Principal–Agent contracts

Principal hires Agent to perform a task

Outcome can be $\$X_1$ or $\$X_2$ with

$X_1 < X_2$

Contract : (w_1, w_2)

w_1 → payment to Agent if X_1

w_2 → payment to Agent if X_2

Probabilities :

X_1	X_2
p	$1-p$

p is fixed

Moral hazard in Principal-Agent relationships

PRINCIPAL	AGENT	AGENT'S ACTION NOT OBSERVED BY THE PRINCIPAL
Owner of firm	Manager	Amount of time/effort spent running the firm
Client	Lawyer	Amount of time/care devoted to case
Client	Doctor	Amount of time/care devoted to study of patient's symptoms
Land owner	Farmer	Farming effort
Landlord	Renter	Upkeep of building

The outcome is uncertain and is affected by the level of effort exerted by the Agent.

Two possible outcomes: $\$X_1 < \$X_2$

Two possible levels of effort for the Agent: e_L low effort

e_H high effort

probability of $X_1 =$

$$0 < p_1^H < p_1^L < 1$$

↓

probability of X_1 if Agent chooses e_H

- the Principal is risk neutral: $U_P(\$m) = m$

- the Agent is risk averse and dislikes effort: $U_A(m, e)$

$\frac{\partial U_A}{\partial m} > 0$, $\frac{\partial^2 U_A}{\partial m^2} < 0$, $\frac{\partial U_A}{\partial e} < 0$ $U_A(m, e_H) < U_A(m, e_L)$

The analysis of optimal risk-sharing taught us that when the Principal is risk neutral and the Agent is risk averse, Pareto efficiency requires that the Agent be paid a fixed wage. Every fixed-wage contract is Pareto efficient.

EXAMPLE

$$X_1 = 3{,}000 \quad \text{and} \quad X_2 = 6{,}000 \qquad\qquad e_L = 1 \quad \text{and} \quad e_H = 1.1$$

$$\text{probability of } X_1 = \begin{cases} \frac{1}{2} & \text{if } e = 1 \\ \frac{1}{40} & \text{if } e = 1.1 \end{cases} \qquad U_P(\$m) = m \qquad U_A(m,e) = \tfrac{1}{e}\ln(m)$$

A contract is a pair (w_1, w_2)

- w_1 is the payment to the Agent if the outcome is X_1
- w_2 is the payment to the Agent if the outcome is X_2

$w_1 \quad w_2$

Fixed-wage contract: $C = (920\ ,\ 920)$

Agent's expected utility: $U_A(m,e) = \frac{1}{e}\ln(m)$

- if Agent chooses $e = 1$ then $\boxed{p_1 = \frac{1}{2}}$ $U_A = \ln(920) = \boxed{6.82}$
- " $e = 1.1$ " $p_1 = \frac{1}{40}$ $U_A = \frac{1}{1.1}\ln(920) = 6.2$

$\frac{1}{1.1} = \frac{10}{11}$

The Agent will choose $e = 1$ (low effort)

The Principal's expected utility is

$$\text{probability of } X_1 = \begin{cases} \frac{1}{2} & \text{if } e = 1 \\ \frac{1}{40} & \text{if } e = 1.1 \end{cases}$$

$$\frac{1}{2}(3{,}000 - 920) + \frac{1}{2}(6{,}000 - 920) = 3580$$

Variable-wage contract: $D = (200\ ,\ 2{,}000)$

Agent's expected utility: $U_A(m,e) = \frac{1}{e}\ln(m)$

• if $e = 1$ $\begin{pmatrix} 200 & 2{,}000 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $EU_A = \frac{1}{2}\,\frac{1}{1}\ln(200) + \frac{1}{2}\,\frac{1}{1}\ln(2000) = 6.45$

• if $e = 1.1$ $\begin{pmatrix} 200 & 2000 \\ \frac{1}{40} & \frac{39}{40} \end{pmatrix}$ $EU_A = \frac{1}{40}\,\frac{1}{1.1}\ln(200) + \frac{39}{40}\,\frac{1}{1.1}\ln(2000) = \boxed{6.86}$

