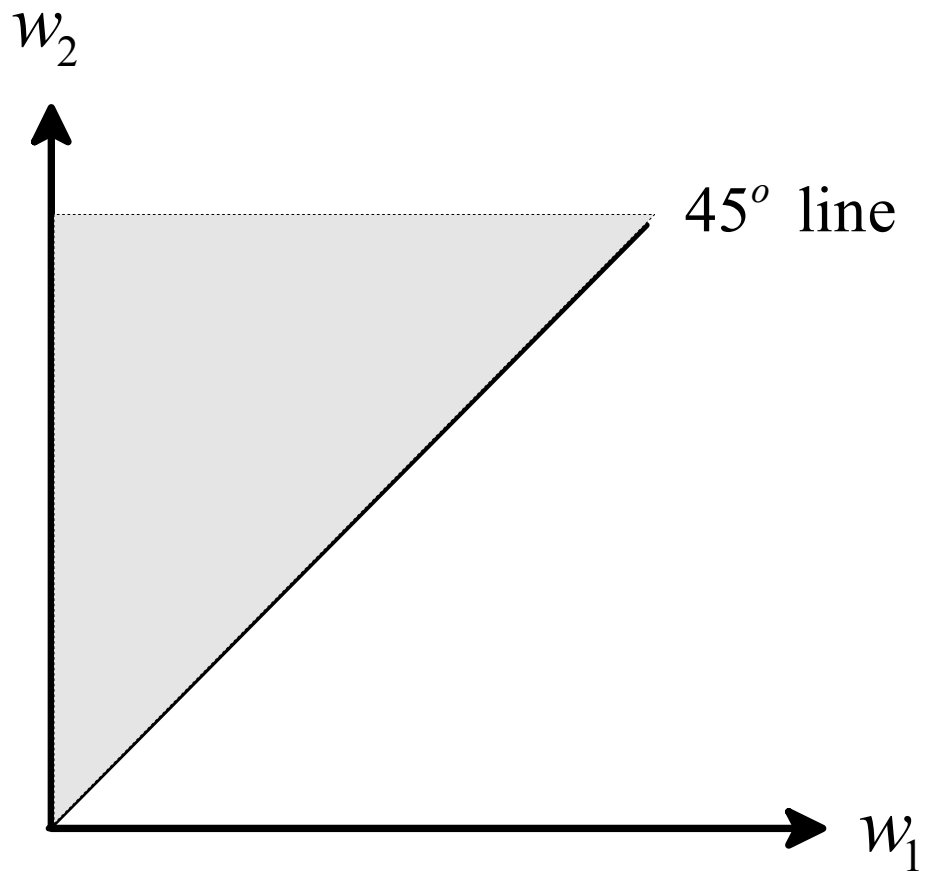


$$u_A(m, e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$$



Identify those contracts that are

1. individually rational and
2. Pareto efficient.

A contract  $C$  is **individually rational** if, for each party, signing the contract is at least as good as not signing it.

$\hat{r}_P$  = reservation utility of the Principal

$\hat{r}_A$  = reservation utility of the Agent.

$C$  is individually rational if

(1)

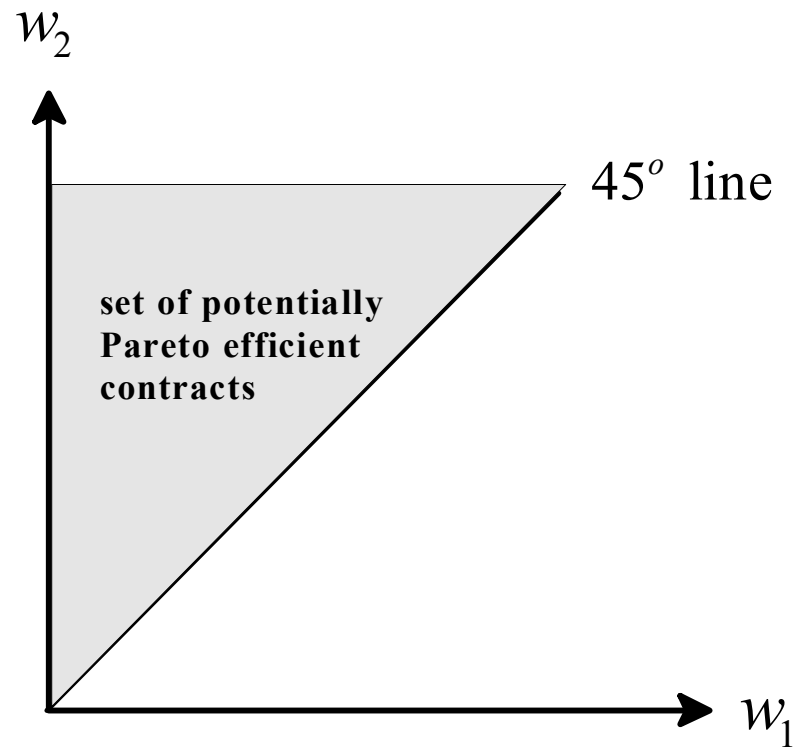
(2)

Contract  $C$  is **Pareto efficient** if, for every other contract  $D$ ,

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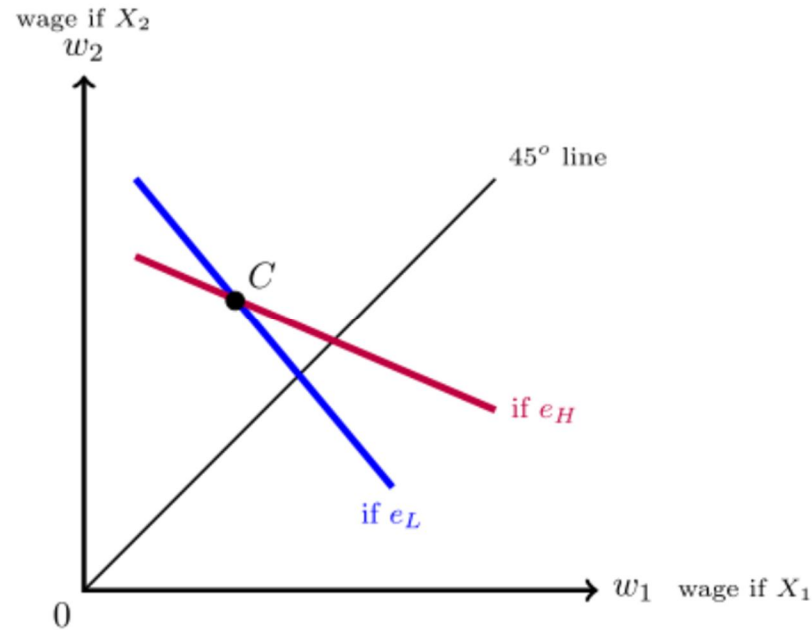
To simplify, assume that  $\hat{r}_P = \hat{r}_A = 0$  so that every contract  $(w_1, w_2)$  with  $0 \leq w_1 \leq X_1$  and  $0 \leq w_2 \leq X_2$  is individually rational. This assumption allows us to concentrate on the issue of Pareto efficiency.



