$C_H = (h_H, d_H)$, $C_L = (h_L, d_L)$ $p_H > p_L$

Monopolist's problem is to

$$\underset{h_H, d_H, h_L, d_L}{Max} \; \pi_3 = q_H N\left[h_H - p_H(L-d_H)\right] + (1-q_H)N\left[h_L - p_L(L-d_L)\right]$$

subject to

(IR_L) C_L is acceptable to L type $EU_L(C_L) \geq EU_L(NI)$

(IC_L) C_L is not worse than C_H for L type : $EU_L(C_L) \geq EU_L(C_H)$

(IR_H) C_H is acceptable to H type $EU_H(C_H) \geq EU_H(NI)$

(IC_H) C_H is not worse than C_L for H type : $EU_H(C_H) \geq EU_H(C_L)$

(IR_H) follows from (IR_L) and (IC_H)

redundant

- (IR_L) $EU_L(C_L) \geq EU_L(NI)$
- if a contract is acceptable to L then it is acceptable also to H
  since $EU_L(C_L) \geq EU_L(NI)$ it follows $EU_H(C_L) \geq EU_H(NI)$
  put this together with $EU_H(C_H) \geq EU_H(C_L)$
  to get (IR_H)

$$EU_L(C_H) = p_L U(W - h_H - d_H) + (1-p_L) U(W - h_H)$$

Thus, the problem can be reduced to

$$\underset{h_H, d_H, h_L, d_L}{Max} \; \pi_3 = \underbrace{q_H N\left[h_H - p_H(L - d_H)\right]}_{\text{increasing in } h_H} + \overbrace{(1-q_H)N\left[h_L - p_L(L - d_L)\right]}^{\text{independent of } h_H}$$

subject to

(IR_L) $\quad EU_L[C_L] \geq EU_L[NI]$ $\quad$ independent of h_H

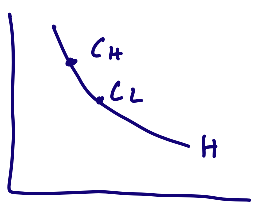
(IC_L) $\quad EU_L[C_L] \geq EU_L[C_H]$ $\quad$ LHS independent of h_H, RHS decreasing in h_H

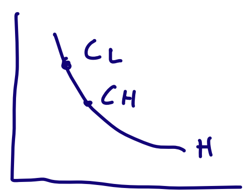
(IC_H) $\quad EU_H[C_H] \geq EU_H[C_L]$

Suppose we are at a point where $EU_H(C_H) > EU_H(C_L)$ and (IR_L) and (IC_L) are satisfied

(IC_H) must be satisfied as an equality.

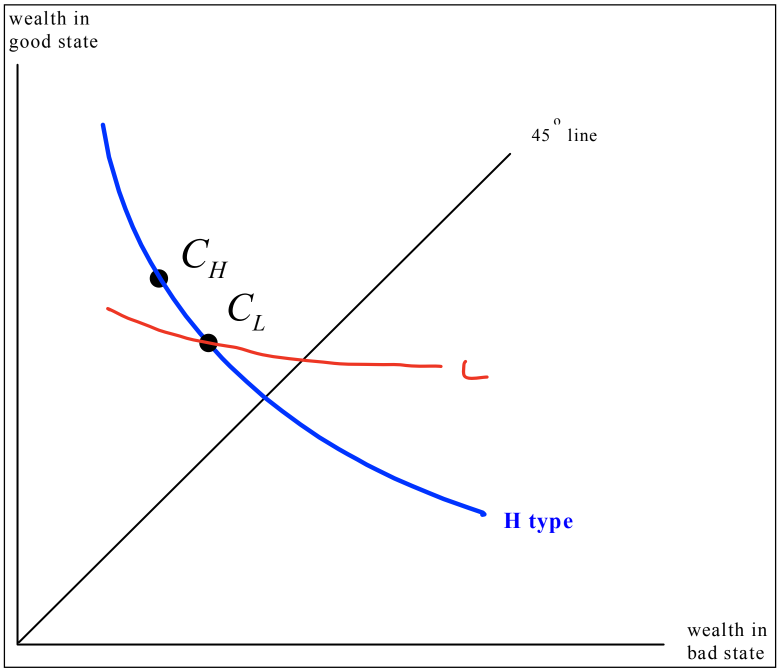
At a maximum $\quad EU_H(C_H) = EU_H(C_L)$





