wage if X_2
w_2

utility $\hat{u}$

utility $> \hat{u}$

utility $> \hat{u}$

45° line

A

E

C

D

utility $\hat{u}$

$\hat{m}$

$\hat{D}$

indifference curve with high effort e_H

indifference curve with low effort e_L

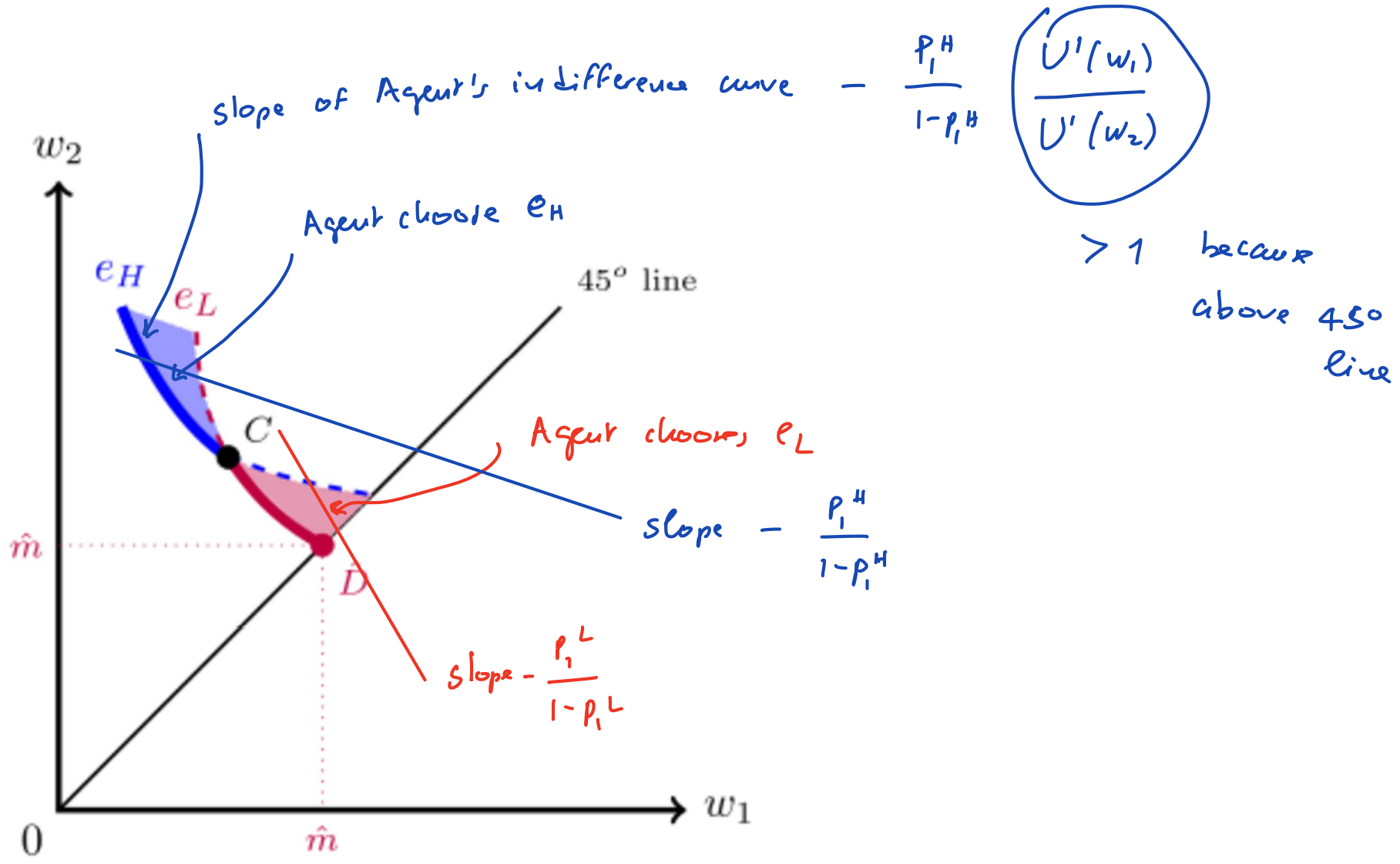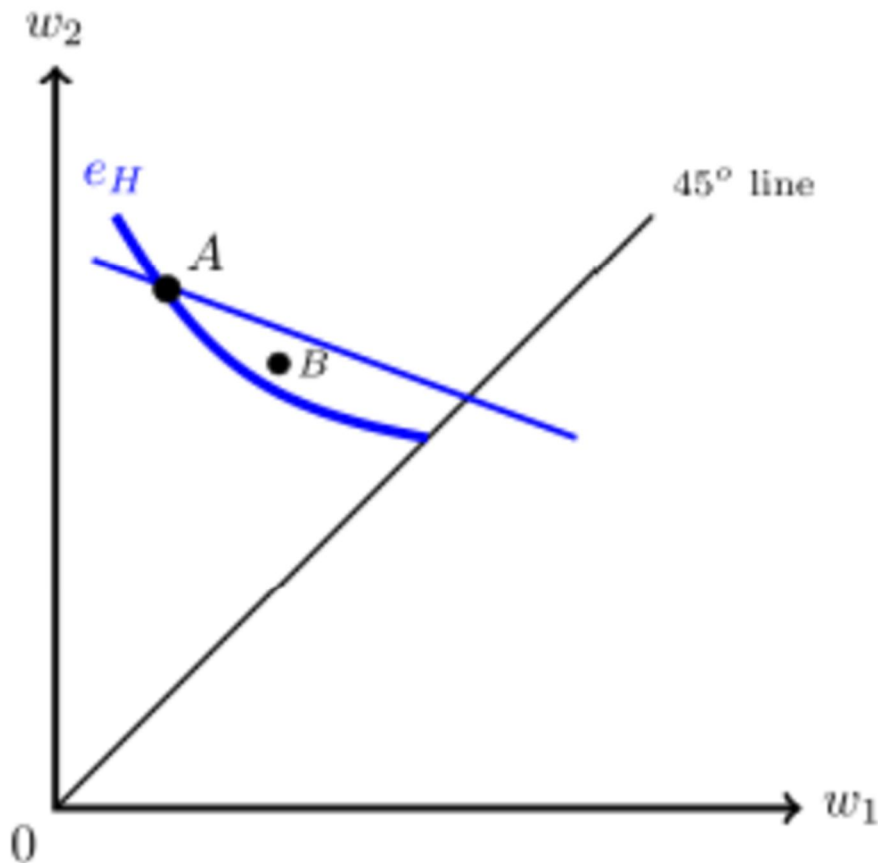w_1 wage if X_1