Fix any contract  $C$  in the shaded area. Then, for each individual, there are two indifference curves that go through point  $C$ : one corresponding to the case where the Agent chooses  $e_L$  and the other corresponding to the case where the Agent chooses  $e_H$ .

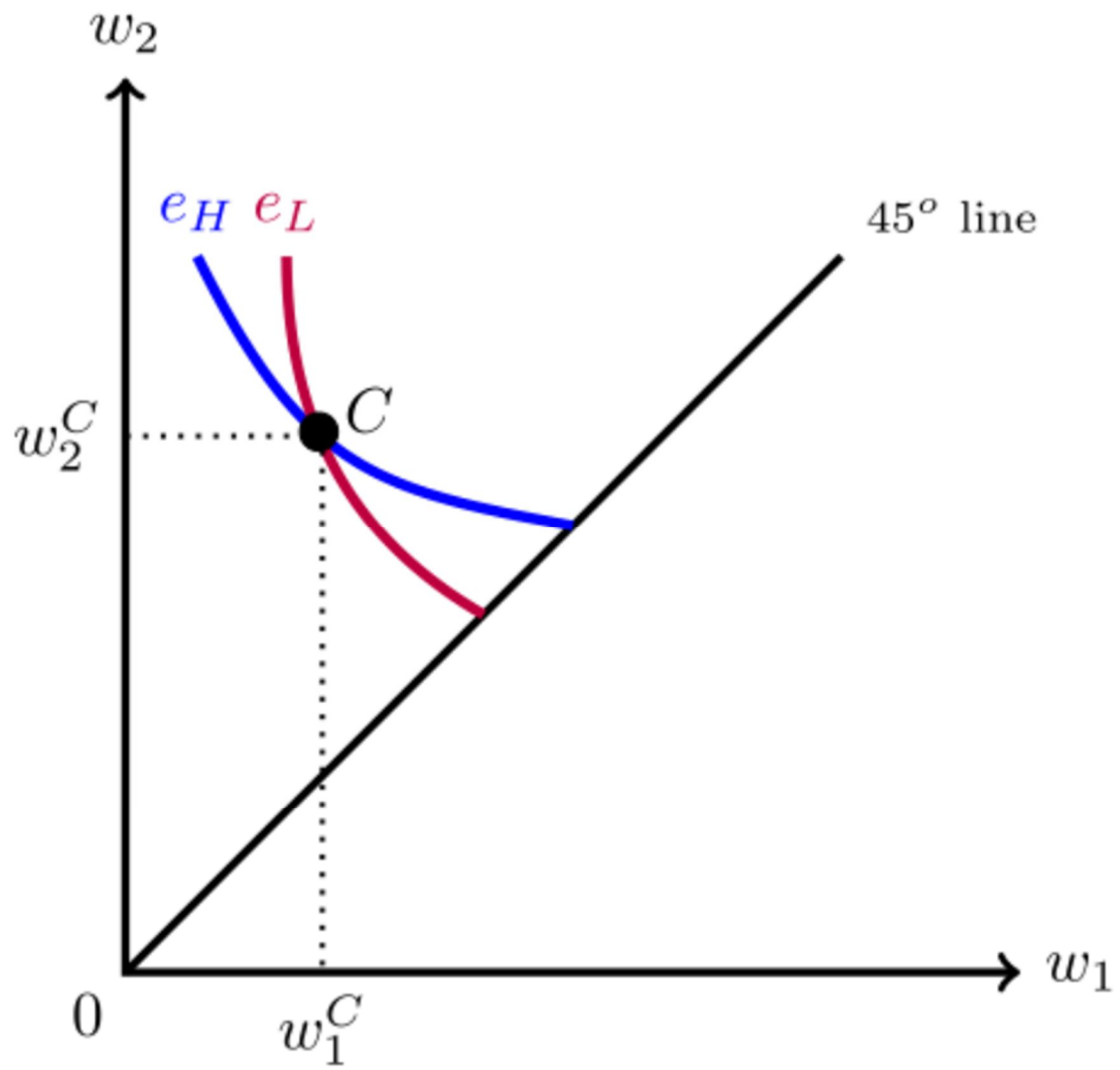
Let us begin with the risk-neutral Principal. Let  $C = (w_1^C, w_2^C)$  and  $D = (w_1^D, w_2^D)$  be two contracts. Let  $\bar{X}_L = p_1^L X_1 + (1 - p_1^L) X_2$  and  $\bar{X}_H = p_1^H X_1 + (1 - p_1^H) X_2$

- **Conditional on the Agent choosing  $e_L$** , the Principal is indifferent between  $C$  and  $D$  if and only if
  
- **Conditional on the Agent choosing  $e_H$** , the Principal is indifferent between  $C$  and  $D$  if and only if



Now the Agent, who is risk averse with utility-of-money function  $u_A(m, e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$  with  $c > 0$ . Through any contract  $C = (w_1^C, w_2^C)$  there are two indifference curves:

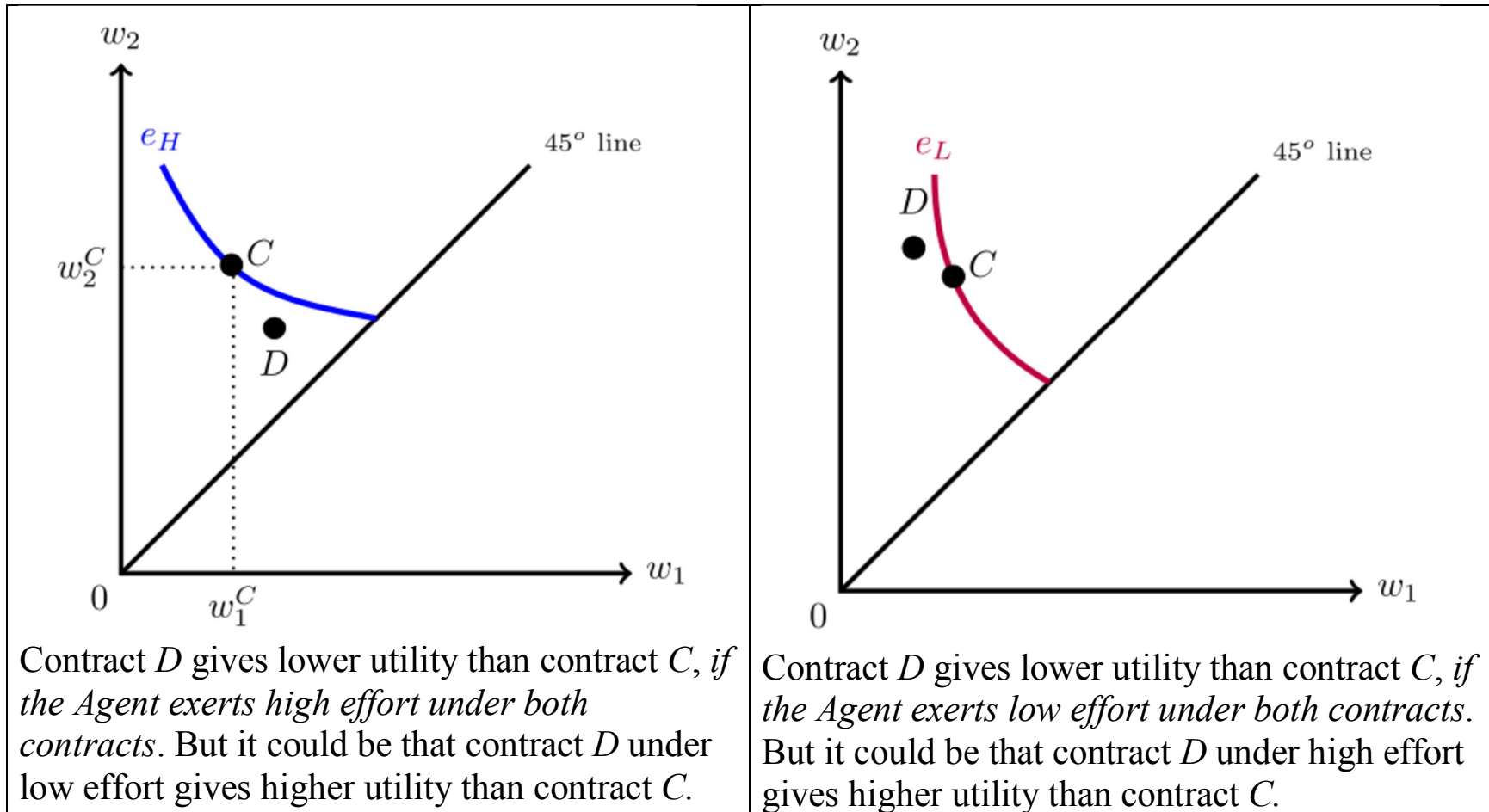
- a steeper one, corresponding to the case where the Agent exerts low effort  $e_L$ , whose slope at  $C$  is
- a less steep one, corresponding to the case where the Agent exerts high effort  $e_H$ , whose slope at  $C$  is



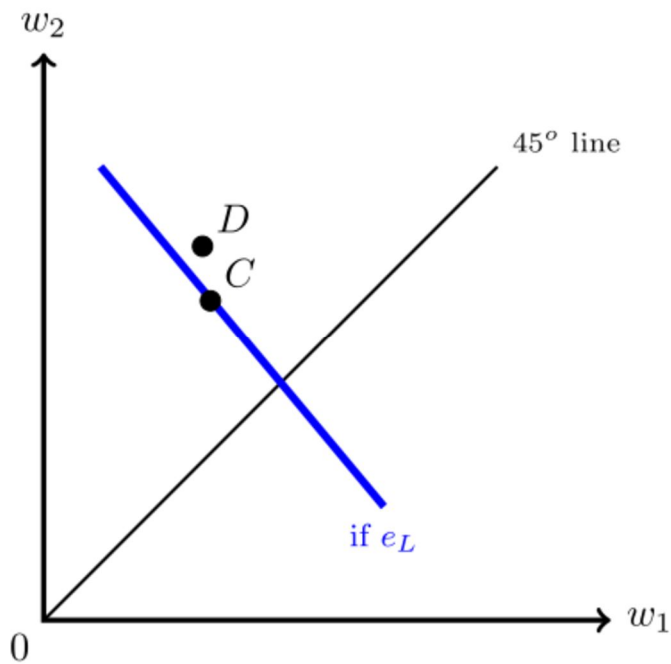


How can we tell which of two contracts,  $C$  and  $D$ , gives higher utility?

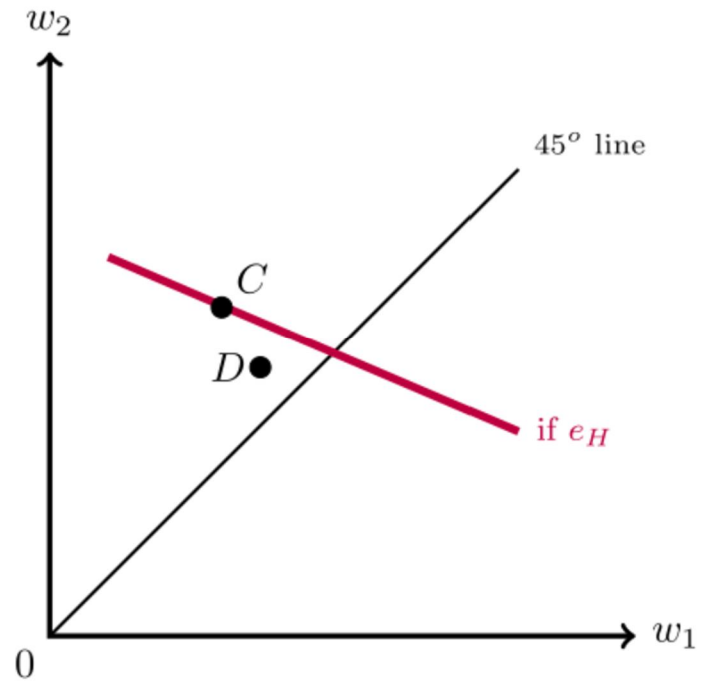
**For the Agent the direction of increasing utility is the North-East direction.**



**For the Principal the direction of increasing utility is the South-West direction.**



Contract  $D$  gives **lower** utility to the Principal than contract  $C$ , if the Agent exerts low effort under both contracts. But it could be that contract  $D$  induces the Agent to choose high effort and gives the Principal higher utility than contract  $C$ .



