$v(q) > u(q)$

What if there is **asymmetric information**: only the owner knows the quality q?

Publicly available information:

Quality q	best: A	B	C	D	E	worst: F	
Number of cars	120	200	100	240	320	140	Total: 1,120
Proportion	$\frac{120}{1120}$	$\frac{200}{1120}$	$\frac{100}{1120}$	$\frac{240}{1120}$	$\frac{320}{1120}$	$\frac{140}{1120}$	
$v(q)$ (seller)	720	630	540	450	360	270	
$u(q)$ (buyer)	800	700	600	500	400	300	

Buyer: if a car is offered to me at price p should I buy it?

Assume that buyers are risk neutral → $U(\$m) = m$

Buying a car at price p is playing the lottery

$$L = \begin{pmatrix} \$(800-p) & \$(700-p) & \$(600-p) & \$(500-p) & \$(400-p) & \$(300-p) \\ \frac{120}{1120}=\frac{3}{28} & \frac{200}{1120}=\frac{5}{28} & \frac{100}{1120}=\frac{5}{56} & \frac{240}{1120}=\frac{3}{14} & \frac{320}{1120}=\frac{2}{7} & \frac{140}{1120}=\frac{1}{8} \end{pmatrix}$$

$$E[L] = \frac{3}{28}(800-p) + \frac{5}{28}(700-p) + \dots + \frac{1}{8}(300-p) =$$

$= 523.21 - p$ as long as $p < 523$ I should buy

Suppose $p = 460$

not offered for sale

Quality q	best: A	B	C	D	E	worst: F
$v(q)$ (seller)	720	630	540	450	360	270

initial prob.

$\frac{3}{28}$	$\frac{5}{28}$	$\frac{5}{56}$	$\frac{3}{14}$	$\frac{2}{7}$	$\frac{1}{8}$
$\frac{6}{56}$	$\frac{10}{56}$	$\frac{5}{56}$	$\frac{12}{56}$	$\frac{16}{56}$	$\frac{7}{56}$

new prob. given $\{D, E, F\}$

0	0	0	$\frac{12}{35}$	$\frac{16}{35}$	$\frac{7}{35}$

$12 + 16 + 7 = 35$

Buying a car at price $P = 460$ means playing

$$M = \begin{pmatrix} (500-460) & (400-460) & (300-460) \\ \frac{12}{35} & \frac{16}{35} & \frac{7}{35} \end{pmatrix}$$

$$E[M] = 414.29 - 460 = -45.71$$

buyers should not buy if $p = 460$

If $270 < p < 300$ then the market is active but only quality F is traded

Suppose $p = 380$

Quality q	best: A	B	C	D	E	worst: F
$v(q)$ (seller)	720	630	540	450	360	270
initial prob.	.	.	.	.	$\frac{16}{56}$	$\frac{7}{56}$
new prob given $\{E, F\}$	0	0	0	0	$\frac{16}{23}$	$\frac{7}{23}$

Suppose $270 < p < 360$ only quality F is offered for sale

buyer realizes that she plays $\begin{pmatrix} 300-p \\ 1 \end{pmatrix}$

$300 - p > 0$ $\quad p < 300$

$16 + 7 = 23$

Buying at price $p = 380$ means $N = \begin{pmatrix} (400-380) & (300-380) \\ \frac{16}{23} & \frac{7}{23} \end{pmatrix}$

$$E[N] = \frac{16}{23}(400-380) + \frac{7}{23}(300-380) = 369.57 - 380$$

$$= -10.43$$

Buyers not willing to buy if $p = 380$

	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$
Quality	L	M	H
probability	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$
seller's value	900	1,200	1,400
buyer's value	1,020	1,320	1,500

Buyers are risk neutral

For every price p determine if there is a second-hand market.

- $P \geq 1{,}400$ all qualities offered for sale

  Buyer : $\begin{pmatrix} 1{,}020 & 1{,}320 & 1{,}500 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{pmatrix}$ exp. value $= 1{,}300$

  nobody willing to buy.

- $1{,}200 \leq P < 1{,}400$ Only qualities L and M offered for sale

  Buyer : $\begin{pmatrix} 1{,}020 & 1{,}320 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $1+4=5$

  Exp. value : $\frac{1}{5}\, 1{,}020 + \frac{4}{5}\, 1{,}320 = 1{,}260$

  if $1{,}200 \leq P \leq 1{,}260$ then there is trading and all cars of qualities L and M are traded. No trading of quality H

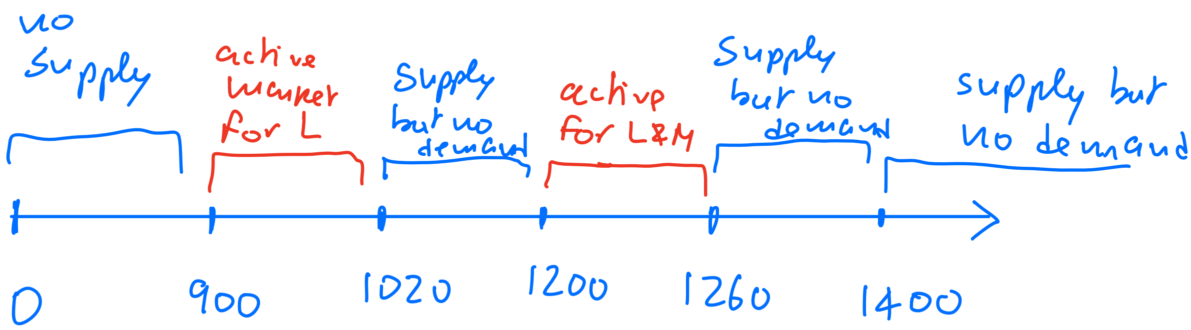
  if $1{,}260 < p < 1{,}400$ buyers not willing to buy

- $900 \leq P < 1{,}200$ M and H not offered for sale

  L is

  Buyer : $\begin{pmatrix} 1{,}020 \\ 1 \end{pmatrix}$

  - $1{,}020 < P < 1{,}200$ no buyer
  - $900 \leq P \leq 1{,}200$ active market but only for L



ADVERSE SELECTION