ECON 106 : DECISION MAKING Professor Giacomo Bonanno

PRACTICE FINAL: ANSWERS

1.

(a) The possible outcomes are:

z_1	Stays home, gets A
z_2	Stays home, gets C
z_3	At party, good time with Kate, gets C
z_4	At party, good time with Kate, gets F
z_5	At party, rejected, gets C
z_6	At party, rejected, gets F
z_7	At party, not approached by Kate, gets C
z_8	At party, not approached by Kate, gets F

(b) Each state specifies whether the exam is easy or difficult, whether Kate is attracted to him or not and whether Kate is shy or not. Thus there are 8 states:

$s_{e,a,s}$	easy, attracted, shy
$s_{e,a,ns}$	easy, attracted, not shy
$s_{e,na,s}$	easy, not attracted, shy
$s_{e,na,ns}$	easy, not attracted, not shy
$s_{ne,a,s}$	not easy, attracted, shy
$s_{ne,a,ns}$	not easy, attracted, not shy
$s_{ne,na,s}$	not easy, not attracted, shy
$s_{ne,na,ns}$	not easy, not attracted, not shy

The decision problem can the be written as follows:

	$s_{e,a,s}$	$s_{e,a,ns}$	$s_{e,na,s}$	$s_{e,na,ns}$	$s_{ne,a,s}$	$s_{ne,a,ns}$	$s_{ne,na,s}$	$s_{ne,na,ns}$
Stay home	z_1	z_1	z_1	z_1	z_2	z_2	z_2	z_2
Go to party, approach	z_3	z_3	z_5	z_5	z_4	z_4	z_6	z_6
Go to party, be cool	z_7	z_3	z_7	z_7	z_8	z_4	z_8	z_8

(c) We can take values from 0 to 5 as follows:

Outcome	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
Utility	4	2	5	3	0	0	2	1

(d)

	$s_{e,a,s}$	$s_{e,a,ns}$	$s_{e,na,s}$	$s_{e,na,ns}$	$s_{ne,a,s}$	$s_{ne,a,ns}$	$s_{ne,na,s}$	$s_{ne,na,ns}$
Stay home	4	4	4	4	2	2	2	2
Go to party, approach	5	5	0	0	3	3	0	0
Go to party, be cool	2	5	2	2	1	3	1	1

No: for every two acts x and y, there is a state where x is better than y and there is another state where y is better than x.

2.

(a) $E = \{s_{e,a,s}, s_{e,a,ns}, s_{e,na,ns}\}$, $\neg E = \{s_{ne,a,s}, s_{ne,a,ns}, s_{ne,a,ns}, s_{ne,na,ns}\}$

$S = \{s_{e,a,s}, s_{e,na,s}, s_{ne,a,s}, s_{ne,na,s}\}$, $\neg S = \{s_{e,a,ns}, s_{e,na,ns}, s_{ne,a,ns}, s_{ne,na,ns}\}$

$A = \{s_{e,a,s}, s_{e,a,ns}, s_{ne,a,s}, s_{ne,a,ns}\}$, $\neg A = \{s_{e,na,s}, s_{e,na,ns}, s_{ne,na,s}, s_{ne,na,ns}\}$.

(b) $P(E \mid S) = P(E)$, $P(E \mid \neg S) = P(E)$, $P(E \mid A) = P(E)$, $P(E \mid \neg A) = P(E)$, $P(S \mid E) = P(S)$, $P(S \mid \neg E) = P(S)$, etc.

(c) $P(E \cap S) \underset{\text{by independence}}{=} P(E)P(S) = 0.4(0.8) = 0.32$

$P(E \cap A) \underset{\text{by independence}}{=} P(E)P(A) = 0.4(0.5) = 0.2$

$P(A \cap S) \underset{\text{by independence}}{=} P(A)P(S) = 0.5(0.8) = 0.4$

3.

(e) Stay home = $\begin{pmatrix} z_1 & z_2 \\ 0.4 & 0.6 \end{pmatrix}$, To party/approach = $\begin{pmatrix} z_3 & z_4 & z_5 & z_6 \\ 0.2 & 0.3 & 0.2 & 0.3 \end{pmatrix}$

To party/cool = $\begin{pmatrix} z_3 & z_4 & z_7 & z_8 \\ 0.04 & 0.06 & 0.36 & 0.54 \end{pmatrix}$.