0

$\hat{m}$

$\hat{u}$-utility locus = set of contracts that give the Agent at most $\hat{u}$ (and exactly $\hat{u}$ if effort is chosen appropriately)

Slope of Agent's indifference curve $-\dfrac{P_1^H}{1-P_1^H}\left(\dfrac{U'(w_1)}{U'(w_2)}\right)$

>1 because above 45⁰ line

w_2

Agent choose e_H

e_H e_L

45° line

C

Agent chooses e_L

slope $-\dfrac{P_1^H}{1-P_1^H}$

$\hat{m}$

D

slope $-\dfrac{P_1^L}{1-P_1^L}$

w_1

0

$\hat{m}$

The image shows a graph with w_2 on the vertical axis and w_1 on the horizontal axis, a 45° line, an indifference curve labeled e_H, and two points A and B.
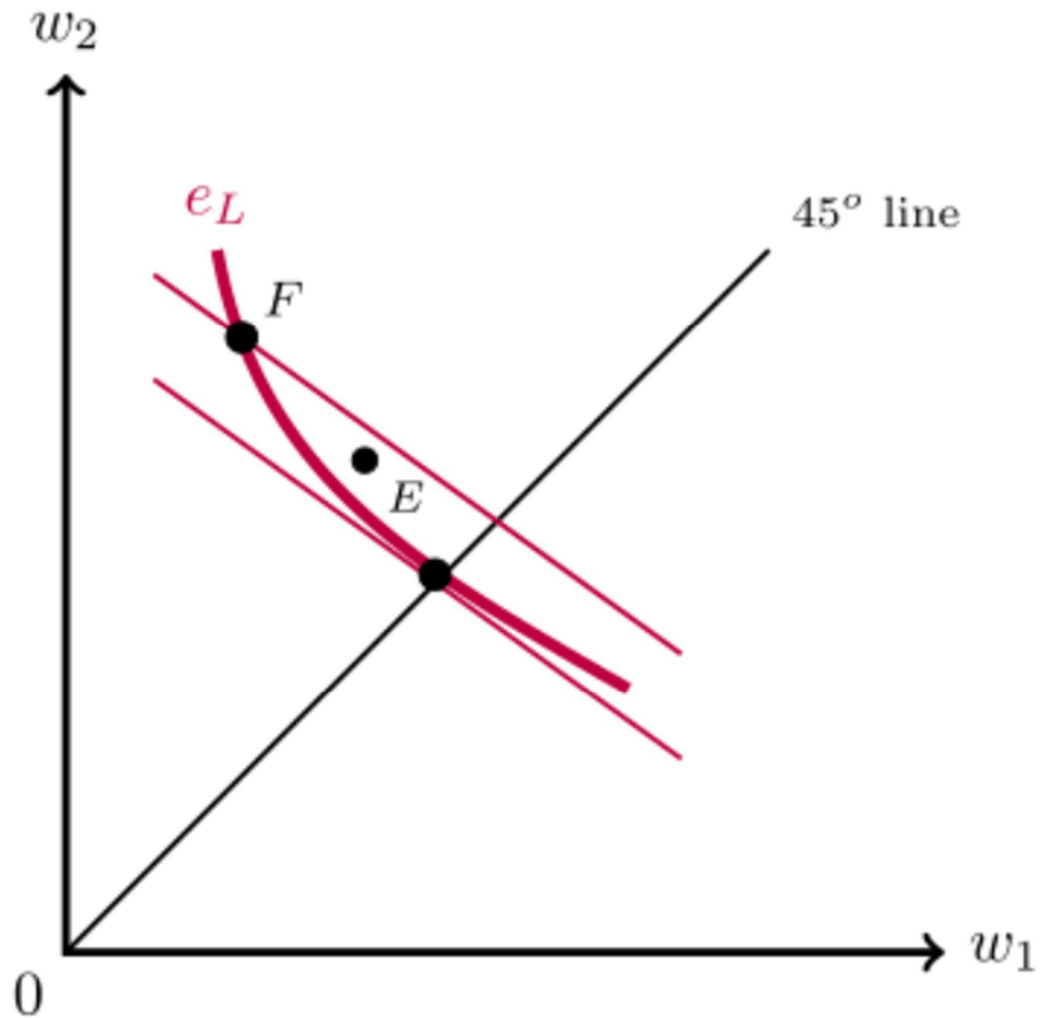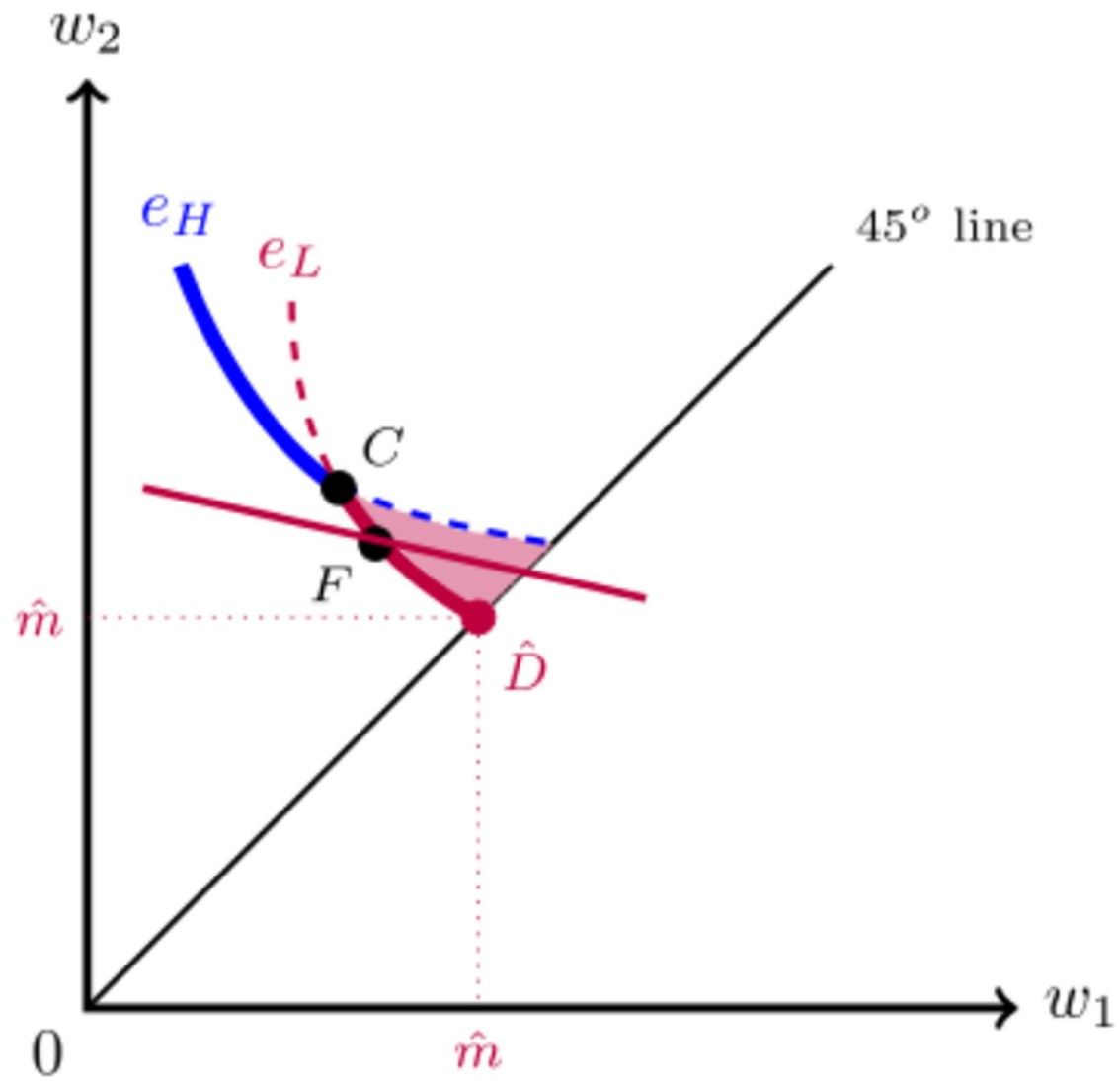
Handwritten note to the right:

Both Principal and Agent prefer B to A
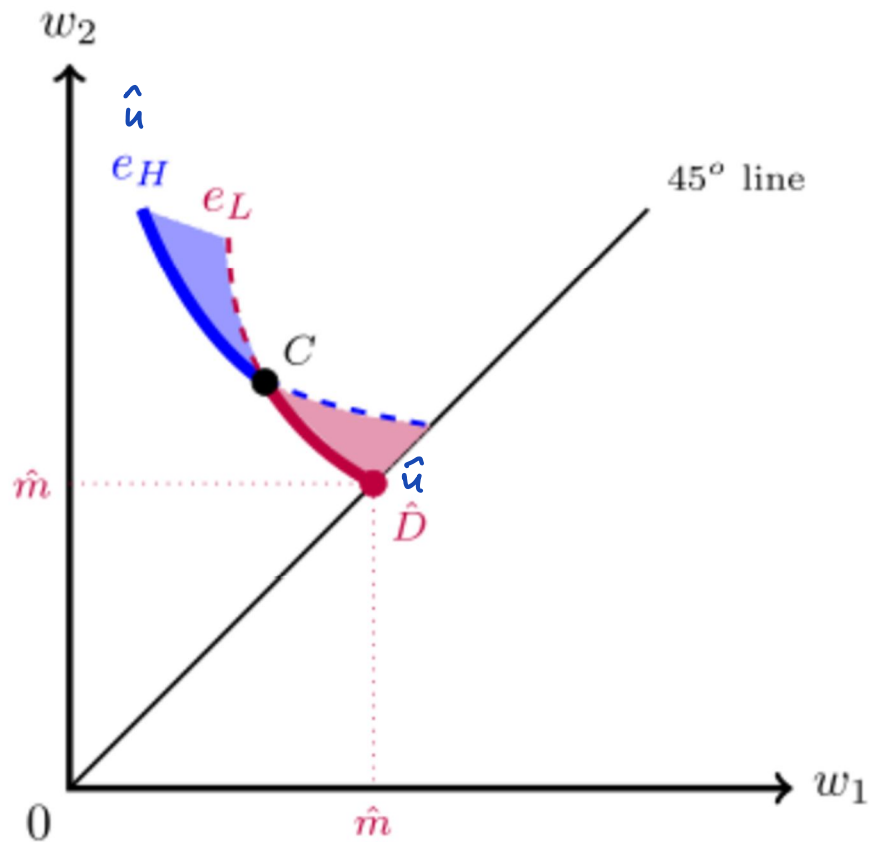
So A is not Pareto efficient

