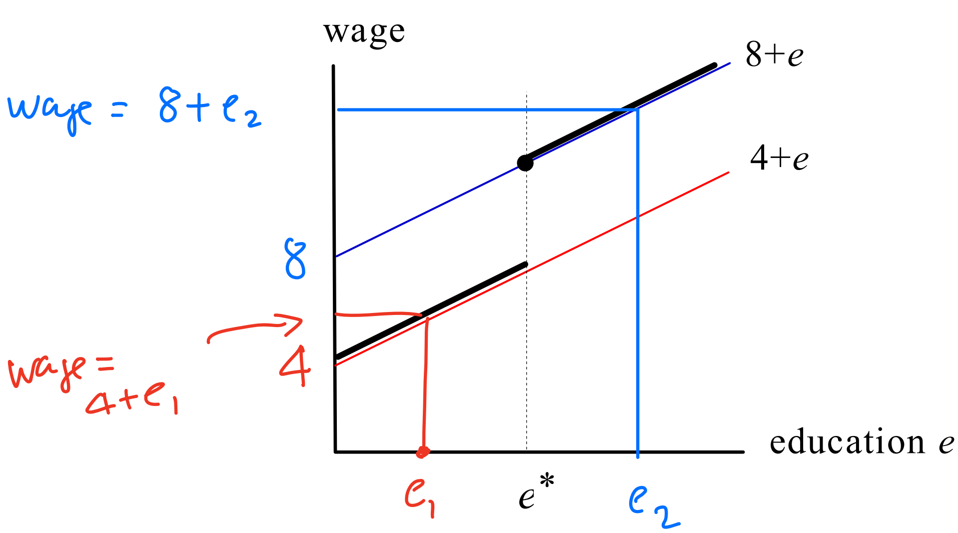
Example of a signaling equilibrium when education does increase productivity

Type *L*: $\begin{cases} \text{productivity: } 4+e \\ \text{cost: } C_L(e) = 4e \end{cases}$ and **Type *H*:** $\begin{cases} \text{productivity: } 8+e \\ \text{cost: } C_H(e) = 2e \end{cases}$



For a signaling equilibrium we need:

for Type L: if $< e^*$ then $e=0$ net $= 4-0=4$

if $\geq e^*$ then $e=e^*$ net $= 8+e^*-4e^*$

for Type H: $e=0$ 4

$e=e^*$ net $8+e^*-2e^*$

$\underbrace{4 > 8+e^*-4e^*}_{\text{L type chooses } e=0}$ $3e^* > 4$ $e^* > \frac{4}{3}$

$8+e^*-2e^* > 4$ $4 > e^*$

$\frac{4}{3} < e^* < 4$

Suppose that 50% of the population is Type L and 50% is Type H.

Consider a signaling equilibrium with $e^* = 3$.