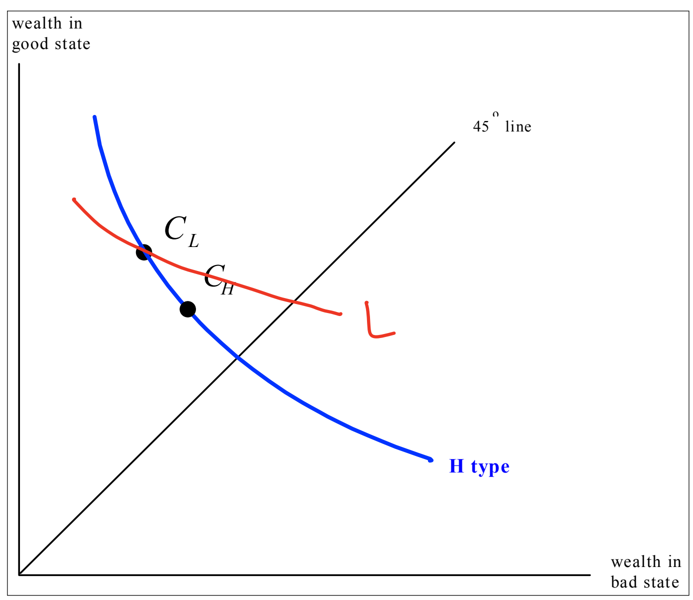
So C_H and C_L be on the same indifference curve for the H type.