Contract  $D$  gives **higher** utility to the Principal than contract  $C$ , if the Agent exerts high effort under both contracts. But it could be that contract  $D$  induces the Agent to choose low effort and gives the Principal lower utility than contract  $C$ .

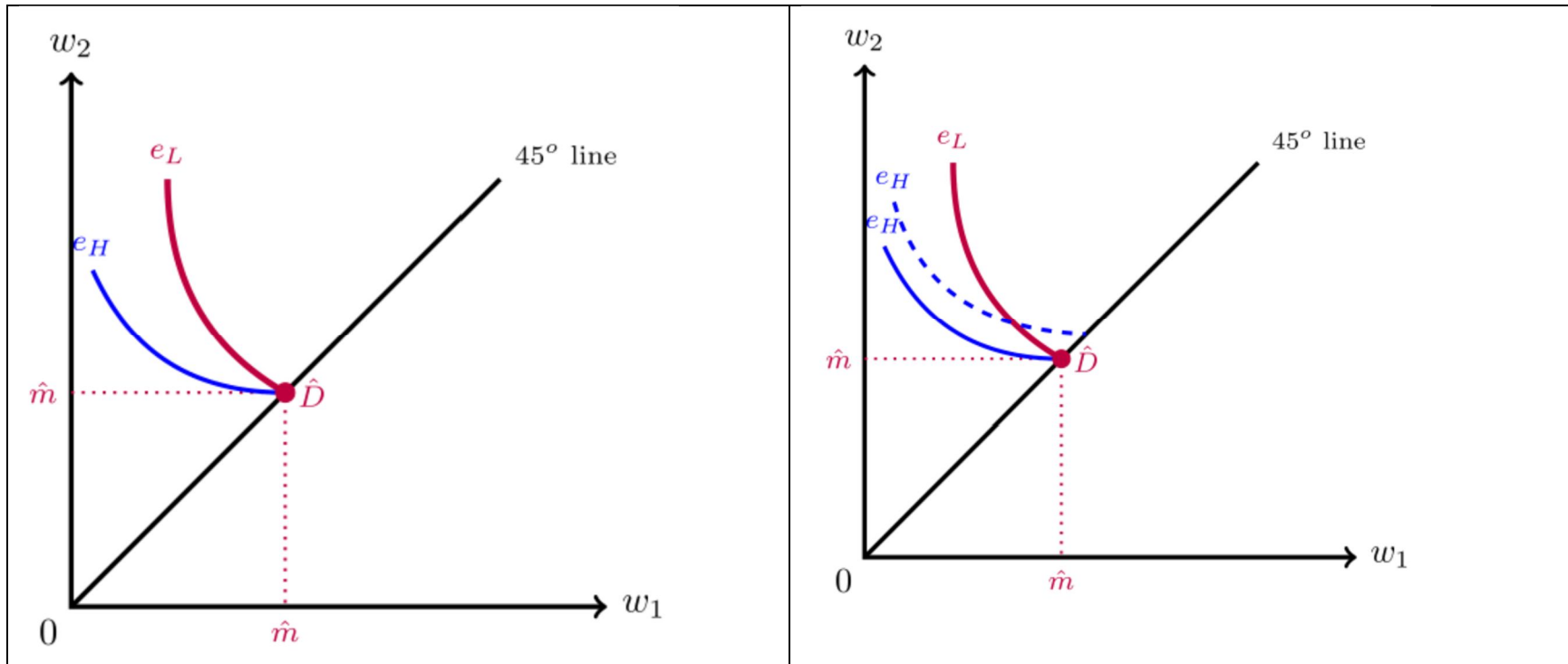
How do we determine which contracts are Pareto efficient?

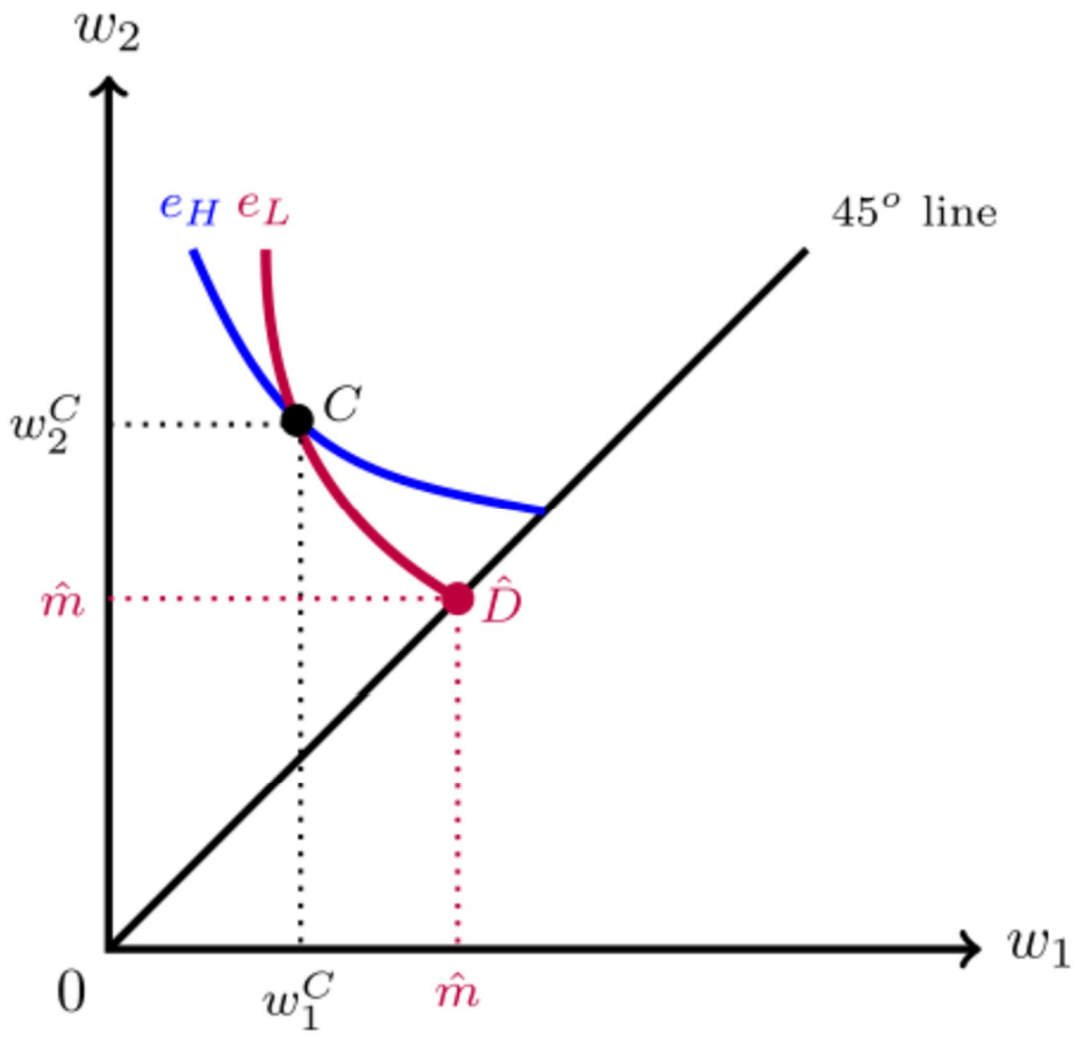
**Step 1.** Pick an arbitrary contract  $\hat{D} = (\hat{m}, \hat{m})$  on the 45° line and let  $\hat{u}$  be the Agent's utility from this contract. Then we know that

