2. A continuum of contracts

Again $W = 900,\ L = 700,\quad p = \frac{1}{50}\ ,\quad U(m) = \sqrt{m}\ ,\quad B = \left(h_B = 60,\ d_B = 100\right).$

The profit from contract B is $\pi(B) = 60 - \frac{1}{50}(700 - 100) = 48$

$\pi(B) =$

$\pi(h, d) = h - \frac{1}{50}(700 - d) = 48$

Suppose that the insurance company tells the consumer that she can choose any other contract that guarantees a profit of \$48 to the insurer,

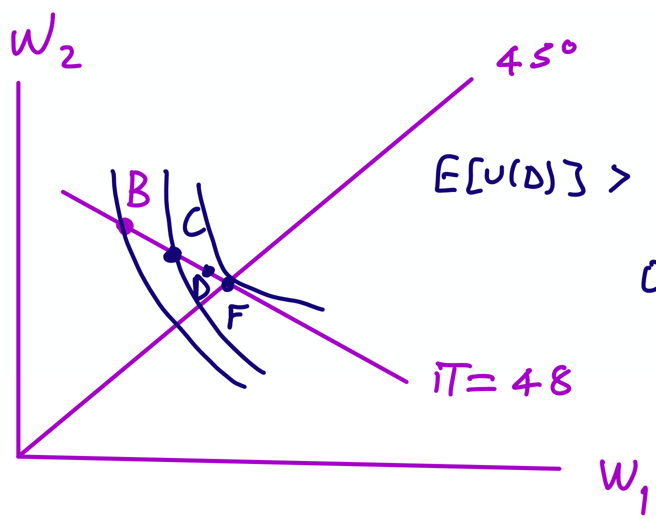
$$h(d) = 62 - \frac{1}{50} d$$

Examples: $h(50) = 61$, $h(100) = 60$ (this is contract B), $h(150) = 59$, $h(200) = 58$

Will the consumer still choose contract $B = \left(h_B = 60,\ d_B = 100\right)$?

