ISOPROFIT LINES

Assume that the **insurance company** is **risk neutral** so that it considers selling an insurance contract $C = (h,d)$, corresponding to the lottery

$$C = \begin{pmatrix} h-(L-d) & h \\ p & 1-p \end{pmatrix},$$ as equivalent to getting its expected value for sure:

$$\mathbb{E}[C] = p\,[h-(L-d)] + (1-p)h = h - \underbrace{p(L-d)} = \underbrace{h - pL + pd}$$

We denote the expected profit from a single contract (h,d) by $\pi(h,d)$. Thus

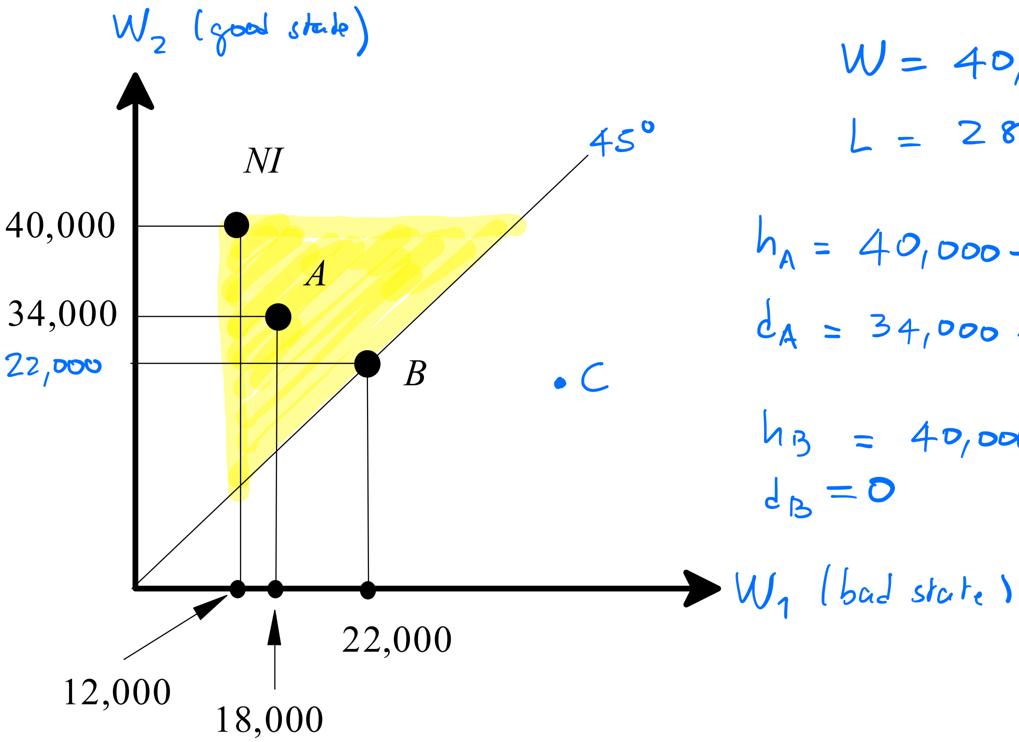
$$\pi(h,d) = h - p(L-d) = \boxed{\underbrace{h} - pL + pd}$$

If the contract is expressed as a point (W_1, W_2) in wealth space then

$$h = W - W_2 \qquad\qquad d = W_2 - W_1$$

$$\Pi = \underbrace{h}_{W-W_2} - pL + p\underbrace{d}_{W_2-W_1} = W - W_2 - pL + p(W_2 - W_1)$$

$$= \boxed{W - pL - (1-p)W_2 - pW_1}$$



$W = 40,000$

$L = 28,000$

$h_A = 40,000 - 34,000 = 6,000$

$d_A = 34,000 - 18,000 = 16,000$

$h_B = 40,000 - 22,000 = 18,000$

$d_B = 0$

Suppose that $p = \frac{1}{10}$. What is $\pi(A)$? What is $\pi(B)$?