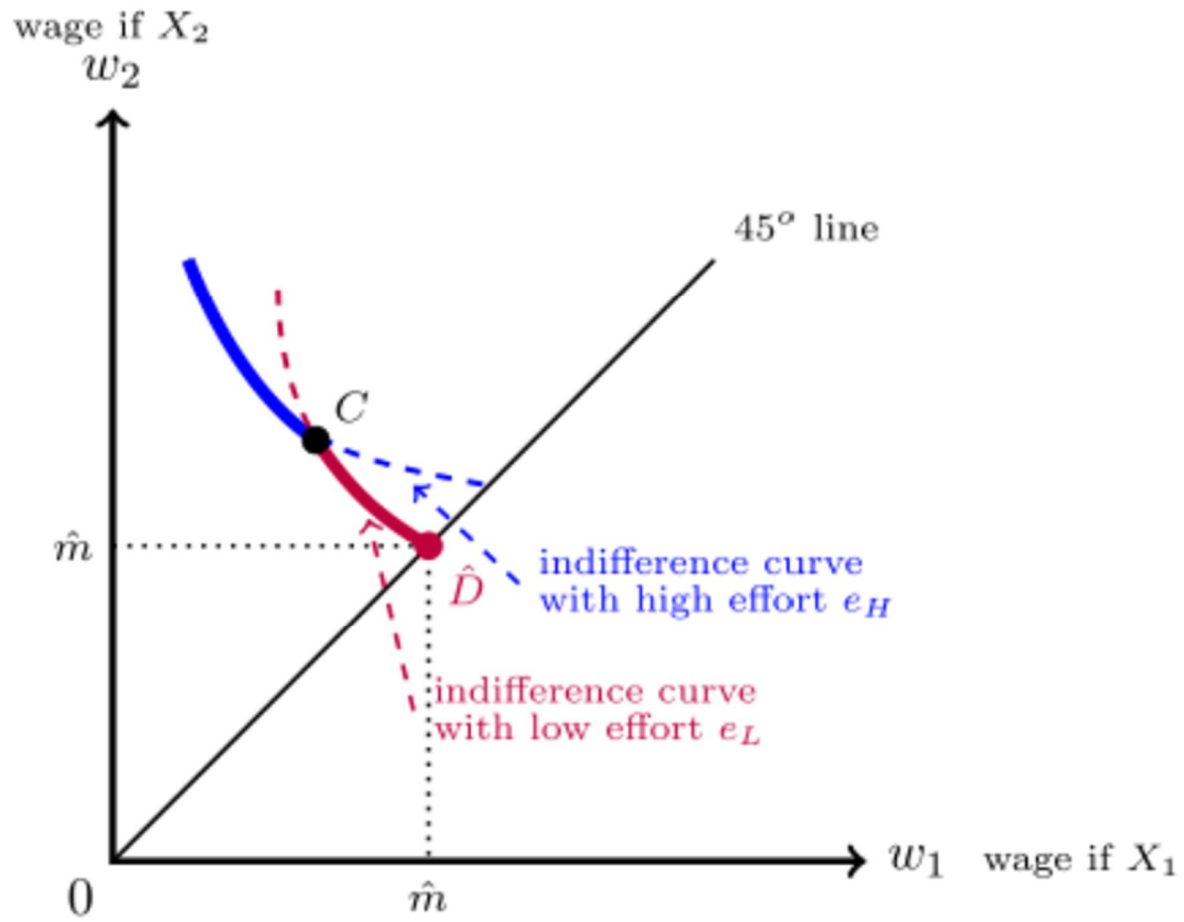
**Step 2.** Determine the set of contracts that give the Agent utility  $\hat{u}$  when she chooses the best level of effort for each contract. Call this set the  $\hat{u}$ -*utility locus* for the Agent.