On this indifference curve, contract C_H cannot be above contract C_L

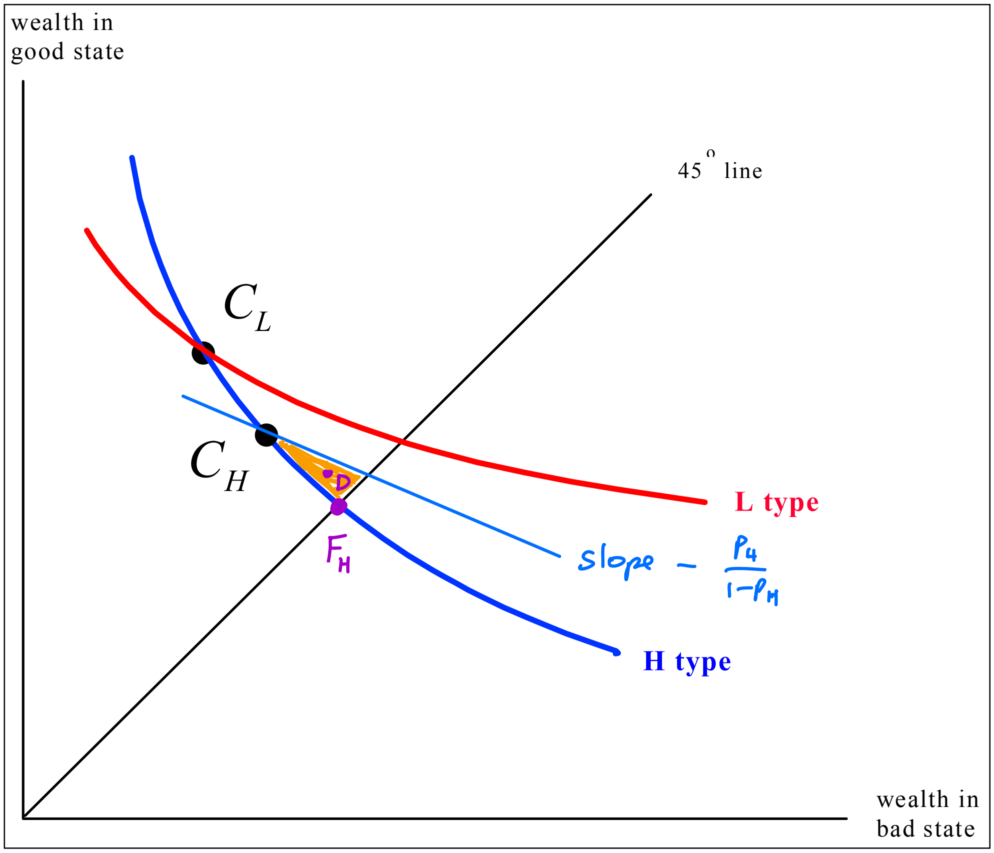


In this case the L type prefer C_H to C_L so (IC_L) is violated

So it must be:

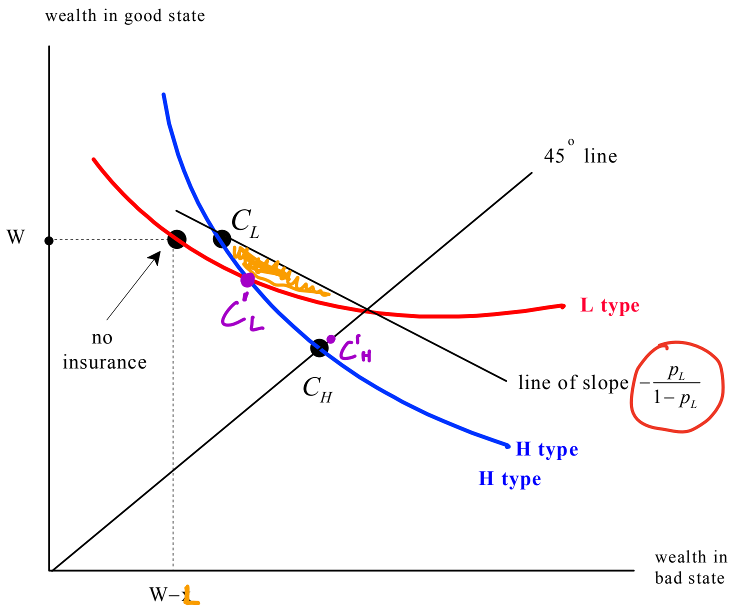


C_H must be a full insurance contract



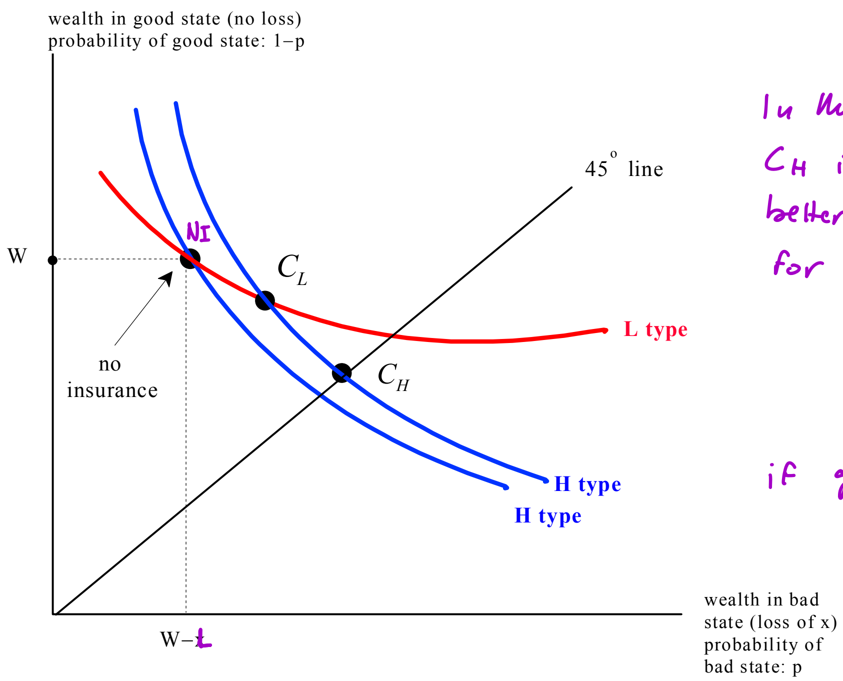
L type prefers C_L to C_H

(IR_L) must be satisfied as an equality.