If the Agent chooses e_H with both contracts A and B, then both Principal and Agent strictly prefer *B* to *A*.
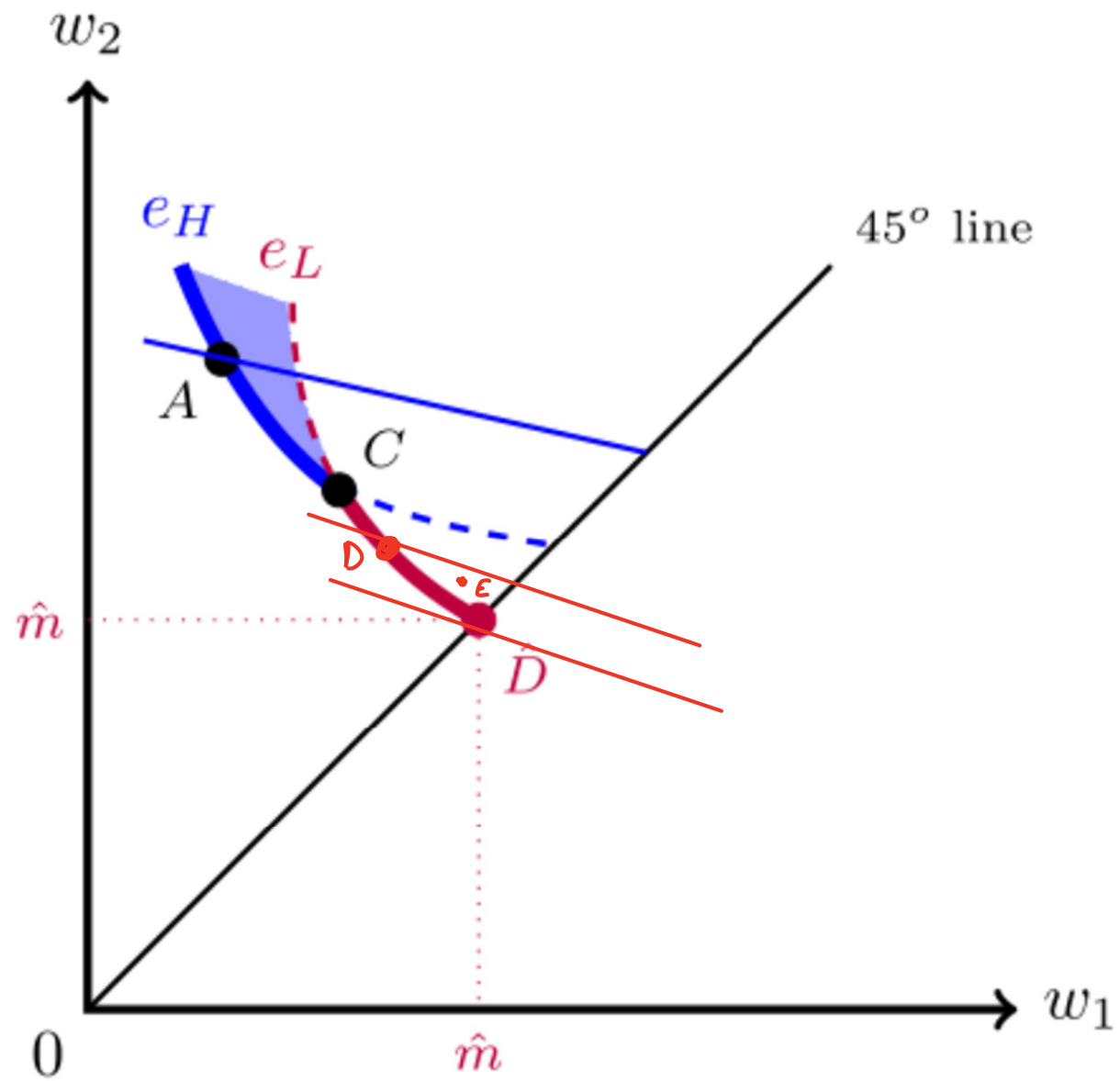
If the Agent chooses e_L with both contracts E and F, then both Principal and Agent strictly prefer E to F.