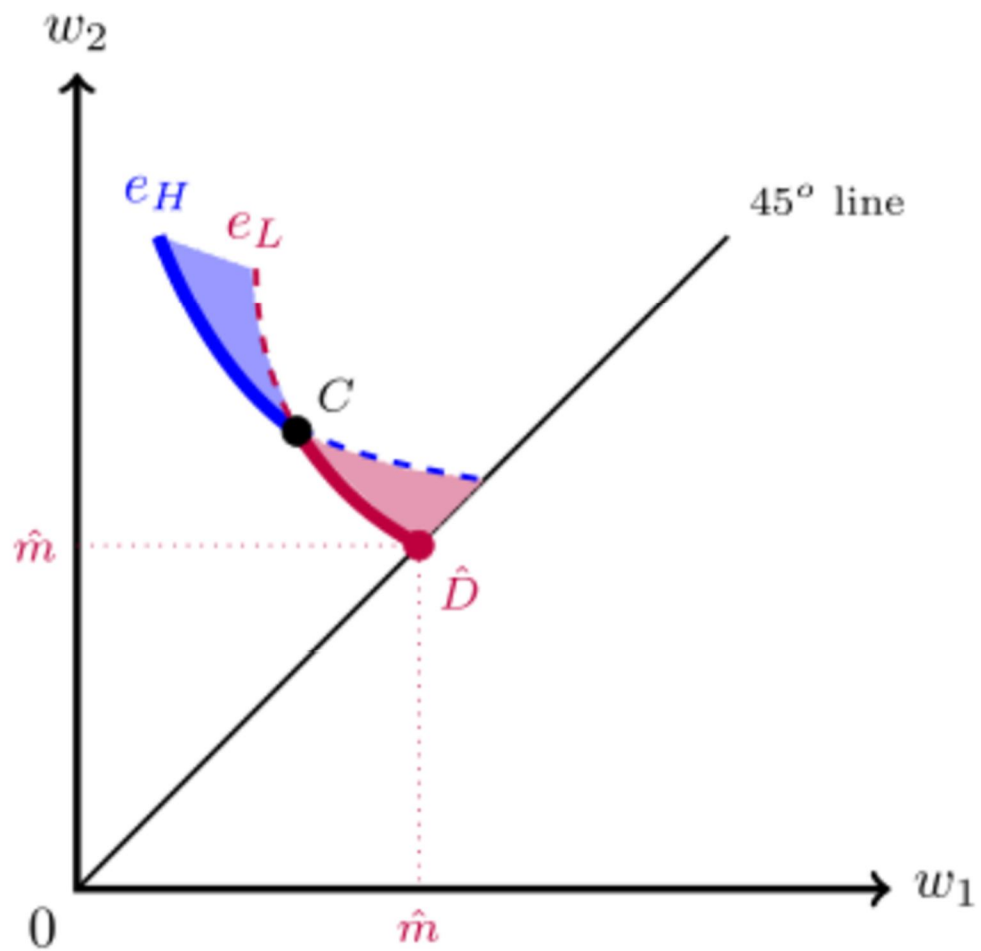
**Step 3.** Find which contracts on the  $\hat{u}$ -*utility locus* are Pareto efficient.

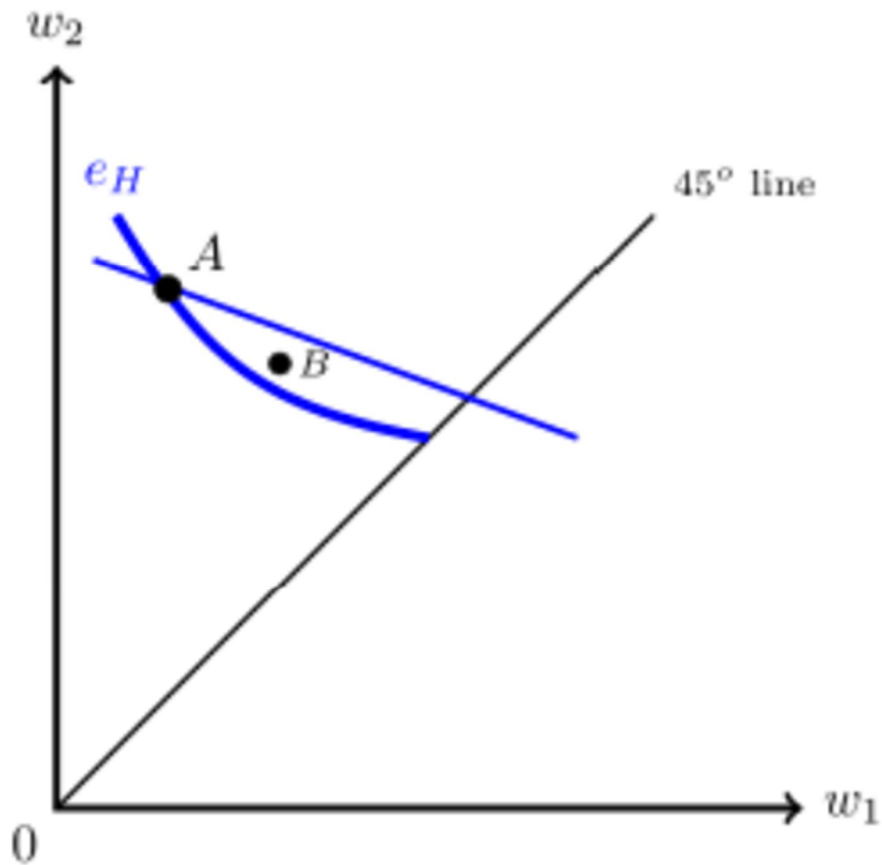
The indifference curve corresponding to  $e_H$  that goes through contract  $\hat{D}$  corresponds to a level of utility less than  $\hat{u}$  (in fact, equal to  $\hat{u} - c$ ).





