ECN 106 : Decision Making Professor Giacomo Bonanno

SPRING 2025 - FIRST MIDTERM EXAM: $\boxed{\text{ANSWERS for VERSION 1}}$

1. **(a)** 3 points per correct answer, –2 points per wrong answer. Since *a* weakly (but not strictly) dominates *b* it must be one of the following two cases.

Case 1: $z_1 \succ z_3$ and $z_2 \sim z_4$. **Case 2:** $z_1 \sim z_3$ and $z_2 \succ z_4$. Thus only rankings 7, 8 and 9 are compatible

1. $z_1 \succ z_2 \succ z_3 \succ z_4$
2. $z_1 \succ z_3 \succ z_2 \succ z_4$
3. $z_1 \succ z_4 \succ z_3 \sim z_2$
4. $z_1 \sim z_2 \succ z_3 \sim z_4$
5. $z_4 \succ z_1 \sim z_2 \succ z_3$
6. $z_2 \succ z_1 \succ z_3 \sim z_4$
7. $\boxed{z_1 \sim z_3 \succ z_2 \succ z_4}$
8. $\boxed{z_2 \succ z_1 \sim z_3 \succ z_4}$
9. $\boxed{z_2 \succ z_1 \sim z_3 \sim z_4}$

(b) 2 points per correct answer, –1 point per wrong answer. The second statement implies that $z_1 \succ z_3$ so that it must be that $z_2 \sim z_4$. Thus there are only 5 possible rankings:

$$\begin{array}{l} \text{best} \\ \\ \text{worst} \end{array} \begin{pmatrix} z_2, z_4 \\ z_1 \\ z_3 \end{pmatrix}, \begin{pmatrix} z_1, z_2, z_4 \\ z_3 \end{pmatrix}, \begin{pmatrix} z_1 \\ z_2, z_4 \\ z_3 \end{pmatrix}, \begin{pmatrix} z_1 \\ z_3, z_2, z_4 \end{pmatrix}, \begin{pmatrix} z_1 \\ z_3 \\ z_2, z_4 \end{pmatrix}$$

or, in the alternative notation, $z_2 \sim z_4 \succ z_1 \succ z_3$, $z_1 \sim z_2 \sim z_4 \succ z_3$, $z_1 \succ z_2 \sim z_4 \succ z_3$, $z_1 \succ z_2 \sim z_3 \sim z_4$, $z_1 \succ z_3 \succ z_2 \sim z_4$.

2. **(a)** The expected value of the lottery following *M* is $\frac{3}{10}(320) + \frac{2}{10}(80) = 112$. The expected value of the lottery following *T* is $320p$. Thus it must be that $320p \geq 112$, that is, $p \geq \frac{7}{20} = 0.35 = 35\%$.

(b) When p = 40%, the expected value of the lottery following *T* is $320\left(\frac{4}{10}\right) = 128$. Thus it must be that $X \geq 128$.

(c) When $X = 120$, choosing *A* and then *M* is strictly dominated by choosing *D*. Thus
(1) if $320p < 120$, that is, if $p < 0.375$, Julia will choose *D*,
(2) if $320p > 120$, that is, if $p > 0.375$, Julia will choose first *A* and then *T*
(3) if $320p = 120$, that is, if $p = 0.375$, Julia will either choose *D* or she will choose first *A* and then *T* (she will be indifferent between the two courses of action)

(d) When p = 20% the expected value of the lottery following *T* is $320\left(\frac{2}{10}\right) = 64$, so that choosing first *A* and then *T* is strictly dominated by choosing first *A* and then *M*. Thus
(1) if $X > 112$, Julia will choose *D*,
(2) if $X < 112$ Julia will choose first *A* and then *M*
(3) if $X = 122$ Julia will either choose *D* or she will choose first *A* and then *M* (she will be indifferent between the two courses of action)

3. You can answer this question with or without a utility function.

A. Answer without using a utility function

(a)
- a_3 strictly dominates a_1
- a_4 weakly dominates a_1, but the converse is not true

[For every other pair of actions *x* and *y* it is neither the case that *x* dominates *y* nor the case that *y* dominates *x*.]

(b) The worst outcome with a_1 is z_1, the worst outcome with a_2 is z_5, the worst outcome with a_3 is z_7 and the worst outcome with a_4 is z_{10}. The best of these are z_5 and z_{10}. Thus the Maximin criterion selects $\{a_2, a_4\}$.

(c) The Leximin solution is a_2.

B. Answer using a utility function

Using the following utility function:

z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}
0	3	5	6	2	9	1	4	7	2	3	8

re-write the decision problem as follows:

states → / acts ↓	s_1	s_2	s_3
a_1	0	3	5
a_2	6	2	9
a_3	1	4	7
a_4	2	3	8

(a) • a_3 strictly dominates a_1

• a_4 weakly dominates a_1, but the converse is not true

[For every other pair of actions x and *y* it is neither the case that *x* dominates *y* nor the case that *y* dominates *x*.]

(b) The lowest utility from a_1 is 0, the lowest utility from a_2 is 2, the lowest utility from a_3 is 1 and the lowest utility from a_4 is 2. Thus the Maximin criterion selects $\{a_2, a_4\}$.

(c) The Leximin solution is a_2.

4. (a) For Bob, the expected utility of choice A is $\frac{1}{6}5+\frac{3}{6}10+\frac{2}{6}4=\frac{43}{6}=7.167$. The expected utility of choice B is $\frac{1}{6}8+\frac{3}{6}9+\frac{2}{6}3=\frac{41}{6}=6.833$. The expected utility of choice C is $\frac{1}{6}2+\frac{3}{6}6+\frac{2}{6}7=\frac{34}{6}=5.667$. Thus Bob, too, will choose A.

(b) The utility function for Bob was

outcome	\$100	\$81	\$64	\$49	\$36	\$25	\$16	\$9	\$4
utility	10	9	8	7	6	5	4	3	2

. First subtract 2 to get

basic outcome	\$100	\$81	\$64	\$49	\$36	\$25	\$16	\$9	\$4
utility	8	7	6	5	4	3	2	1	0

. Then divide by 8:

basic outcome	\$100	\$81	\$64	\$49	\$36	\$25	\$16	\$9	\$4
utility	1	$\frac{7}{8}$	$\frac{6}{8}$	$\frac{5}{8}$	$\frac{4}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0