The only two candidates for Pareto efficiency on the $\hat{u}$-utility locus are C and $\hat{D}$.
Which of the two is Pareto efficient depends on how the Principal ranks them:

- if $\hat{D} \succ_P C$ then $\hat{D}$ is the only Pareto efficient contract among the ones that give the Agent utility $\hat{u}$

- if $C \succ_P \hat{D}$ " C "   "   "

- if $C \sim_P \hat{D}$ both C and $\hat{D}$ are the only Pareto efficient contracts

EXAMPLE.

$$X_1 = 300 \quad \text{and} \quad X_2 = 500 \qquad e_L = 1 \quad \text{and} \quad e_H = 2$$

$$\underbrace{U_P(\$m) = m}_{\text{risk neutral}} \qquad U_A(m,e) = \sqrt{m} - e$$

$$\text{probability of } X_1 = \begin{cases} \frac{1}{2} & \text{if } e = 1 \\ \frac{1}{12} & \text{if } e = 2 \end{cases}$$

Find a Pareto efficient contract that gives utility 8 to the Agent. $\quad \hat{u} = 8$

$$\hat{D} : (\hat{m}, \hat{m}) \longrightarrow \underline{\text{Agent choose } e = 1} \qquad U_A(\hat{D}) = \sqrt{\hat{m}} - 1$$

$$\sqrt{\hat{m}} - 1 = 8 \qquad \sqrt{\hat{m}} = 9$$

$$\hat{D} : (81, 81) \qquad\qquad \hat{m} = 81$$

$C = (w_1, w_2)$ belongs to both blue and red ind. curves for utility 8

on blue (i.e $e = 2$) $\quad \begin{cases} \frac{1}{12}\left(\sqrt{w_1} - 2\right) + \frac{11}{12}\left(\sqrt{w_2} - 2\right) = 8 \\ \\ \frac{1}{2}\left(\sqrt{w_1} - 1\right) + \frac{1}{2}\left(\sqrt{w_2} - 1\right) = 8 \end{cases}$

on red (i.e $e = 1$)