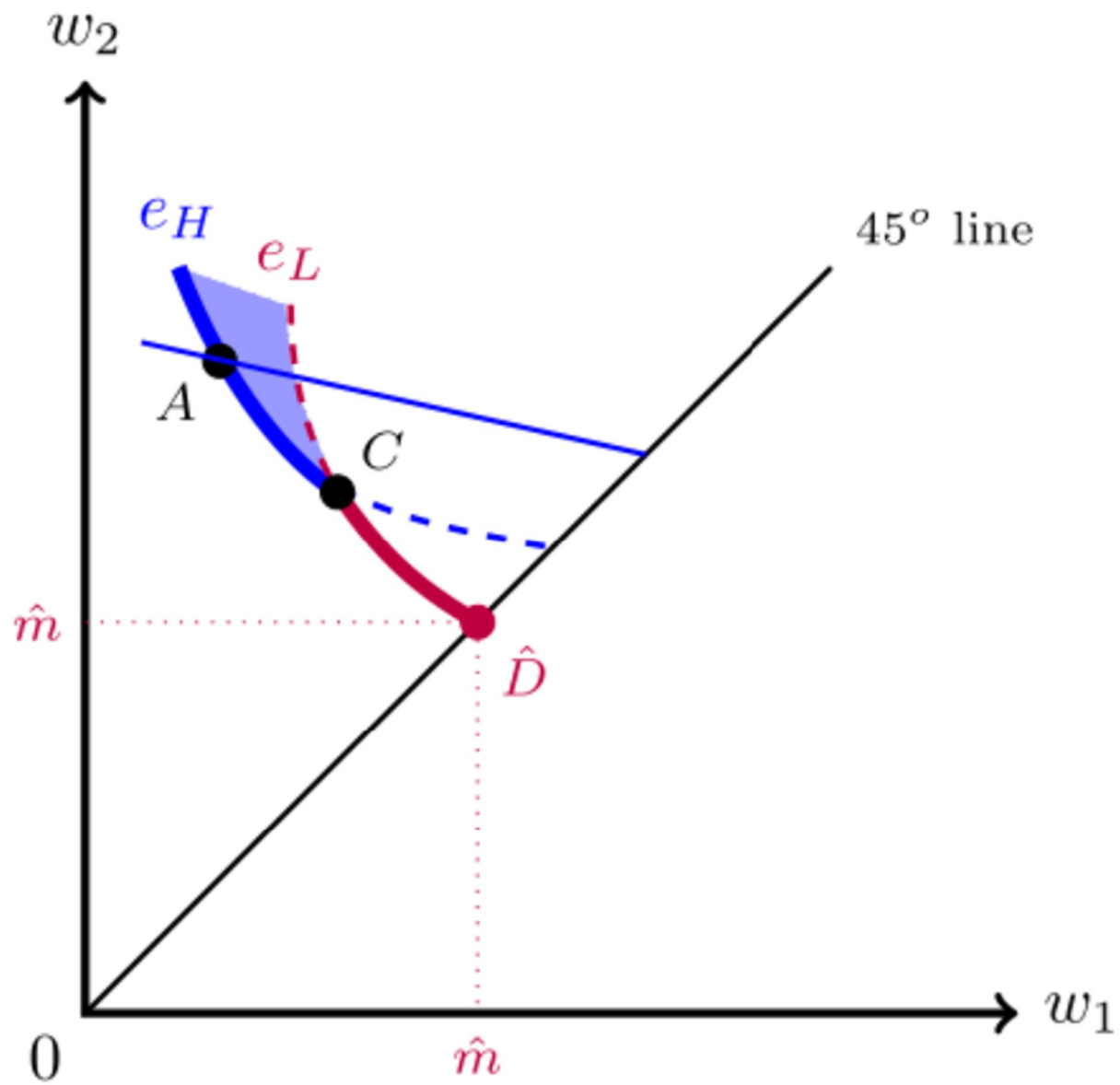


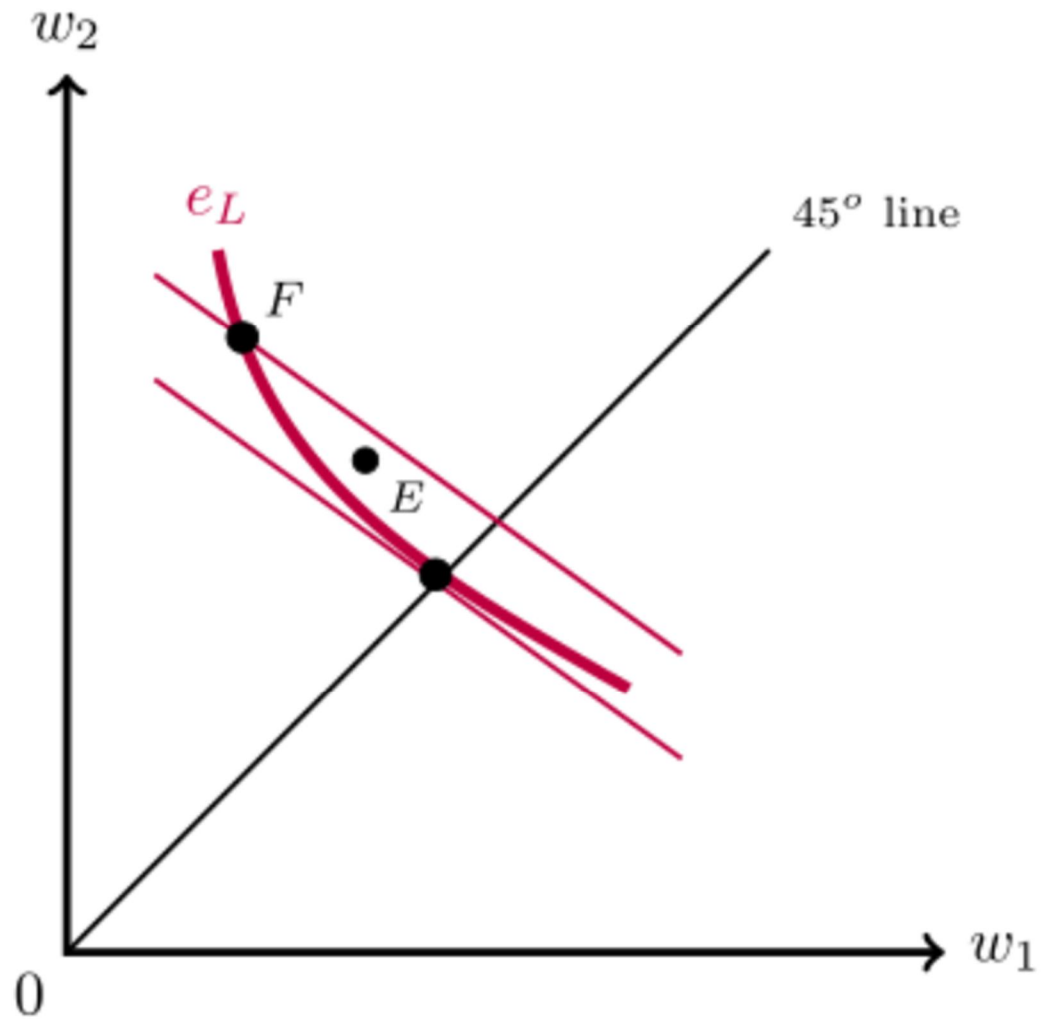




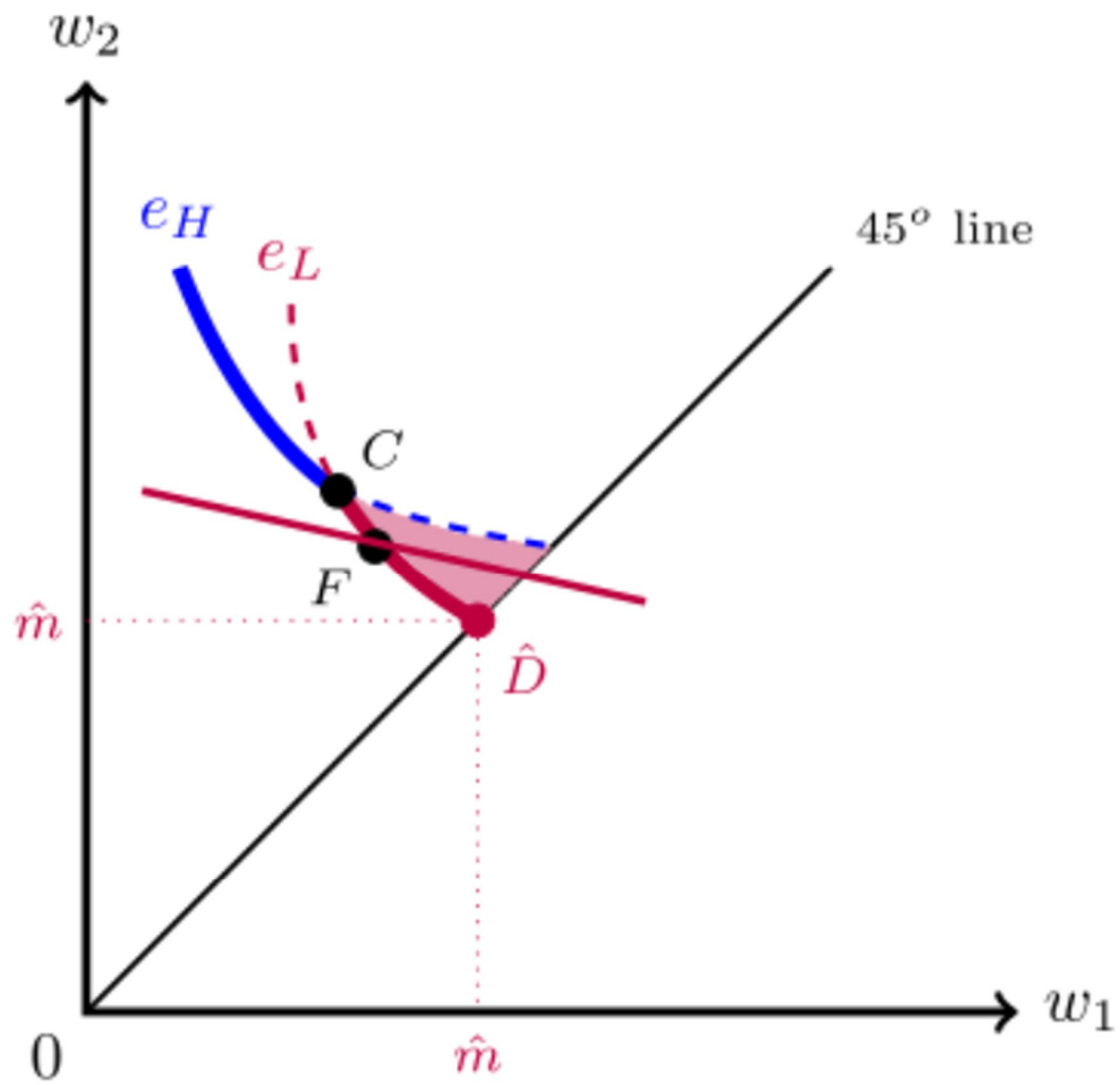
*If the Agent chooses  $e_H$  with both contracts A and B, then both Principal and Agent strictly prefer B to A.*

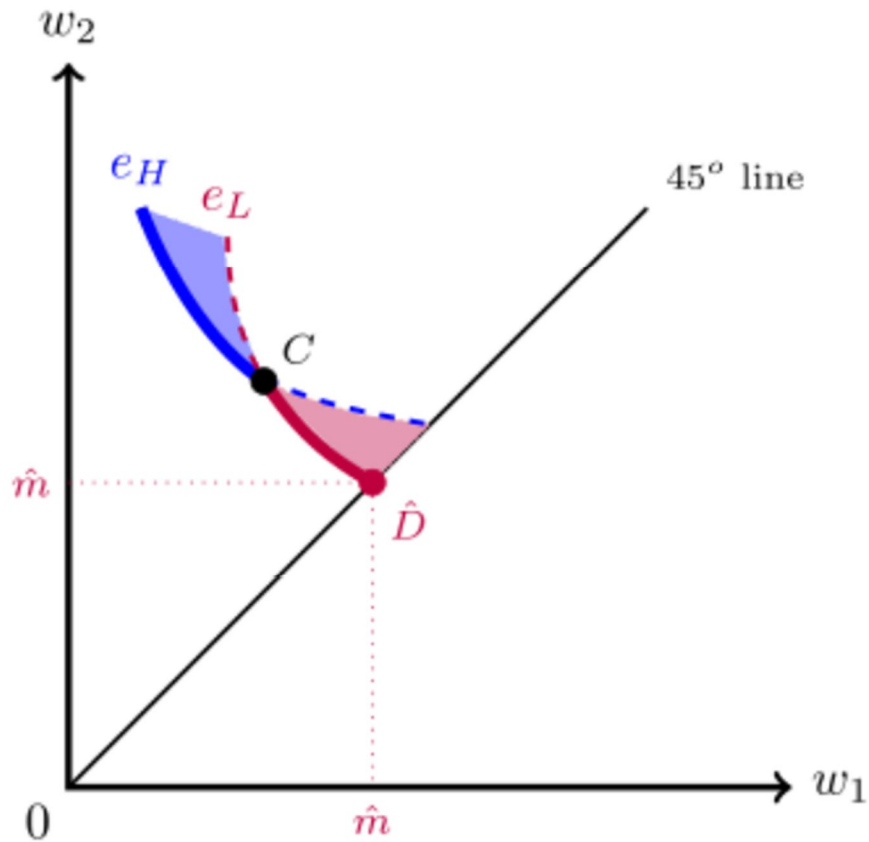






If the Agent chooses  $e_L$  with both contracts  $E$  and  $F$ , then both Principal and Agent strictly prefer  $E$  to  $F$ .





The only two candidates for Pareto efficiency on the  $\hat{u}$ -utility locus are  $C$  and  $\hat{D}$ . Which of the two is Pareto efficient depends on how the Principal ranks them:

- if  $\hat{D} \succ_P C$
- if  $C \succ_P \hat{D}$