$F = (h = 62,\ d = 0)$

$E[U(F)] = \sqrt{900 - 62}$
$= 28.9482$

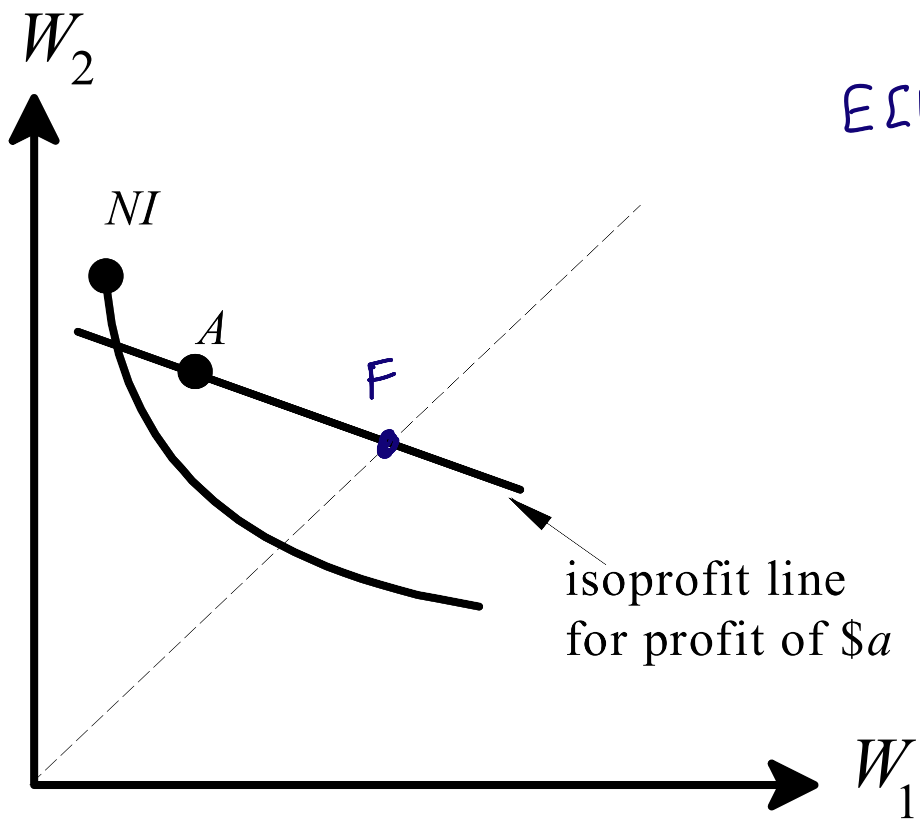


$E[U(D)] > E[U(C)] > E[U(B)]$

Of all these contracts, the best for the consumer is F

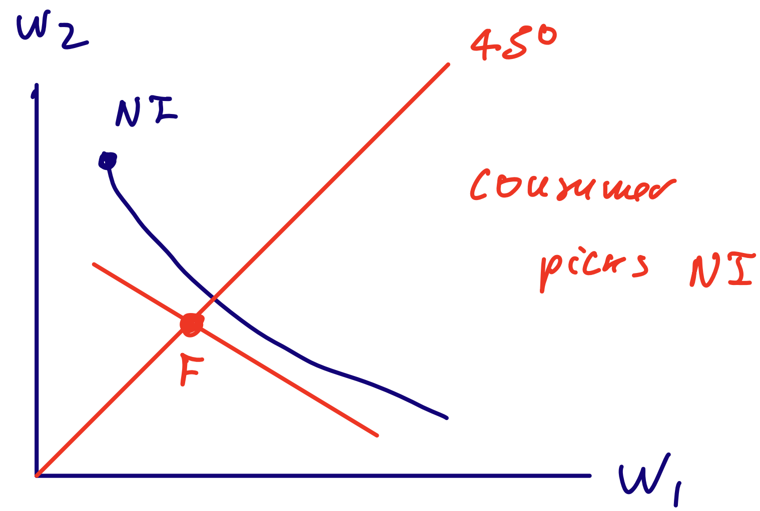
$E[U(NI)] = \frac{1}{50}\sqrt{900 - 700} + \frac{49}{50}\sqrt{900} = 29.68$

By the familiar slope argument…



E[U(F)] > E[U(NI)]

consumer picks F



Principal-Agent relationships

Contractual relationships between two individuals: Principal and Agent. Examples:

Consider first case where Agent's effort is not an issue

Principal	Agent	Contract
Owner of firm	Manager	Division of profits
Client	Lawyer	Lawyer's fee
Land-owner	Farmer	Division of crop
Patient	Doctor	Doctor's fee

Assume that neither individual has any additional wealth to draw from.
The outcome of the relationship is uncertain:

Two possible outcomes (dollar amounts)

