Practice Problems Set # 3

Topic: probability, expected value

VERY IMPORTANT: do not look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will not give you more practice problems later on.

1. Let the universal set be \( U = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8\} \). Let \( A = \{z_2, z_4, z_5, z_7\} \), \( B = \{z_3, z_6, z_8\} \), \( C = \{z_2, z_6\} \), \( D = \{z_3, z_4\} \), \( E = \{z_7, z_8\} \).

You are given the following data: \( P(A \cup B) = \frac{21}{24} \), \( P(A \cap C) = \frac{5}{24} \), \( P(B \cap C) = \frac{3}{24} \), \( P(A \cap D) = \frac{2}{24} \), \( P(B \cap D) = \frac{3}{24} \), \( P(B) = \frac{7}{24} \) and \( P(E) = \frac{2}{24} \).

(a) Find the probability \( P(z_i) \) for each \( i = 1, ..., 8 \).

(b) Calculate \( P((A \cup B) \cap (C \cup D)) \).

2. You toss a fair coin three times.

(a) What is the set of possibilities (or universal set or sample space)?

(b) Let \( E \) be the event that you will get at least one Heads. What is \( E \)?

(c) What is the probability of event \( E \)?

(d) Let \( F \) be the event that you will get a Tails either in the first toss or in the third toss? [Note: this is not an exclusive ‘or’.] What is event \( F \)?

(e) What is the probability of event \( F \)?

3. Let \( U \) be the universal set (or sample space) and \( E \) and \( F \) be two events. Let the complement of \( E \) be denoted by \( \neg E \) and the complement of \( F \) by \( \neg F \). Suppose that \( P(E) = \frac{3}{10} \), \( P(F) = \frac{3}{5} \) and \( P(\neg E \cup \neg F) = \frac{4}{5} \). What is the probability of \( E \cup F \)?

4. Consider the following probability distribution: \( \frac{3}{12} \) \( \frac{1}{12} \) \( \frac{3}{12} \) \( \frac{1}{12} \) \( \frac{3}{12} \) \( \frac{2}{12} \) \( \frac{2}{12} \) \( \frac{1}{12} \) \( \frac{3}{12} \).

What is the probability of the event \( \{z_2, z_3, z_6, z_7\} \)?
5. Consider the following money lottery:

\[
\begin{array}{ccccccc}
$10 & $15 & $18 & $20 & $25 & $30 & $36 \\
\frac{1}{12} & \frac{1}{12} & 0 & \frac{3}{12} & \frac{3}{12} & 0 & \frac{3}{12} \\
\end{array}
\]

What is its expected value?

6. Consider the following lottery:

\[
z_1, \frac{z_2}{10}, \frac{z_3}{10}, \text{ where } z_1 \text{ is the outcome where you get } $100 \text{ and a slap on your face, } z_2 \text{ is the outcome where you get a free trip to Disneyland and a C in this class and } z_3 \text{ is the outcome where you get a free one-year subscription to a newspaper of your choice.}
\]

(a) What is its expected value of this lottery?

(b) Given the choice between getting $20 for sure or playing the above lottery, Amelia chooses to get the $20. Is she risk averse, risk neutral or risk loving?

7. Given the choice between getting $18 for sure or playing the money lottery

\[
\begin{array}{cccc}
$10 & $20 & $30 \\
\frac{3}{10} & \frac{5}{10} & \frac{2}{10} \\
\end{array}
\]

James chooses to get $18. Is he risk averse, risk neutral or risk loving?