probability of loss

$$\pi(A) = 6,000 - \frac{1}{10}(28,000 - 16,000) = 6000 - \frac{1}{10} 12,000$$

$$= 4,800$$

$$\boxed{W - pL - (1-p)W_2 - pW_1}$$

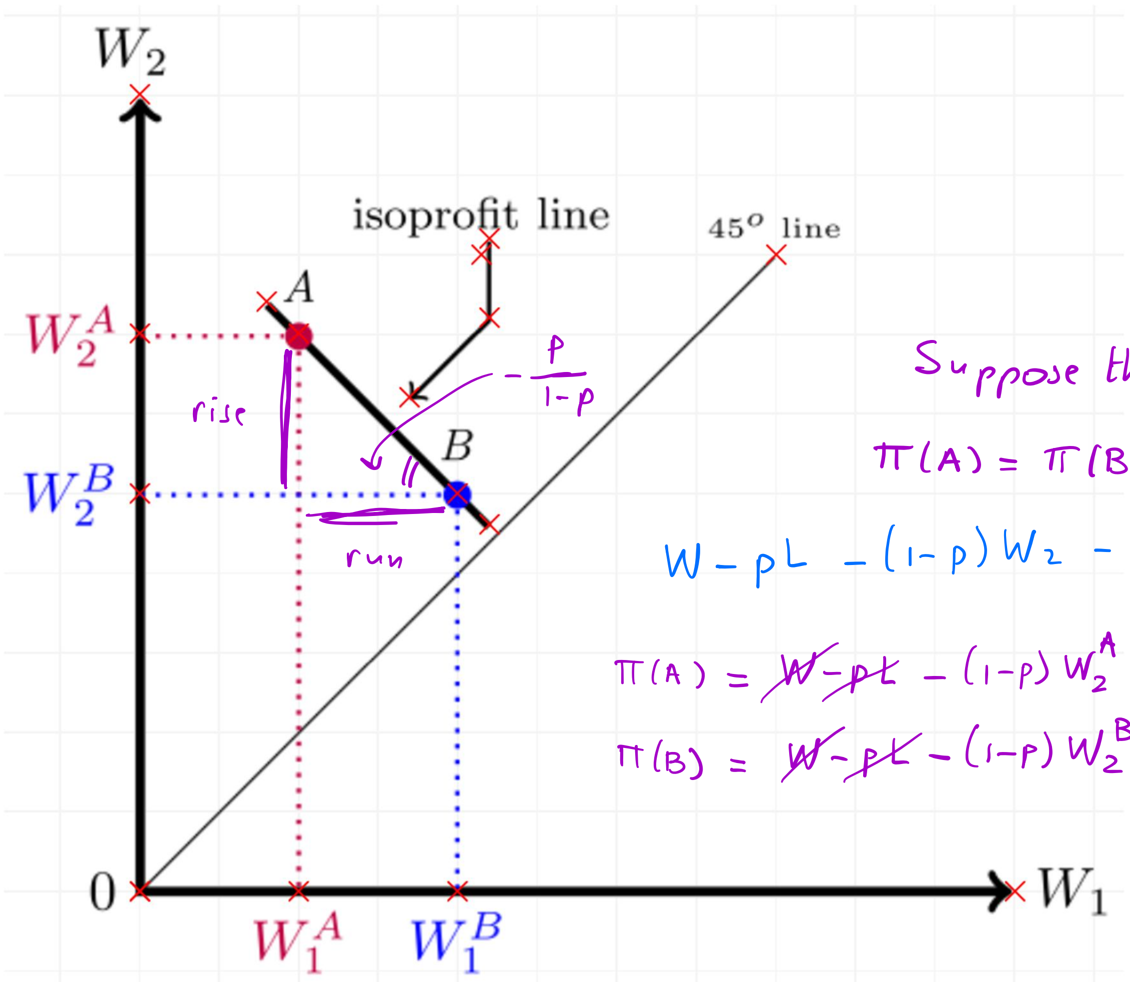
$$\pi(A) = 40,000 - \frac{1}{10} 28,000 - \frac{9}{10} 34,000 - \frac{1}{10} 18,000$$

$$= 4,800$$

$$\pi(B) = 18,000 - \frac{1}{10} 28000 = 18,000 - 2800 = 15,200$$

$$\pi(B) = 40000 - \frac{1}{10} 28000 - \frac{9}{10} 22000 - \frac{1}{10} 22,000 = 15,200$$

An **isoprofit line** is defined as a line joining contracts that give the same expected profit. Let $A = (W_1^A, W_2^A)$ and $B = (W_1^B, W_2^B)$ be such that $\pi(A) = \pi(B)$