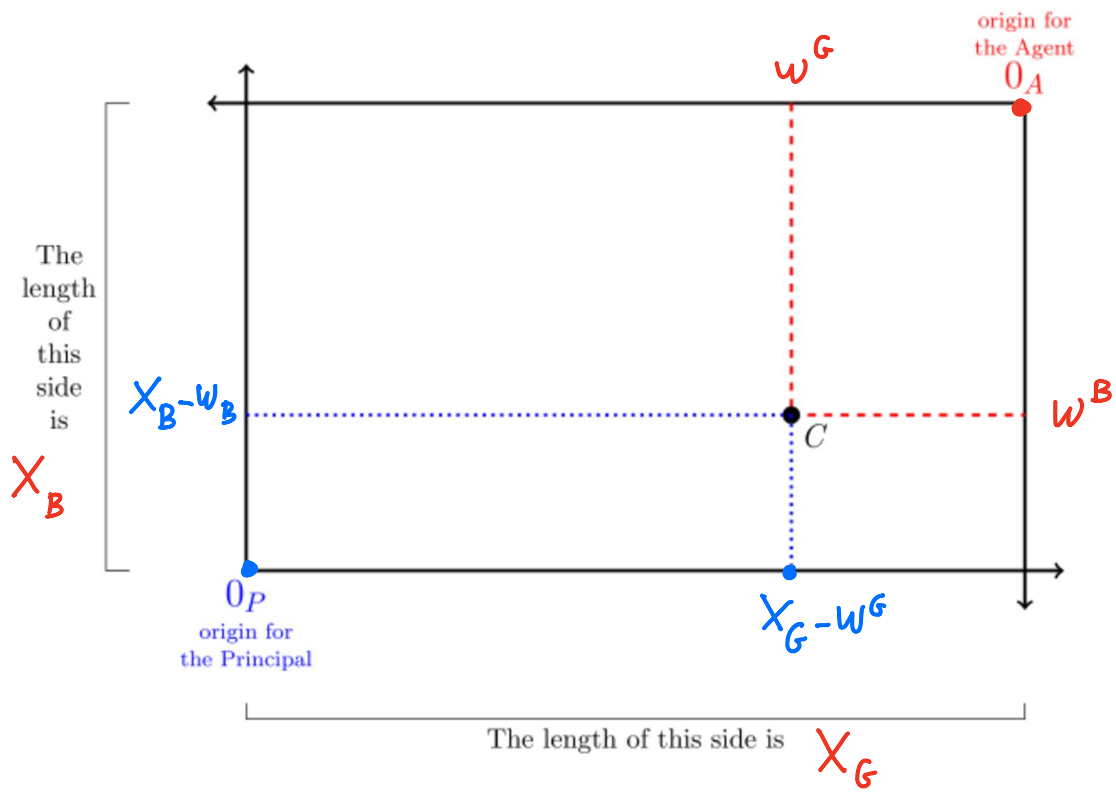
	good		bad	
	X_G	$>$	X_B	also known
probability	p		$1-p$	fixed and known

A contract is specified as a pair (w^G, w^B)

w^G payment to Agent if outcome X_G
so that Principal will get $X_G - w^G$

w^B payment to Agent if outcome is X_B $\quad 0 \le w_G \le X_G$
So Principal gets $X_B - w^B$ $\quad 0 \le w^B \le X_B$

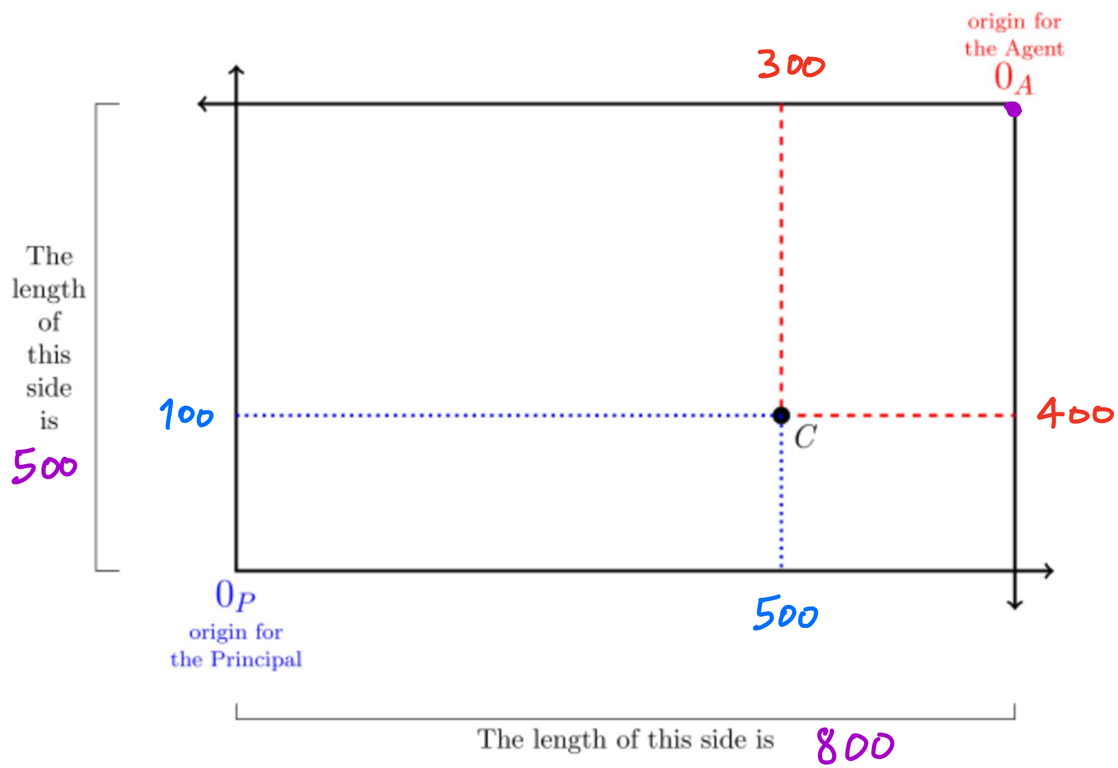
The set of possible contracts can be represented graphically by means of an Edgeworth box