Then Type L have a net wage of 4

Type H a net wage of $8 + 3 - 6 = 5$

productivity of $L = 4 + 0 = 4$

" $H = 8 + 0 = 8$

Force everybody to choose $e = 0$ and force employers to pay

everybody w = average productivity: $\frac{1}{2} 4 + \frac{1}{2} 8 = 6$

L types are better off: $6 > 4$

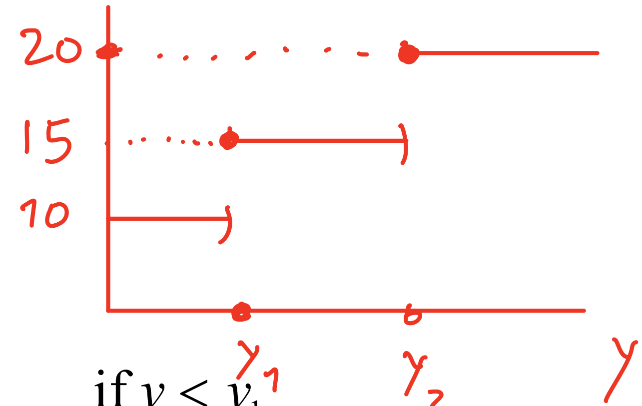
H types " : $6 - 0 > 5$

An example with three types

Type A: productivity 10, cost $C_A(y)=ay$

Type B: productivity 15, cost $C_B(y)=by$

Type C: productivity 20, cost $C_C(y)=cy$



$$0 < c < b < a$$

$$Wage\ offer: \begin{cases} 10 & \text{if } y < y_1 \\ 15 & \text{if } y_1 \le y < y_2 \\ 20 & \text{if } y_2 \le y \end{cases}$$

For a separating signaling equilibrium we need:

Type A to choose $y < y_1$ i.e. $y = 0$

Type B to choose $y_1 \le y < y_2$ i.e. $y = y_1$

Type C to choose $y \ge y_2$ i.e $y = y_2$

Necessary conditions for Type A: $10 > 15 - a y_1$ $y = 0$ better than $y = y_1$

$10 > 20 - a y_2$ $y = 0$ better than $y = y_2$

Necessary conditions for Type B: $15 - b y_1 > 10$ $y = y_1$ better than $y = 0$

$15 - b y_1 > 20 - b y_2$ $y = y_1$ better than $y = y_2$

Necessary conditions for Type C:

$20 - c y_2 > 10$ $y = y_2$ better than $y = 0$

$20 - c y_2 > 15 - c y_1$ $y = y_2$ " " $y = y_1$

For example $y_1 = 5$, $y_2 = 12$, $a = 2$, $b = \frac{3}{4}$, $c = \frac{1}{4}$

we have a signaling equilibrium

Index *vs* signal

	Women, L	Women, H	Men, L	Men, H
productivity	1	2	1	2
proportion	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Cost of acquiring y units of education	y	$\frac{y}{2}$	y	$\frac{y}{2}$

indepent of y

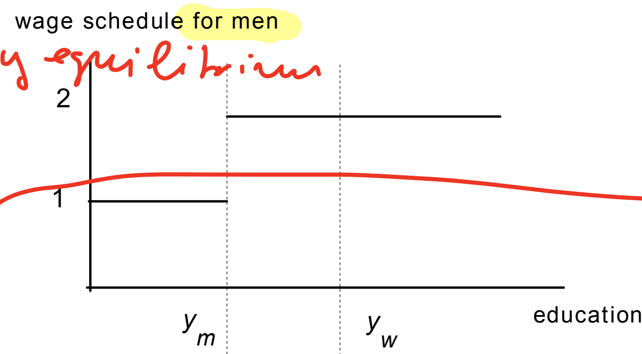
" of gender

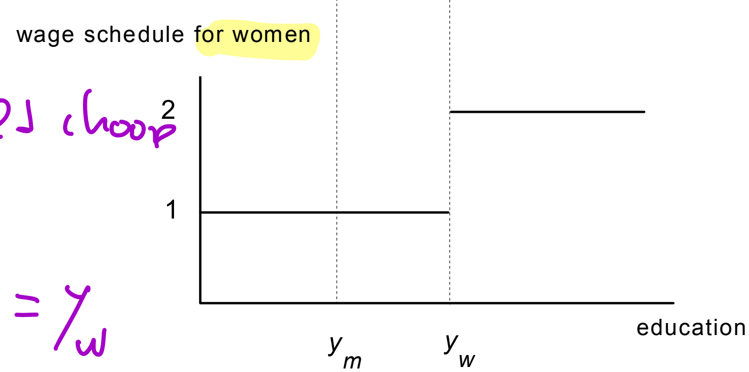
In order to have a signaling equilibrium

L-type men should $y = 0$

H-type men $y = y_m$

$1 < y_m < 2$





L-type women should choose $y = 0$

H-type women $y = y_w$

$1 < y_w < 2$

if $1 < y_m < y_w < 2$