Since C_L is above 45° line,

- L indiff. curve through C_L is steeper than isoprofit line with slope $-\frac{p_L}{1-p_L}$
- At every point (in particular at C_L) H ind. curve is steeper than L-ind. curve

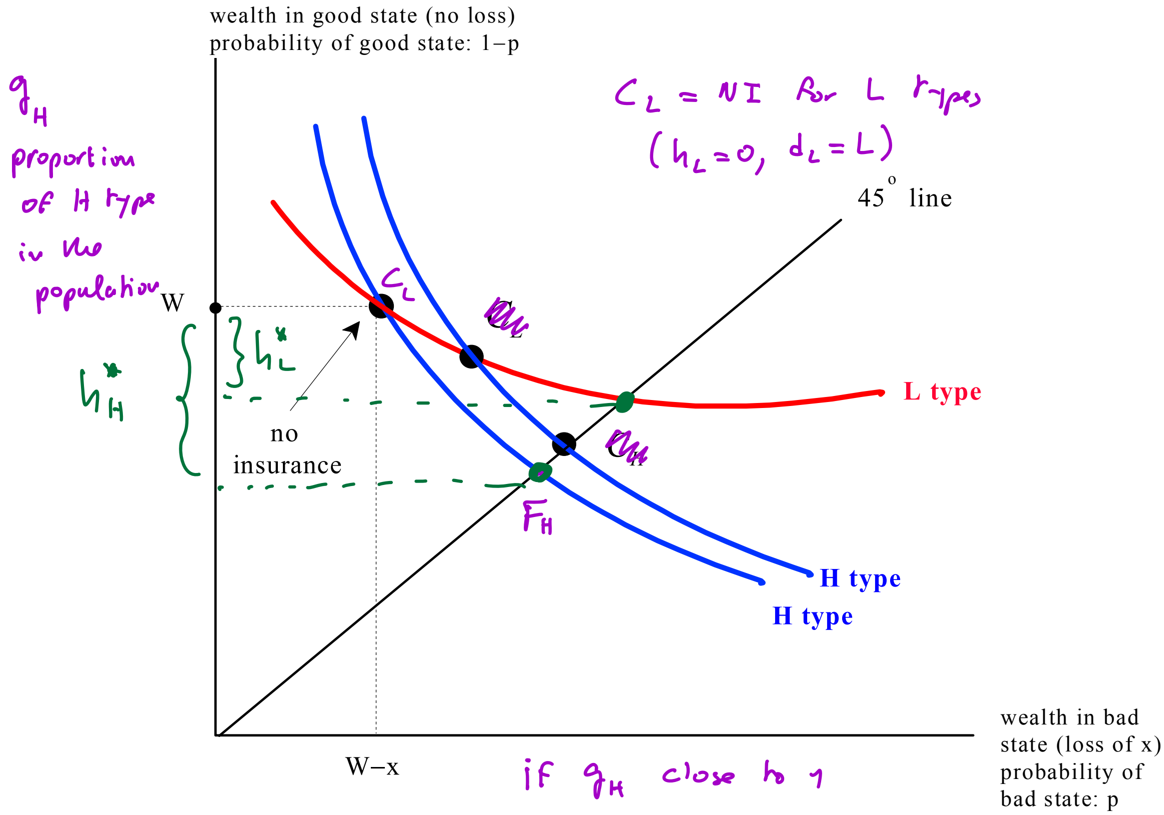


In this case C_H is strictly better than NI for H type

if p_H not close to 1

(IC_L) is not binding: it is always satisfied as a strict inequality.