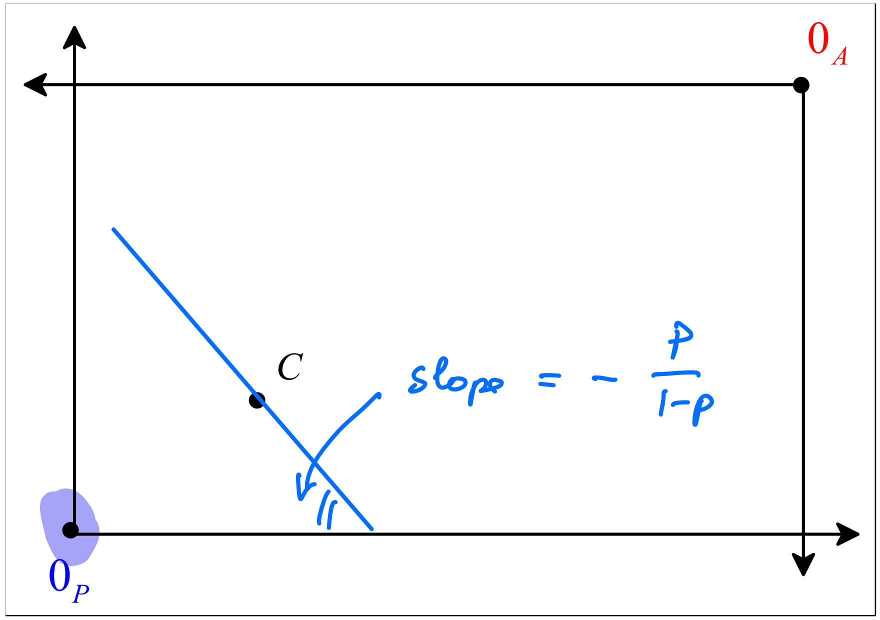
C viewed by Agent: (w^G, w^B)

C viewed by Principal $(X_G - w^G, X_B - w^B)$

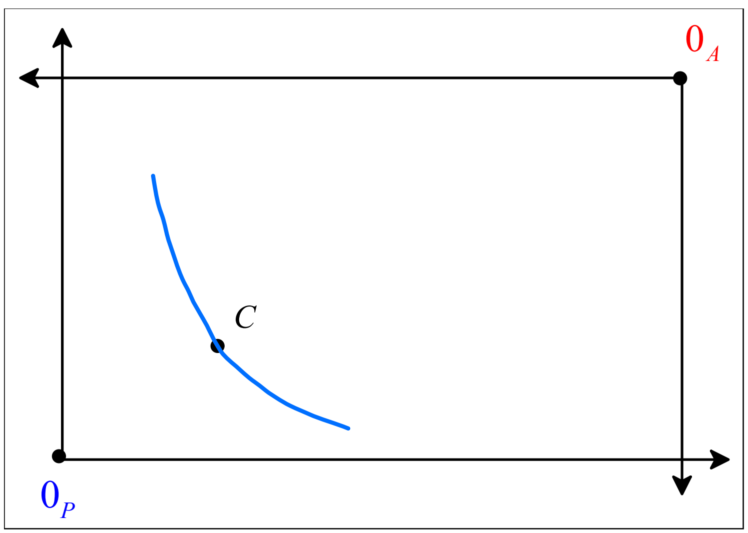
Example: $X^G = \$800,\ X^B = \$500,\ C = (w^G = 300, w^B = 400)$