Solution $W_1 = 60.84$ $W_2 = 104.04$

$$EU_P(C) = \frac{1}{12}(300 - 60.84) + \frac{11}{12}(500 - 104.04) = \underline{\underline{382.89}}$$

$$EU_P(\hat{D}) = \frac{1}{2}(300 - 81) + \frac{1}{2}(500 - 81) = 319$$

C is the only Pareto efficient contract
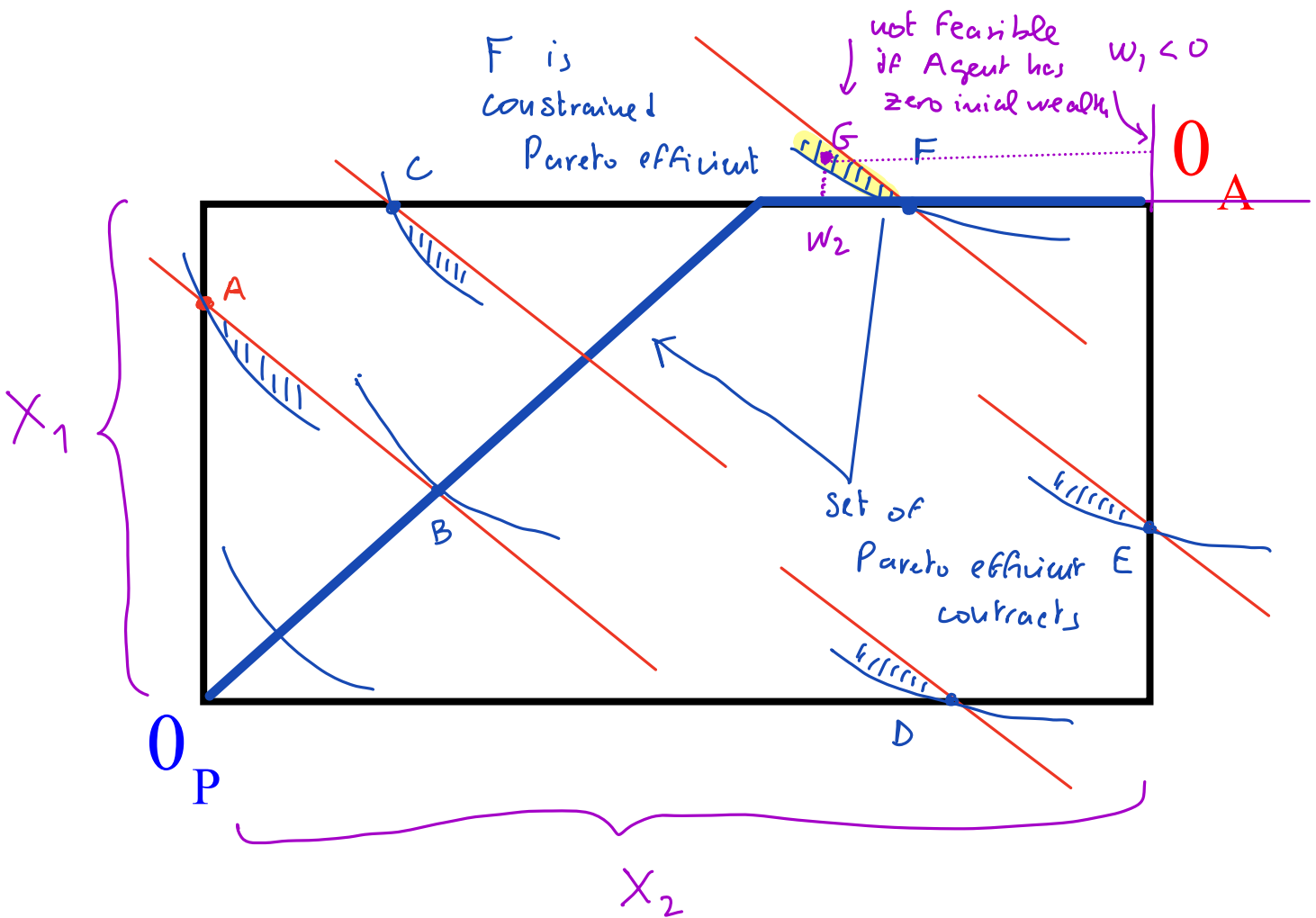among those that give the Agent utility 8