Suppose that

$\pi(A) = \pi(B)$

$W - pL - (1-p) W_2 - p W_1$

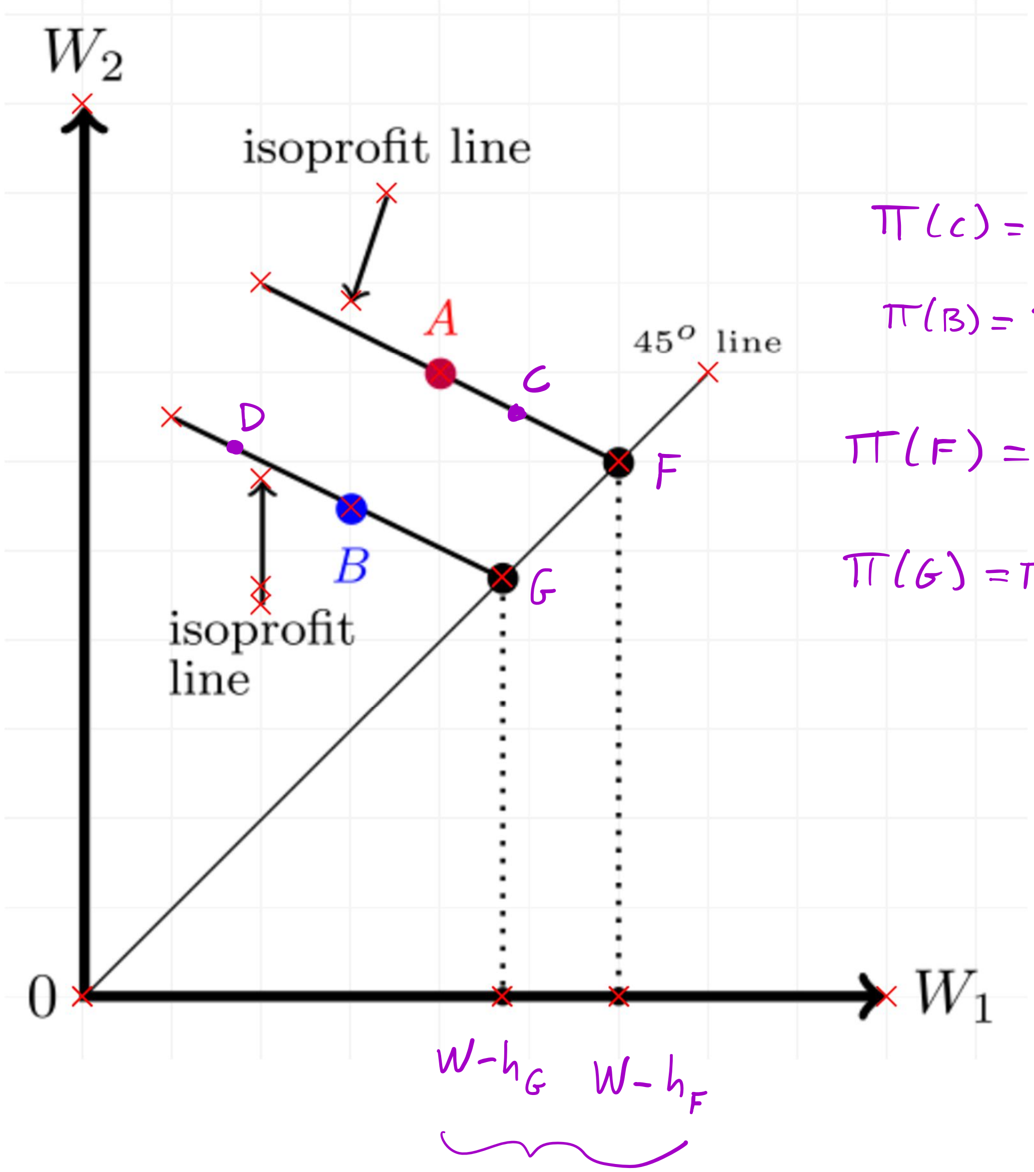
$\pi(A) = \cancel{W - pL} - (1-p) W_2^A - p W_1^A$