The Agent chooses

$e = 1.1$

The Principal's expected utility is probability of $X_1 = \begin{cases} \frac{1}{2} & \text{if } e = 1 \\ \frac{1}{40} & \text{if } e = 1.1 \end{cases}$

$$\frac{1}{40}(3{,}000 - 200) + \frac{39}{40}(6{,}000 - 2{,}000) = 3{,}970$$

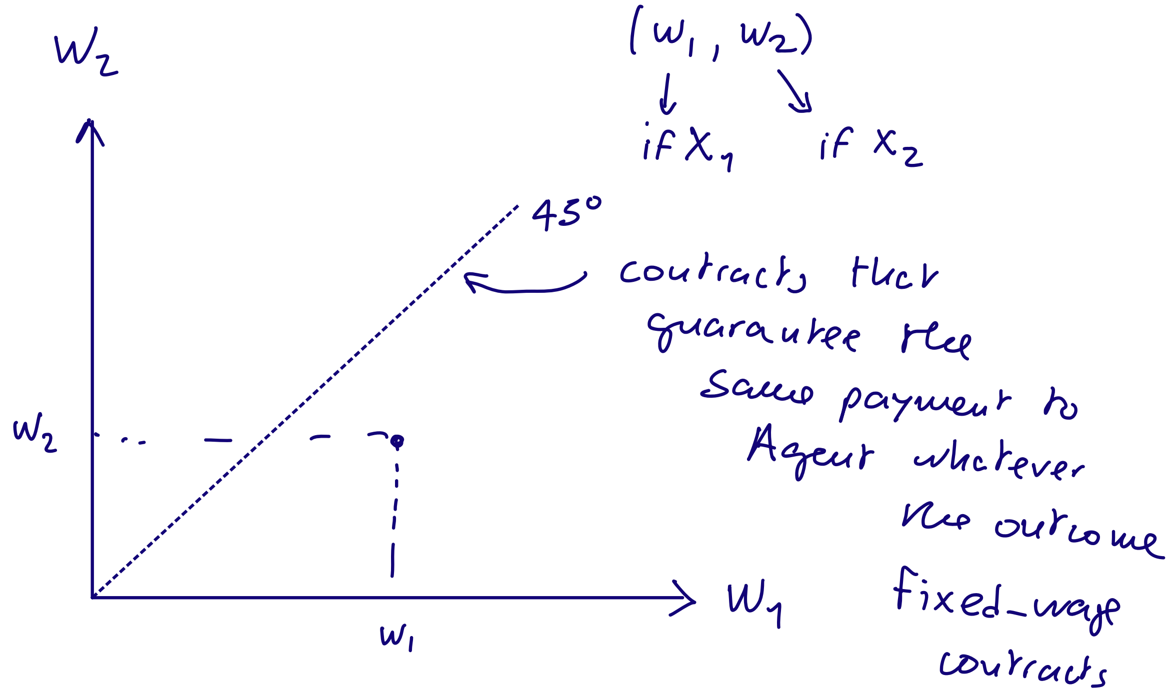
$D \succ_A C$

$D \succ_P C$

D Pareto dominates C so C is not Pareto efficient

	Expected utility of Agent if she chooses $e = 1$	Expected utility of Agent if she chooses $e = 1.1$	Thus the agent will choose $e =$	Thus the Principal's expected utility is
CONTRACT C (fixed wage of \$920)	6.82	6.2	1	3,580
CONTRACT D ($w_1 = 200$, $w_2 = 2{,}000$)	6.45	6.86	1.1	3,970

Contract *D* Pareto dominates contract *C* even though it does not guarantee a fixed income to the risk-averse person (the Agent).