$X_1 < X_2$

$P \qquad 1-P \qquad$ fixed: no moral hazard

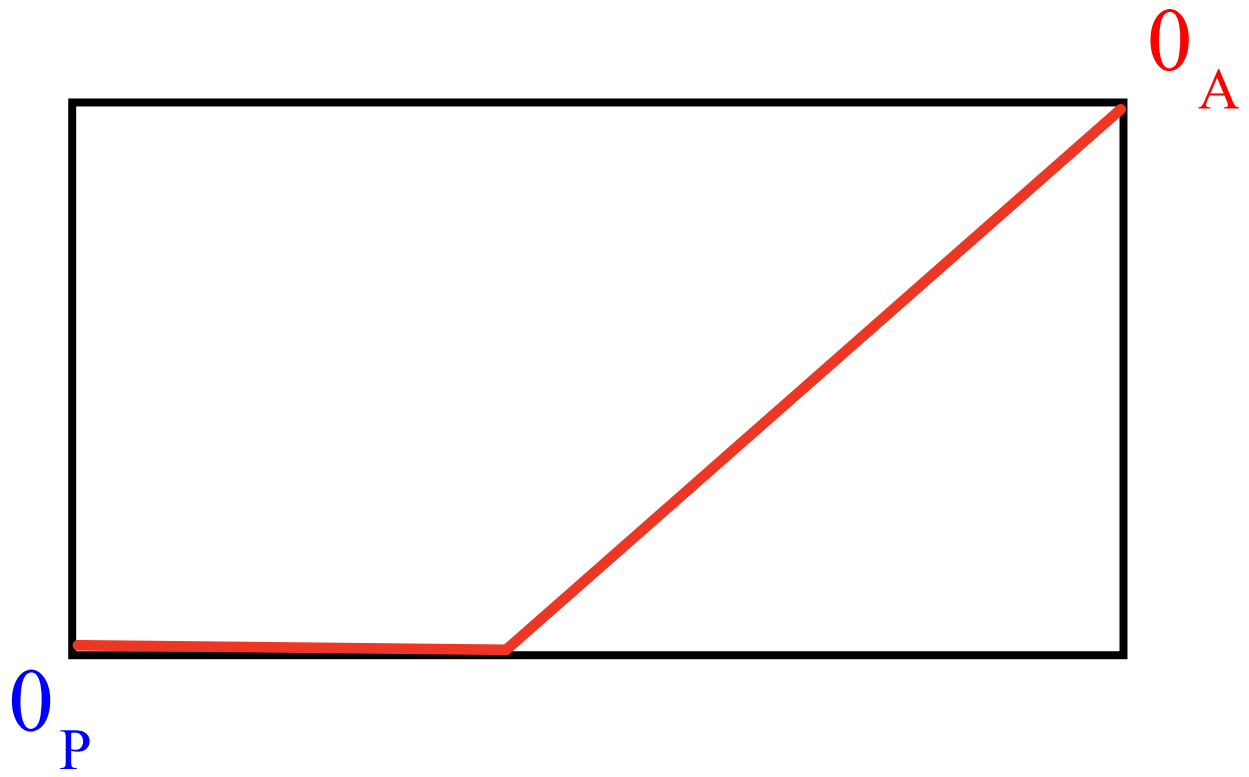# Principal-Agent optimal risk sharing with zero initial wealth

Constrained Pareto-efficient contracts
on the sides of the Edgeworth box

CASE 1: the Principal is risk averse, the Agent risk neutral

F is constrained Pareto efficient

not feasible if Agent has zero initial wealth

$w_1 < 0$

C

G

F

0
A

w_2

A

X_1

B

Set of Pareto efficient contracts

E

D

0_P

X_2

A risk averse
P risk neutral

0 A

0 P

CASE 3: both Principal and Agent are risk averse