$\pi(B) = \cancel{W - pL} - (1-p) W_2^B - p W_1^B$

if $\pi(A) = \pi(B)$ then $(1-p) W_2^A + p W_1^A = (1-p) W_2^B + p W_1^B$

$(1-p)(W_2^A - W_2^B) = -p(W_1^A - W_1^B)$

$$\frac{W_2^A - W_2^B}{W_1^A - W_1^B} = \boxed{-\frac{p}{1-p}}$$



$\pi(C) = \pi(A)$

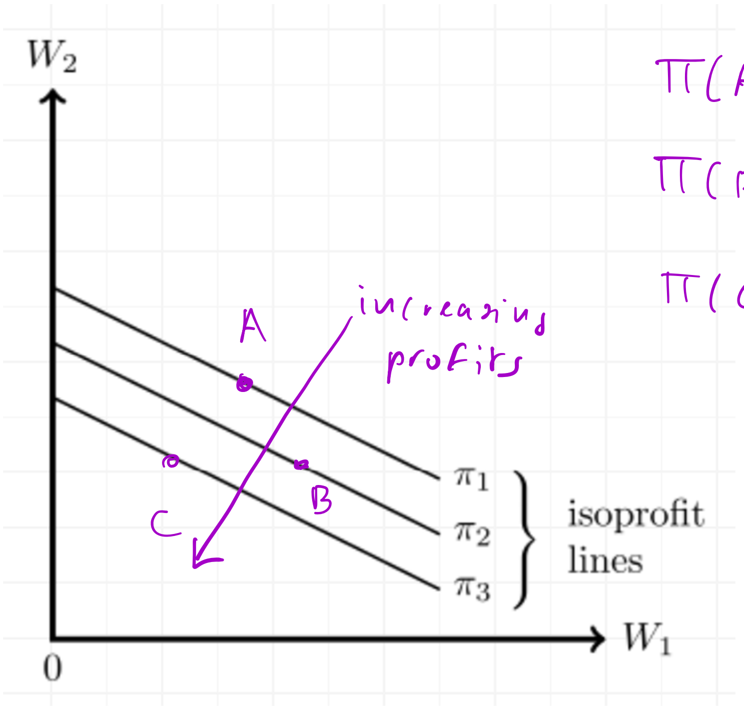
$\pi(B) = \pi(D)$

$\pi(F) = \pi(A)$

$\pi(G) = \pi(B)$

$h_G > h_F$

$$\begin{array}{l} \pi(G) = h_G - pL \\ \pi(F) = h_F - pL \end{array} \Rightarrow \begin{array}{ccc} \pi(G) & > & \pi(F) \\ \parallel & & \parallel \\ \pi(B) & > & \pi(A) \end{array}$$



$\pi(A) = \pi_1$

$\wedge$

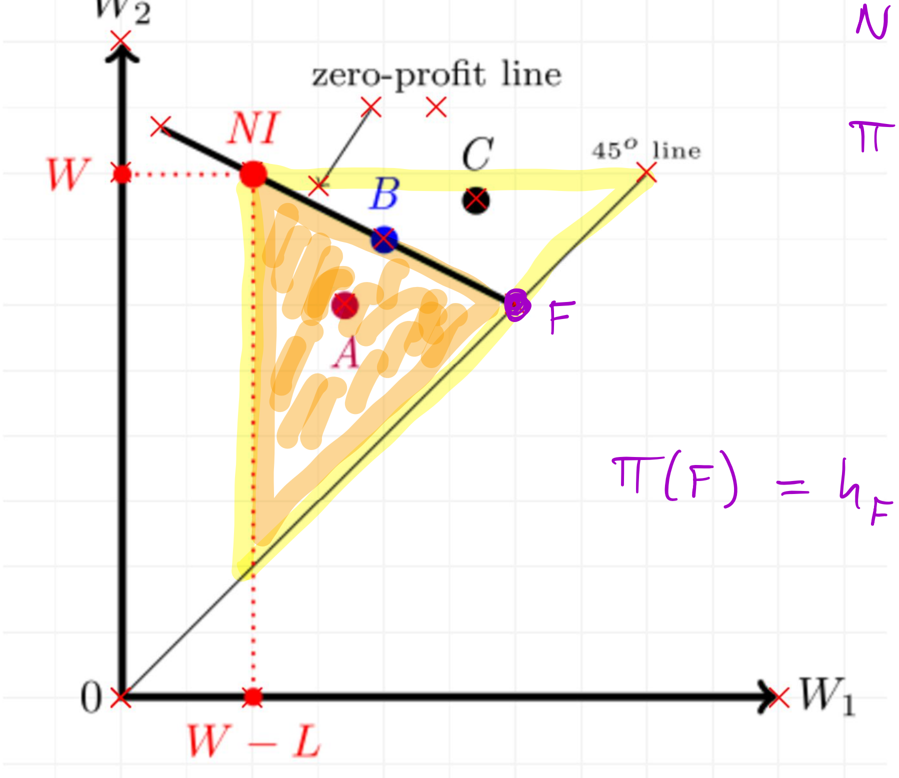
$\pi(B) = \pi_2$

$\wedge$

$\pi(C) = \pi_3$

Since No Insurance can be thought of as the trivial contract *h* = 0 and *d* = *L*, which gives zero profits, the isoprofit line going through the NI point is the zero-profit line:

$\pi(c) < 0$



$NI = (h = 0, d = L)$

$\pi(NI) = h - p(L-d)$

$= 0 - p(L-L)$

$= 0$

$\pi(F) = h_F - pL = 0$

$h_F = pL$

$F = (h = 2{,}800, d = 0)$ $W = 40{,}000$ $L = 28000$ $p = \frac{1}{10}$

$pL = \frac{1}{10} 28000 = 2800$

EXAMPLE

Let $W = 1{,}600,\ L = 360,\ p = \frac{1}{12}$. Consider contract $C = (h_C = 50,\ d_C = 120)$.

(1) Represent NI and C in a wealth diagram.

(2) Calculate $\pi(C)$.

(3) Let D be a contract obtained from C by **reducing** the premium by 20 and increasing the deductible in such a way that $\pi(D) = \pi(C)$. Find the premium and deductible of contract D.

(4) Represent contract D in the wealth diagram.

(5) Find the full-insurance contract, call it F, that lies on the isoprofit line that goes through contracts C and D and represent it in the wealth diagram.

(6) Calculate the slope of the isoprofit line through C in the wealth diagram.

(7) Calculate the equation of the isoprofit line through C in the wealth diagram.

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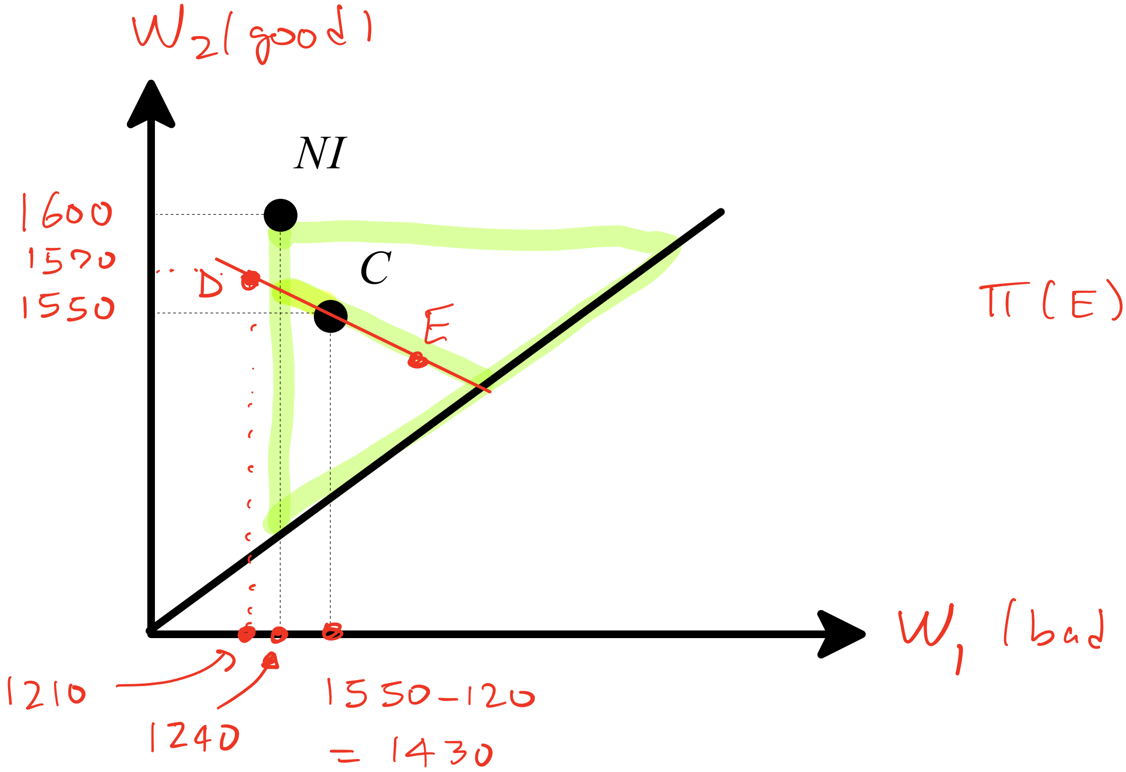
(8) Next prove that given two contracts $A = (h_A, d_A)$ and $B = (h_B, d_B)$, $\pi(A) = \pi(B)$ if and only if the expected value of the wealth lottery (for the insured) corresponding to contact *A* is equal to the expected value of the wealth lottery (for the insured) corresponding to contact *B*