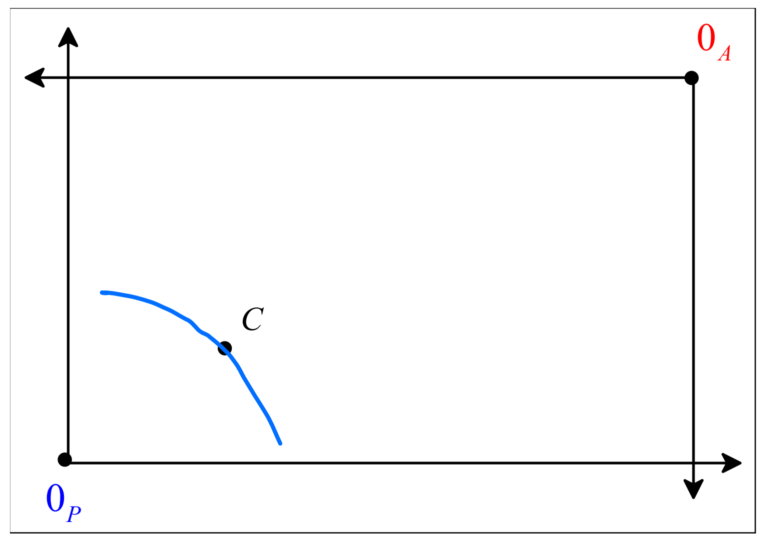


INDIFFERENCE CURVES Start with the Principal



Principal is risk neutral

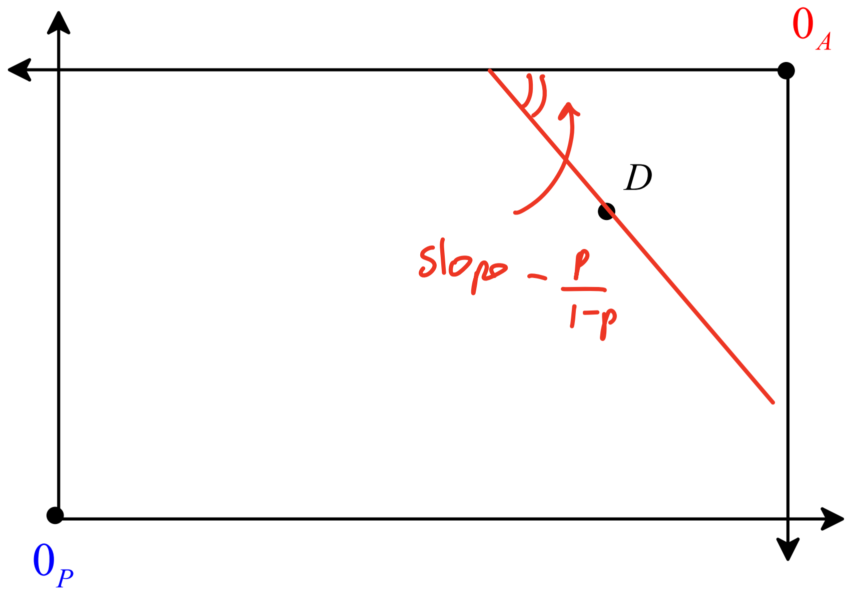




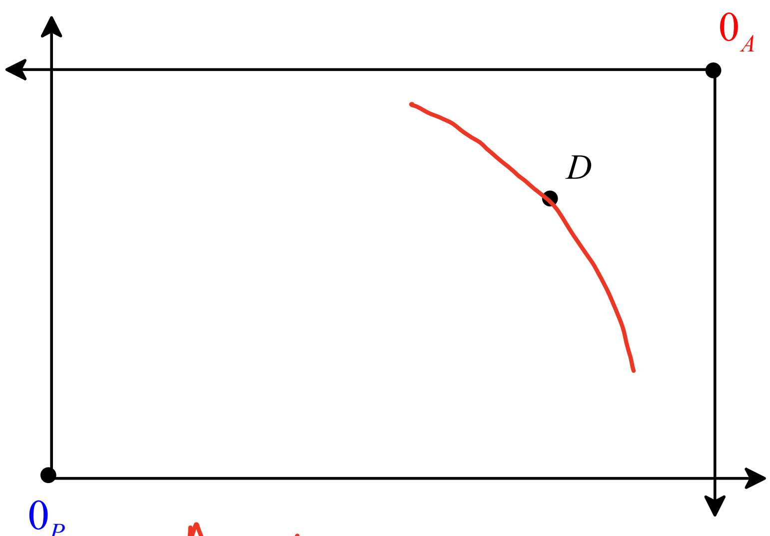
Principal risk-averse

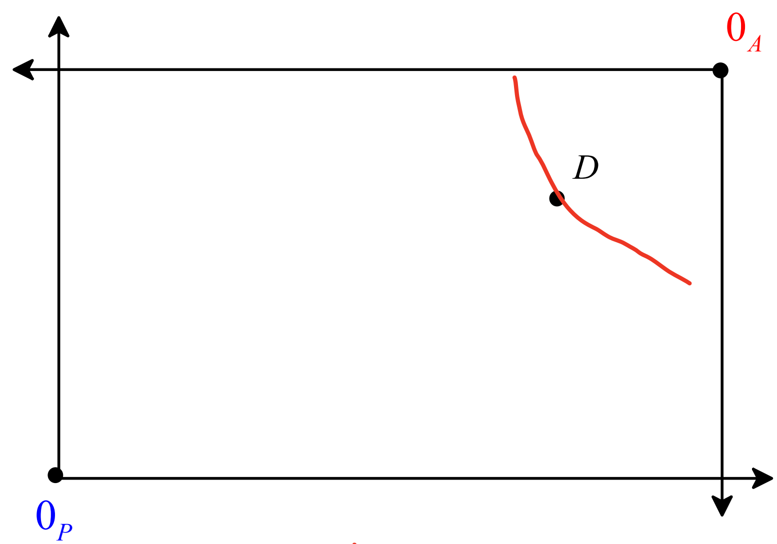
Principal risk-loving

INDIFFERENCE CURVES now the Agent



Agent is risk neutral

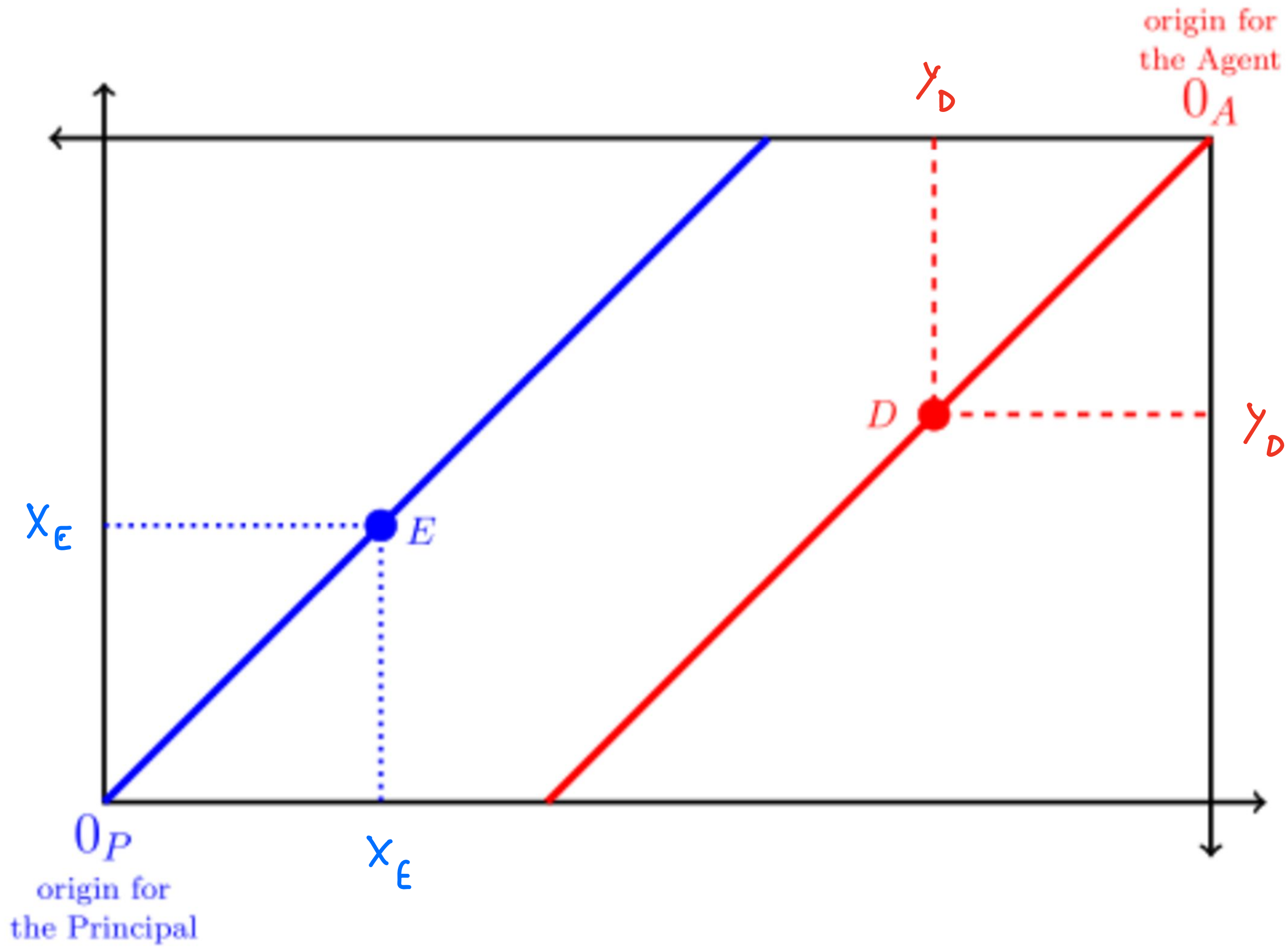




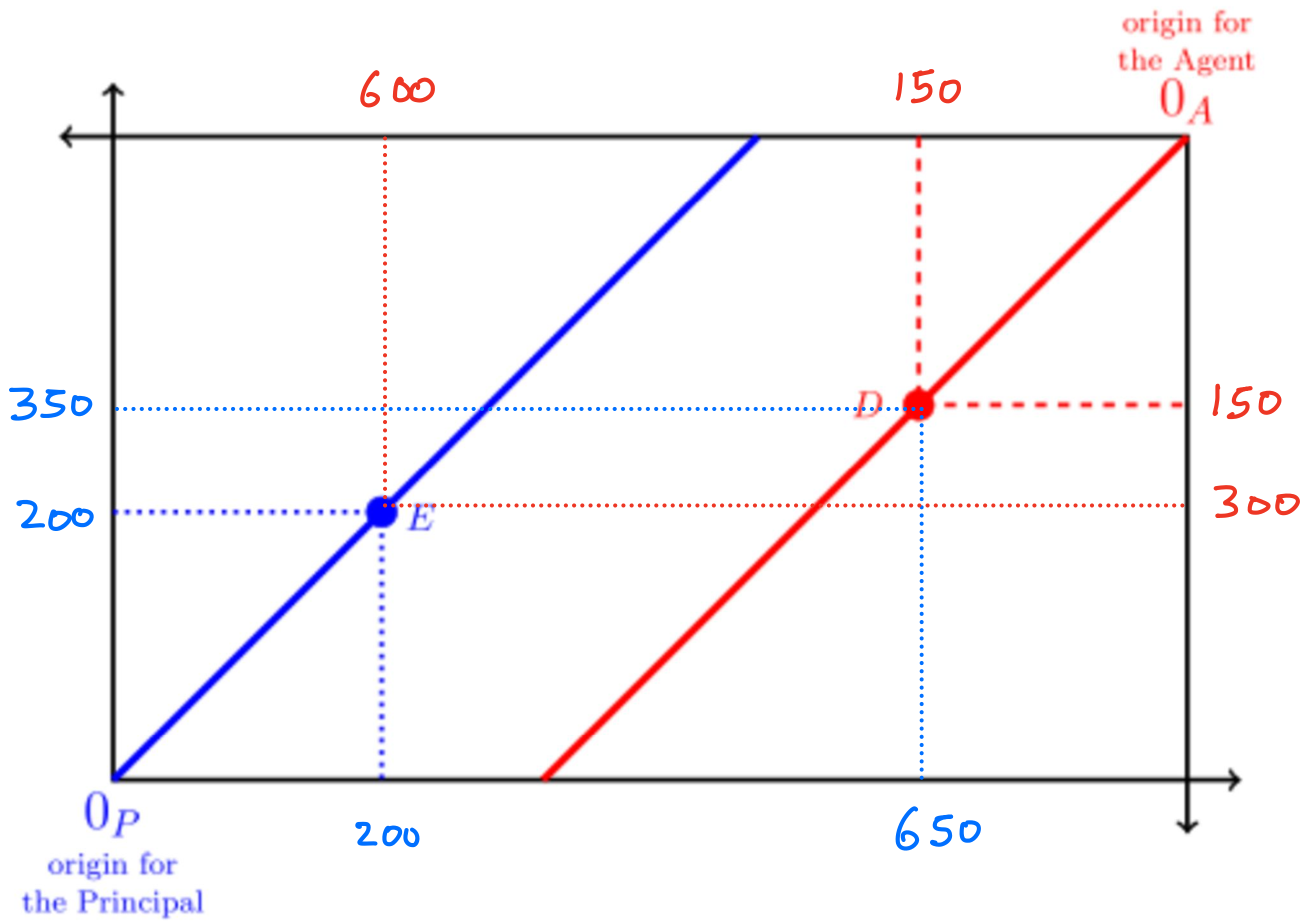
Agent is risk-averse

