be greater than 10. Thus your expected wage if you search once more is (using Bayes’ rule):

\[
\Pr\{y \leq 10\} \cdot 10 + \Pr\{y > 10\} \left[ 20 \Pr\{y=20\} + 30 \Pr\{y=30\} + 40 \Pr\{y=40\} + 50 \Pr\{y=50\} + 60 \Pr\{y=60\} \right]
\]

\[= \Pr\{y \leq 10\} \cdot 10 + \left[ 20 \Pr\{y=20\} + 30 \Pr\{y=30\} + 40 \Pr\{y=40\} + 50 \Pr\{y=50\} + 60 \Pr\{y=60\} \right] =
\]
\[= \frac{1}{12} \cdot 10 + \left[ 20 \cdot \frac{3}{12} + 30 \cdot \frac{2}{12} + 40 \cdot \frac{4}{12} + 50 \cdot \frac{1}{12} + 60 \cdot \frac{1}{12} \right] = 33.33.
\]

CASE 2: the highest offer so far is w=20. Searching once more means playing the lottery
\[
\left( \begin{align*}
20 & 30 & 40 & 50 & 60 \\
\frac{4}{12} & \frac{2}{12} & \frac{4}{12} & \frac{1}{12} & \frac{1}{12}
\end{align*} \right)
\]
which has an expected value of 34.16. [Alternatively, the expected wage from one more search can be computed as: \( \frac{4}{12} \cdot 20 + \left[ 30 \cdot \frac{2}{12} + 40 \cdot \frac{4}{12} + 50 \cdot \frac{1}{12} + 60 \cdot \frac{1}{12} \right] = 34.16. \]

CASE 3: the highest offer so far is w=30. Searching once more means playing the lottery
\[
\left( \begin{align*}
30 & 40 & 50 & 60 \\
\frac{6}{12} & \frac{4}{12} & \frac{1}{12} & \frac{1}{12}
\end{align*} \right)
\]
which has an expected value of 37.5. [Alternative computation:

\[
\frac{6}{12} \cdot 30 + \left[ 40 \cdot \frac{4}{12} + 50 \cdot \frac{1}{12} + 60 \cdot \frac{1}{12} \right] = 37.5. \] Thus it pays to search once more (expect to increase the wage by 7.5 at a cost of 0.25).

CASE 4: the highest offer so far is w=40. Searching once more means playing the lottery
\[
\left( \begin{align*}
40 & 50 & 60 \\
\frac{10}{12} & \frac{5}{12} & \frac{1}{12}
\end{align*} \right)
\]
which has an expected value of 42.5. [Alternative computation: \( \frac{10}{12} \cdot 40 + \left[ 50 \cdot \frac{1}{12} + 60 \cdot \frac{1}{12} \right] = 42.5. \] Thus it pays to search once more (expect to increase the wage by 2.5 at a cost of 0.25).
CASE 5: the highest offer so far is \( w=50 \). Searching once more means playing the lottery with the distribution \( \begin{pmatrix} 50 \\ \frac{11}{12} \\ \frac{1}{12} \\ \end{pmatrix} \), which has an expected value of 50.83. [Alternative computation: \( \frac{11}{12} \times 50 + \left[ \frac{60}{12} \times \frac{1}{12} \right] = 50.83 \).] Thus it pays to search once more (expect to increase the wage by 0.83 at a cost of 0.25).

Thus the optimal strategy is to continue searching until offered a wage of 60, that is set a reservation wage of 60.

**PROBLEM # 44: ANSWERS**

Let \( F(x) \) be the cumulative distribution function, that is, \( F(x) = \int_{-\infty}^{x} f(t) \, dt \). Thus

\[
F(x) = \begin{cases} 
0 & \text{if } x < 10 \\
\frac{x - 10}{50} & \text{if } x \in [10,60] \\
1 & \text{if } x > 60
\end{cases}
\]

The optimal strategy is to keep searching until a wage of at least \( x^* \) is found, where \( x^* \) solves:

\[
\int_{x^*}^{\infty} [1 - F(x)] \, dx = 0.25.
\]