$W = 1{,}600,\ L = 360,\ p = \frac{1}{12}$ $\qquad C = \left(h_C = 50,\ \underbrace{d_C = 120}\right)$

(1) Represent NI and C in a wealth diagram.



$\pi(E) = 30$

(2) Calculate $\pi(C) = 50 - \frac{1}{12}\underbrace{(360 - 120)} = 30$

$W = 1{,}600,\ L = 360,\ p = \tfrac{1}{12}$ $\qquad C = \left(h_C = 50, d_C = 120\right)$

$\pi(C) = 30$

(3) Let D be a contract obtained from C by **reducing** the premium by 20 and increasing the deductible in such a way that $\pi(D) = \pi(C)$. Find the premium and deductible of contract D.

$$D = (h_D = \underbrace{50-20}_{30},\ d_D)$$

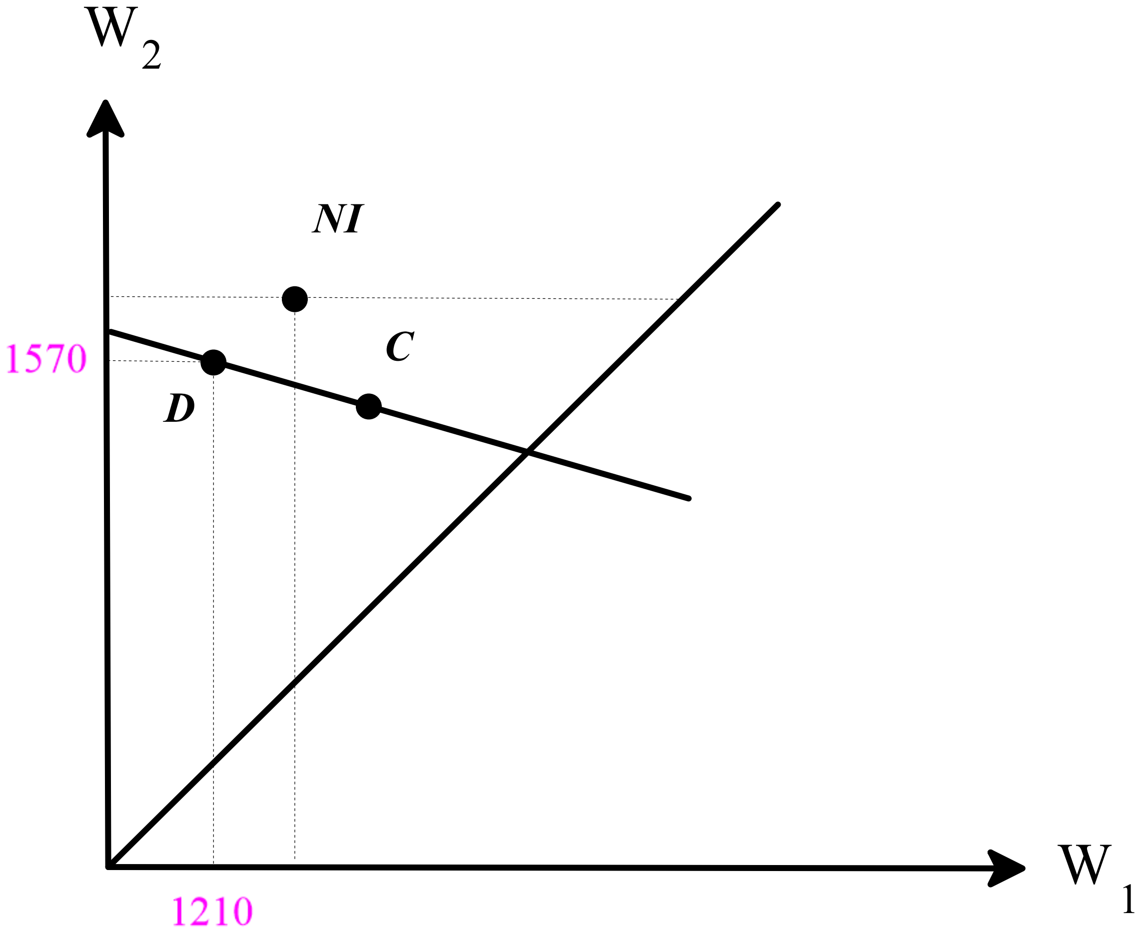
$$\pi(D) = \cancel{30} - \frac{1}{12}(360 - d_D) = \overbrace{\cancel{30}}^{\pi(C)}$$