(f) We can normalize the utility function U so that $U(z_3) = 1$ and $U(z_5) = U(z_6) = 0$. Since Jonathan is indifferent between $\begin{pmatrix} z_4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} z_3 & z_5 \\ 0.6 & 0.4 \end{pmatrix}$, it must be that $U(z_4) = 0.6U(z_3) + 0.4U(z_5) = (0.6)1 + (0.4)0 = 0.6$. Thus the expected utility of party/approach is $0.2U(z_3) + 0.3U(z_4) + 0.2U(z_5) + 0.3U(z_6) = 0.2(1) + 0.3(0.6) + 0.2(0) + 0.3(0) = 0.38$. Hence, since he is indifferent between party/approach and staying home, it must be that the expected utility of staying home is equal to 0.38, that is, $0.4U(z_1) + 0.6U(z_2) = 0.38$. Thus all we know about the utility function is the following, with $1 > x > 0.6 > y > z > 0$ and $0.4x + 0.6y = 0.38$

Outcome	*Utility*
z_3	1
z_1	x
z_4	0.6
z_2	y
z_7	y
z_8	z
z_5	0
z_6	0

(g) Two questions: (1) what value of p would make you indifferent between z_1 for sure and the lottery $\begin{pmatrix} z_3 & z_5 \\ p & 1-p \end{pmatrix}$? (2) what value of q would make you indifferent between z_8 for sure and the lottery $\begin{pmatrix} z_3 & z_5 \\ q & 1-q \end{pmatrix}$? The answer to the first question gives the value of $U(z_1)$ and this, together with the equation $0.4U(z_1)+0.6U(z_2)=0.38$ enables you to figure out the value of $U(z_2)$. The answer to the second question gives the value of $U(z_8)$.

(h) Then Jonathan's utility function is

Outcome	*Utility*
z_3	1
z_1	0.8
z_4	0.6
z_2	0.1
z_7	0.1
z_8	0.05
z_5	0
z_6	0

Thus EU(stay home) = $0.4U(z_1)+0.6U(z_2)=0.4(0.8)+0.6(0.1)=0.38$

EU(party/approach) = $0.2U(z_3)+0.3U(z_4)+0.2U(z_5)+0.3U(z_6)$

$=0.2(1)+0.3(0.6)+0.2(0)+0.3(0)=0.38$

EU(party/cool) = $0.04U(z_3)+0.06U(z_4)+0.36U(z_7)+0.54U(z_8)$

$=0.04(1)+0.06(0.6)+0.36(0.1)+0.54(0.05)=0.139$

Thus Jonathan will either stay home or go to the party and approach Kate.

4. (A) Since the discount rate is $\rho=\frac{1}{9}$, the discount factor is $\delta=\dfrac{1}{1+\rho}=\frac{9}{10}=0.9$. Thus

(a) $U_0(\$100 \text{ in 6 years})=(0.9)^6(100)=53.14$ and

$U_0(\$200 \text{ in 8 years})=(0.9)^8(200)=86.09$. Thus she chooses to get \$200 in 8 years.

(b) $U_6(\$100 \text{ now})=100$ and $U_6(\$200 \text{ in 2 years})=(0.9)^2(200)=162$. Thus she will choose to get \$200 two years later.

(c) Yes, her preferences are time consistent: she ranks the alternatives the same way at date 0 and at date 6.

(B) **(d)** $U_0(\$100 \text{ in 6 years}) = (0.6)(0.9)^6(100) = 31.89$ and

$U_0(\$200 \text{ in 8 years}) = (0.6)(0.9)^8(200) = 51.66$. Thus she chooses to get \$200 in 8 years.

(e) $U_6(\$100 \text{ now}) = 100$ and $U_6(\$200 \text{ in 2 years}) = (0.6)(0.9)^2(200) = 97.2$. Thus she will change her mind and choose \$100 right away.

(f) No, because she changes her initial plan after 6 years.

5. **(a)** With the Borda count and sincere voting *x* gets 22 points, *a* gets 17, *b* gets 16 and *c* gets 15. Thus the social ranking is

x
a
b
c

If, after the election, *x* drops out then the next best candidate will be chosen, that is candidate *a*.