Agent is risk loving

The 45° lines


origin for
the Agent
0_A
Y_D
D
Y_D
X_E
E
X_E
0_P
origin for
the Principal

Example: $X^G = \$800,\ X^B = \$500,\ D = (w^G = 150, w^B = 150),\ E = (w^G = 600, w^B = 300)$



Recall: a contract is a pair (w^G, w^B) where w^G is the payment to the Agent if the outcome is X^G and w^B is the payment to the Agent if the outcome is X^B.

$U_P(m)$ Principal's utility function VNM

$U_A(m)$ Agent's utility function. VNM

Given a contract C $=(w^G, w^B)$, the Principal's expected utility is:

$$\mathbb{E}[U_P(C)] = p\, U_P(X_G - w^G) + (1-p)\, U_P(X_B - w^B)$$

while the Agent's expected utility is:

$$\mathbb{E}[U_A(C)] = p\, U_A(w^G) + (1-p)\, U_A(w^B)$$

We want to characterize the set of **Pareto efficient contracts**.

Definition. Contract ***C* is *Pareto dominated* by contract *B*** if: or B Pareto dominates C

either $\begin{cases} E[U_A(B)] > E[U_A(C)] \\ \text{and} \\ E[U_P(B)] \geq E[U_P(C)] \end{cases}$ that is, $B \succ_A C$ and $B \succsim_P C$

or $\begin{cases} E[U_P(B)] > E[U_P(C)] \\ \text{and} \\ E[U_A(B)] \geq E[U_A(C)] \end{cases}$ that is, $B \succ_P C$ $B \succsim_A C$

Definition. A contract that is not Pareto dominated by any other contract is called ***Pareto efficient* (or Pareto optimal)**. Thus contract C is Pareto efficient if for every other contract D, either

or

or both.