$$d_D = 360 = L!$$

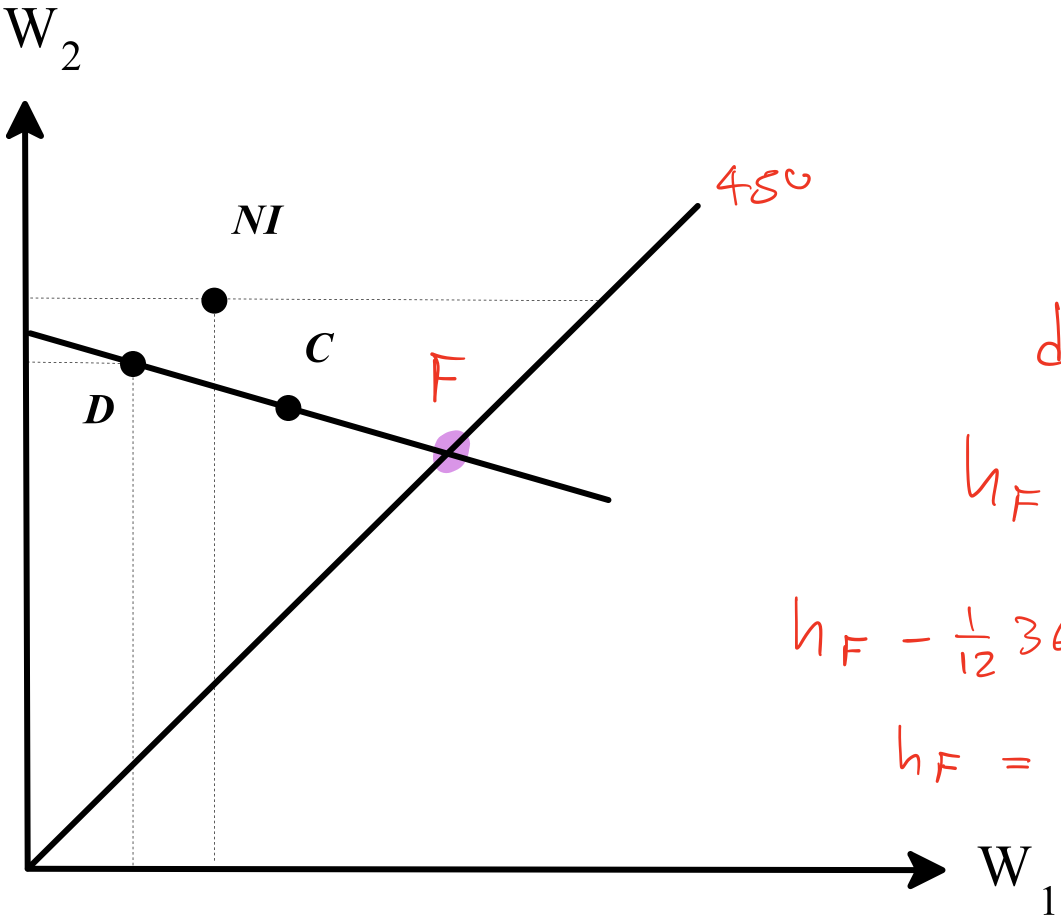
$W = 1{,}600,\ L = 360,\ \ p = \tfrac{1}{12}$ $\qquad C = \left(h_C = 50, d_C = 120\right)$

$\pi(C) = 30 \qquad D = (h_D = 30,\ d_D = 360)$

(4) Represent contract D in the wealth diagram.



(5) Find the full-insurance contract, call it *F*, that lies on the isoprofit line that goes through contracts C and D and represent it in the wealth diagram.



$d_F = 0$

$h_F = ?$

$h_F - \frac{1}{12} 360 = \cancel{0}\ 30$

$h_F = 60$