(b) Eliminating *x* from the above profile we have:

1	**2**	**3**	**4**	**5**	**6**	**7**
c	*a*	*b*	*c*	*a*	*b*	*c*
b	*c*	*a*	*b*	*c*	*a*	*b*
a	*b*	*c*	*a*	*b*	*c*	*a*

and using the Borda count with this profile we have that *a* gets 13 points, *b* gets 14 and *c* gets 15. Thus the social ranking becomes

c
b
a

that is, a complete reversal of the previous one! The winner is now *c*, who was the lowest ranked candidate before!

6. **(a)** When the range of the SCF has only two alternatives, plurality voting satisfies Unanimity, Non-dictatorship and Non-manipulability.

(b)

2's → *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*a*	*a*	*a*	*a*
acb	*a*	*a*	*a*	*a*	*a*	*a*
bac	*a*	*a*	*b*	*b*	*b*	*b*
bca	*a*	*a*	*b*	*b*	*b*	*b*
cab	*a*	*a*	*c*	*c*	*c*	*c*
cba	*a*	*a*	*c*	*c*	*c*	*c*

3 reports abc

2's → *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*b*	*b*	*a*	*a*
acb	*a*	*a*	*b*	*b*	*a*	*a*
bac	*b*	*b*	*b*	*b*	*b*	*b*
bca	*b*	*b*	*b*	*b*	*b*	*b*
cab	*c*	*c*	*b*	*b*	*c*	*c*
cba	*c*	*c*	*b*	*b*	*c*	*c*

3 reports acb

2's → *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*b*	*b*	*a*	*a*
acb	*a*	*a*	*b*	*b*	*a*	*a*
bac	*b*	*b*	*b*	*b*	*b*	*b*
bca	*b*	*b*	*b*	*b*	*b*	*b*
cab	*c*	*c*	*b*	*b*	*c*	*c*
cba	*c*	*c*	*b*	*b*	*c*	*c*

3 reports bac

2's → *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*b*	*b*	*a*	*a*
acb	*a*	*a*	*b*	*b*	*a*	*a*
bac	*b*	*b*	*b*	*b*	*b*	*b*
bca	*b*	*b*	*b*	*b*	*b*	*b*
cab	*c*	*c*	*b*	*b*	*c*	*c*
cba	*c*	*c*	*b*	*b*	*c*	*c*

3 reports bca

` *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*a*	*a*	*c*	*c*
acb	*a*	*a*	*a*	*a*	*c*	*c*
bac	*b*	*b*	*b*	*b*	*c*	*c*
bca	*b*	*b*	*b*	*b*	*c*	*c*
cab	*c*	*c*	*c*	*c*	*c*	*c*
cba	*c*	*c*	*c*	*c*	*c*	*c*

3 reports cab

` *abc* *acb* *bac* *bca* *cab* *cba*

1's ↓

abc	*a*	*a*	*a*	*a*	*c*	*c*
acb	*a*	*a*	*a*	*a*	*c*	*c*
bac	*b*	*b*	*b*	*b*	*c*	*c*
bca	*b*	*b*	*b*	*b*	*c*	*c*
cab	*c*	*c*	*c*	*c*	*c*	*c*
cba	*c*	*c*	*c*	*c*	*c*	*c*

3 reports cba

This SCF satisfies Freedom of Expression, Unanimity and Non-dictatorship but violates Non-manipulability.

7. **(a)** For Ann $U_0(\$100 \text{ in 4 years}) = (0.9)^4(\sqrt{100}) = 6.561$ and $U_0(\$400 \text{ in 6 years}) = (0.9)^6(\sqrt{400}) = 10.629$. Thus she chooses to get \$400 in 6 years.

(b) For Christina, $U_0(\$100 \text{ in 4 years}) = (0.7)(0.8)^4(\sqrt{100}) = 2.867$ and $U_0(\$400 \text{ in 6 years}) = (0.7)(0.8)^6(\sqrt{400}) = 3.67$. Thus she too chooses to get \$400 in 6 years.

(c) For Ann $U_4(\$100 \text{ now}) = \sqrt{100} = 10$ and $U_4(\$400 \text{ in 2 years}) = (0.9)^2(\sqrt{400}) = 16.2$. Thus she chooses \$400 in two years.

(d) For Christina $U_4(\$100 \text{ now}) = \sqrt{100} = 10$ and $U_4(\$400 \text{ in 2 years}) = (0.7)(0.8)^2(\sqrt{400}) = 8.96$. Thus she changes her mind and chooses to get \$100 right away.

(e) Yes, because after 4 years she confirms her earlier choice.

(f) No, because after 4 years she changes her initial plan.