Option 1 is a special case of Option 3



Monopolist chooses: h_H

$d_H = 0$

h_L, d_L must be such that

$$\begin{cases} EU_H(C_L) = EU_H(C_H) \\ EU_L(C_L) = EU_L(NI) \end{cases}$$

solve for h_L and d_L as a function of h_H

EXAMPLE. $W = 1{,}600$, $L = 700$, $p_H = \frac{1}{5}$, $p_L = \frac{1}{10}$, $U(m) = \sqrt{m}$.

h_H^* is given by the solution to $$\sqrt{1{,}600 - h} = \underbrace{\frac{1}{5}\sqrt{1600-700} + \frac{4}{5}\sqrt{1600}}_{EU_H(NI)}$$

Solution is $h_H^* = 156$

Thus under $\boxed{\textbf{Option 1}}$ profits are:

Now **Option 3**. Let $h_H \in [79, 156]$ be the premium for the full-insurance contract targeted to the H type To find c_L solve:

$h_H \rightsquigarrow C_H = (h_H, 0)$

$\rightsquigarrow C_L$ so as to satisfy

$EU_L(C_L) = EU_L(NI)$

$EU_H(C_L) = EU_H(C_H)$

We can solve the two equations in terms of h_H:

$$h_L(h_H) = h_H + 156\sqrt{1,600 - h_H} - 6,084$$
$$d_L(h_H) = 80h_H + 5,460\sqrt{1,600 - h_H} - 219,260$$

Then the monopolist will choose h_H to maximize

$\pi_3 =$

This function is strictly concave and $\left.\frac{d\pi_3}{dh_H}\right|_{h_h=79} = q_H N > 0$ and

$\left.\frac{d\pi_3}{dh_H}\right|_{h_h=156} = \frac{47}{38}q_H - \frac{9}{38}$. This is negative if and only if $q_H < \frac{9}{47}$. Thus,

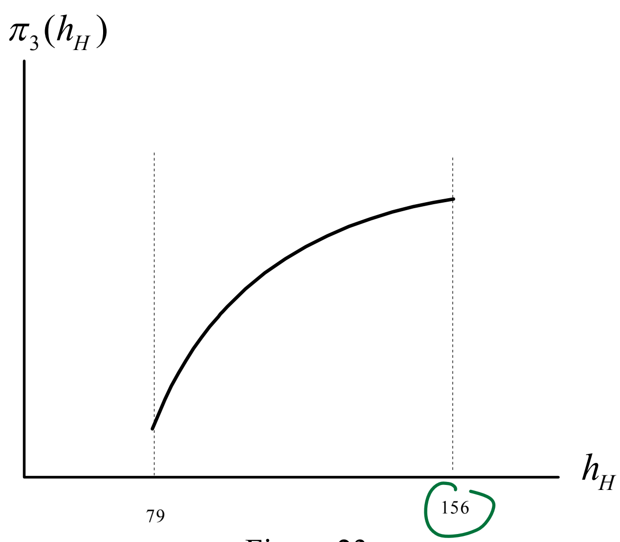


Figure 23a

$\boxed{q_H \geq \frac{9}{47}}$: the optimal solution is to offer only the full insurance contract at premium $h_H^* = 156$ (Option 1)

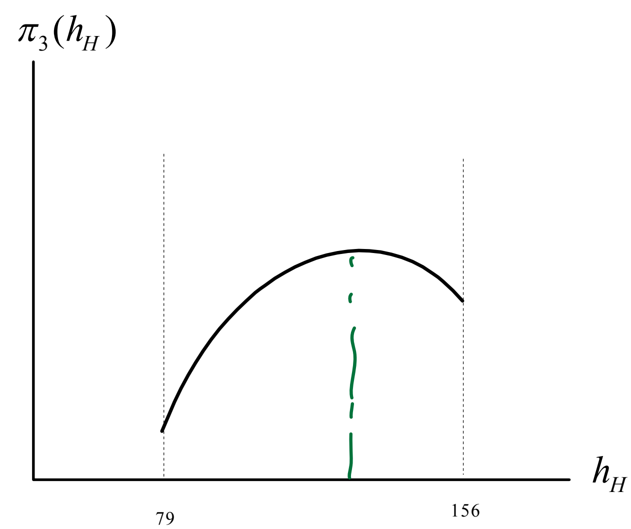


Figure 23b

$\boxed{q_H < \frac{9}{47}}$: the optimal solution is to offer two contracts (Option 3)