$X_1 = 3{,}000 \qquad X_2 = 6{,}000$

$(1{,}500,\ 1{,}000)$ below $45°$ line

For now on we will focus on the following case:

- two possible outcomes: $0 < \$X_1 < \$X_2$

- two possible levels of effort for the Agent: $0 < e_L < e_H$

- probability of $X_1 = \begin{cases} p_1^L & \text{if } e_L \\ p_1^H & \text{if } e_H \end{cases}$ with $0 < p_1^H < p_1^L < 1$

- the Principal is risk neutral: $U_P(\$m) = m$

- the Agent's utility function is: $u_A(m,e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$ with $c > 0$ and

  $U_A(m)$ strictly increasing and strictly concave

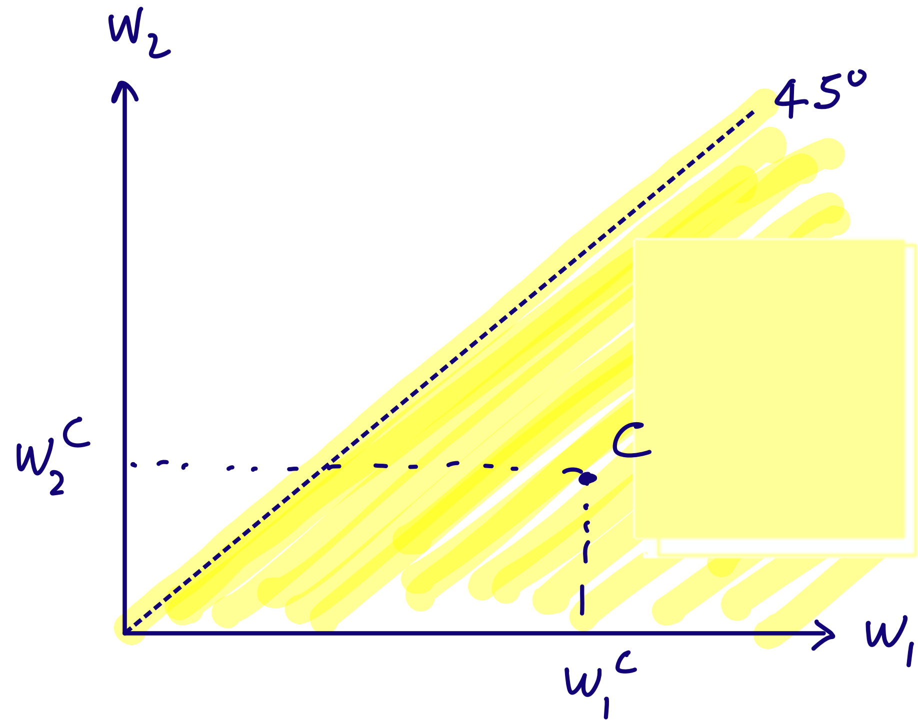
  c is utility cost of effort

  i.e. A is risk averse

on or

Proposition. Any contract below the 45° line, that is, any contract (w_1, w_2) with $w_1 \geq w_2$, is Pareto inefficient: it is Pareto dominated by a contract on the 45° line.

Step 1. Show that if $C = (w_1^C, w_2^C)$ is such that $w_1^C \geq w_2^C$ then the Agent under contract C will choose low effort e_L.



$$u_A(m,e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$$

STEP 1: show that with contract C the Agent chooses e_L

i.e. $EU_{A,e_L}(C) > EU_{A,e_H}(C)$ or $EU_{A,e_L}(C) - EU_{A,e_H}(C) > 0$

$$p_1^L U_A(w_1^C) + (1-p_1^L) U_A(w_2^C) - \underbrace{\left[p_1^H \left[U_A(w_1^C) - c\right] + (1-p_1^H)\left(U_A(w_2^C) - c\right)\right]}_{EU_{A,e_H}(C)}$$

$$= \underbrace{p_1^L U_A(w_1^c) + U_A(w_2^c) - p_1^L U_A(w_2^c)}_{EU_{A,e_2}(c)} -$$

$$\left[p_1^H U_A(w_1^c) - p_1^H c + U_A(w_2^c) - c - p_1^H U_A(w_2^c) + p_1^H c \right]$$

$$= \underbrace{(p_1^L - p_1^H)}_{>0} U_A(w_1^c) - \underbrace{(p_1^L - p_1^H)}_{>0} U_A(w_2^c) + c$$

$$= \underbrace{(p_1^L - p_1^H)}_{>0} \underbrace{\left[U_A(w_1^c) - U_A(w_2^c) \right]}_{} + \underbrace{c}_{>0} \quad > 0$$

c is below 45° line

$w_1^c > w_2^c$

so $U_A(w_1^c) > U_A(w_2^c)$