Now, \( 1 - F(x) = \frac{60 - x}{50} \) and, for every \( z \in [10,60] \),

\[
\int_{z}^{\infty} [1 - F(x)] \, dx = \int_{z}^{60} \frac{60 - x}{50} \, dx = \frac{1}{50} \left( \int_{z}^{60} dx - \int_{z}^{60} x \, dx \right) = \frac{1}{50} \left( 60z - \frac{x^2}{2} \right) \bigg|_{z}^{60} = \frac{z^2}{100} - \frac{6}{5} z + 36.
\]

Thus \( x^* \), the reservation wage, is the solution to: \( \frac{z^2}{100} - \frac{6}{5} z + 36 = 0.25 \). This quadratic equation has two roots: \( z = 55 \) and \( z = 65 \). Only the first one is in the range \([10,60]\) hence \( x^* = 55 \). Thus the optimal search strategy is: keep searching until you find a wage of at least 55 and then accept.
Let \( n \) be the number of searches already undertaken (\( n \) could be 0 or 1 or 2 or 3, etc.), that is, the number of firms approached so far.

<table>
<thead>
<tr>
<th>Highest salary offered so far</th>
<th>20</th>
<th>40</th>
<th>50</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>searching once more means playing the following lottery</td>
<td>20</td>
<td>40</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>50</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>expected value of lottery</td>
<td>44</td>
<td>48</td>
<td>53</td>
<td>80</td>
</tr>
<tr>
<td>expected salary gain</td>
<td>44 – 20 = 24</td>
<td>48 – 40 = 8</td>
<td>53 – 50 = 3</td>
<td>80 – 80 = 0</td>
</tr>
<tr>
<td>expected net gain</td>
<td>24 – x</td>
<td>8 – x</td>
<td>3 – x</td>
<td>– x</td>
</tr>
<tr>
<td>utility if she does not search once more (and accepts highest offer so far)</td>
<td>20 – nx</td>
<td>40 – nx</td>
<td>50 – nx</td>
<td>80 – nx</td>
</tr>
<tr>
<td>expected utility if she searches once more</td>
<td>44 – nx – x</td>
<td>48 – nx – x</td>
<td>53 – nx – x</td>
<td>80 – nx – x</td>
</tr>
<tr>
<td>difference between row 7 and row 6 (Note: same as row 5!)</td>
<td>24 – x</td>
<td>8 – x</td>
<td>3 – x</td>
<td>– x</td>
</tr>
</tbody>
</table>

The optimal strategy, as a function of \( x \), is as follows:
<table>
<thead>
<tr>
<th>VALUE OF x</th>
<th>OPTIMAL STRATEGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>x &gt; 44</td>
<td>Do not search (not even once)</td>
</tr>
<tr>
<td>x = 44</td>
<td>Either do not search or search only once</td>
</tr>
<tr>
<td>44 &gt; x &gt; 24</td>
<td>Search once and accept whatever salary is offered.</td>
</tr>
<tr>
<td>x = 24</td>
<td>Either search once and accept whatever salary is offered, or keep searching until offered a salary of at least 40 then accept.</td>
</tr>
<tr>
<td>24 &gt; x &gt; 8</td>
<td>Keep searching until offered a salary of at least 40 then accept.</td>
</tr>
<tr>
<td>x = 8</td>
<td>Either keep searching until offered a salary of at least 40 and then accept, or keep searching until offered a salary of at least 50 and then accept.</td>
</tr>
<tr>
<td>8 &gt; x &gt; 3</td>
<td>Keep searching until offered a salary of at least 50 and then accept.</td>
</tr>
<tr>
<td>x = 3</td>
<td>Either keep searching until offered a salary of at least 50 and then accept, or keep searching until offered a salary of 80 and then accept.</td>
</tr>
<tr>
<td>3 &gt; x ≥ 0</td>
<td>Keep searching until offered a salary of 80 and then accept.</td>
</tr>
</tbody>
</table>

**PROBLEM # 46: ANSWERS**

(a) We need to solve \( \sqrt{4,624 - p + 20} = \sqrt{4,624} \) for \( p \). The solution is \( p = 2,320 \).

(b) If she buys for $2,200 her utility is \( 4,624 - 2,200 - 80 + 20 = 68.415 \)

If she searches a second time her expected utility is

\[
\frac{2}{4}(\sqrt{4,624 - 2,200 - 160 + 20}) + \frac{1}{4}(\sqrt{4,624 - 1,900 - 160 + 20}) + \frac{1}{4}(\sqrt{4,624 - 1,800 - 160 + 20}) = 72.499
\]

Thus she should search once more.