ECN 106 Final Exam

Wednesday, March 20, 10:30am-12:30pm in this room (Giedt 1003)

Office hours on Tuesday, time TBA

- Four questions. Two questions on the material after the third Midterm (Chapters 11, 12 and 13), two questions on earlier material.

- **What you can skip:**
  - ▸ Chapter 5: No need to memorize the axioms of expected utility (Section 5.3)
  - ▸ Chapter 7: Simpson's paradox (Section 7.3)
  - ▸ Chapter 8: Belief revision and Information and truth (Sections 8.3 and 8.4)
  - ▸ Chapter 9: Different sources of information (Section 9.4)
  - ▸ Chapter 11: Proof of Arrow's theorem (Section 11.3)
  - ▸ Chapter 12: Proof of Gibbard-Satterthwaite's theorem (Section 12.4)
  - ▸ Chapter 13: The confirmation bias and The psychology of decision making (Sections 13.5 and 13.6)

Review

1. Choice under certainty. Completeness and transitivity.
   Ordinal utility function.

2. Choice under **un**certainty: States, outcomes, and acts.
   Strict/weak dominance. Difference between "*a* is a dominant act" and "*a* dominates *b*". MaxiMin. Leximin.

state →	s_1	s_2
act ↓		
a	4	8
b	3	7
c	2	5
d	5	0

a strictly dominates b

a " " c

but a is not a dominant act

state →	s_1	s_2
act ↓		
a	4	8
b	3	7
c	2	5
d	$\boxed{4}$	0

a is a weakly dominant act

state →	s_1	s_2
act ↓		
a	4	$\boxed{8}$
b	3	7
c	2	5
d	4	0
e	4	$\boxed{5}$

MaxiMin = a

$\{a, e\} \longrightarrow$ Leximin a

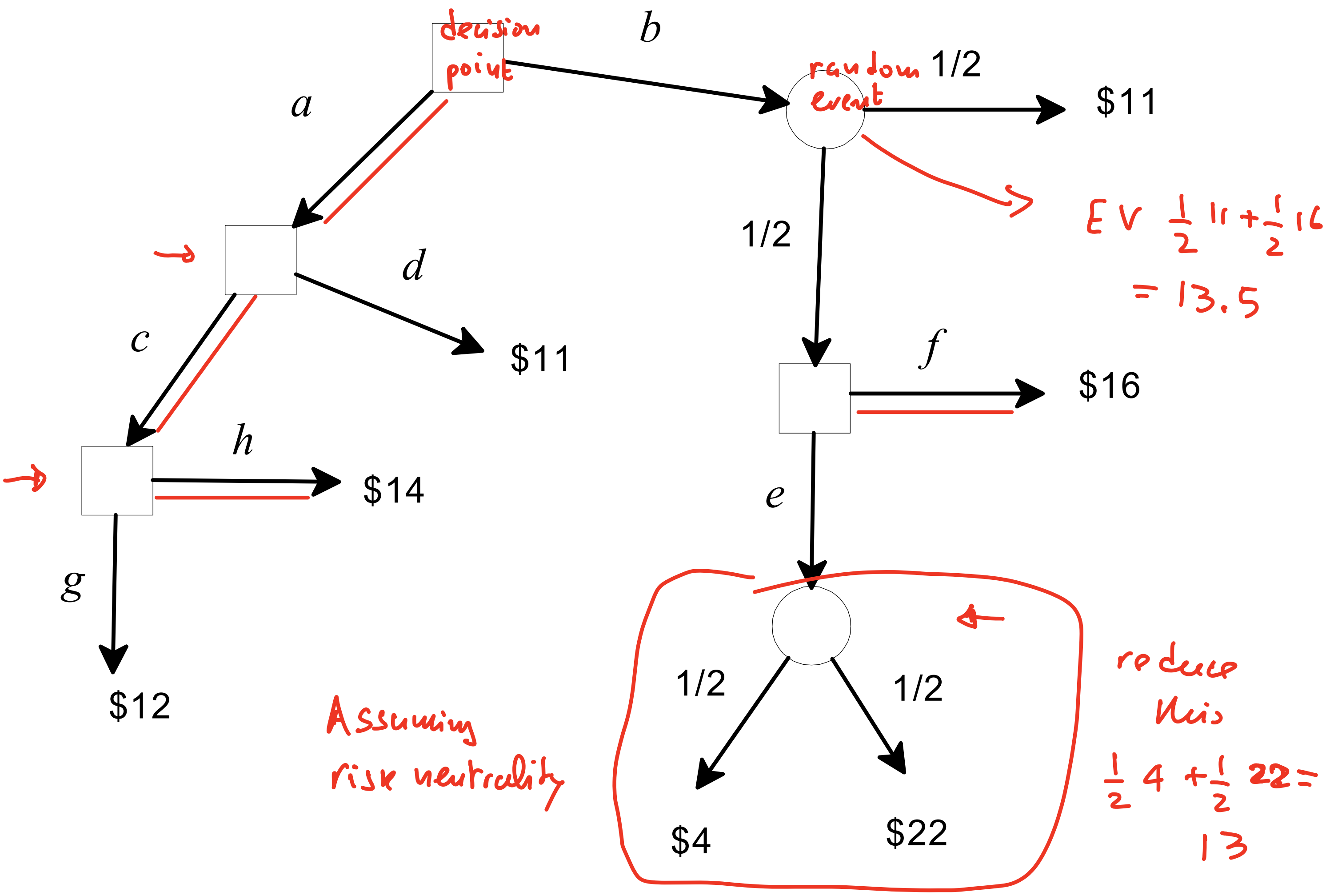
3. Attitudes to risk. Money lotteries, expected value and risk neutrality. Risk aversion. Risk love.

EV(B) = 14

Ann prefers $A = \begin{pmatrix} \$15 \\ 1 \end{pmatrix}$ to $B = \begin{pmatrix} \$8 & \$20 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. What is her attitude to risk?

4. Decision trees. Sequential decisions. Backward induction.

Consider a money-loving individual who faces the following decision:



5. Expected utility: Part 1. von Neumann-Morgenstern utility functions. Normalization.

Suppose there are 6 basic outcomes. What is a utility function?

	z_1	z_2	z_3	z_4	z_5	z_6
U:	3	5	10	1	2	20

Suppose $Z = \{\$9, \$16, \$25, \$36\}$. Suppose the individual is indifferent between $A = \begin{pmatrix} \$16 \\ 1 \end{pmatrix}$ and $B = \begin{pmatrix} \$9 & \$36 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$. Construct a vNM utility function such that $U(\$9) = 3$ and $U(\$36) = 6$.

\$9	\$16	\$25	\$36
3	4	?	6
worst			best

$U(\$16) = \frac{2}{3} 3 + \frac{1}{3} 6 = 2 + 2 = 4$

Is it the case that $U(\$x) = \sqrt{x}$? Not enough information

Suppose $Z = \{\$9, \$16, \$25, \$36\}$. What is the **normalized** utility function of a risk neutral person?

Suppose $U(\$25) = 4.5$

	3	4	4.5	6	
Step 1	0	1	1.5	3	
Step 2	0	$\frac{1}{3}$	$\frac{1.5}{3}$	1	← normalized U

risk-neutral individual	\$9	\$16	\$25	\$36	
	9	16	25	36	←
Step 1	0	7	16	27	
Step 2	0	$\frac{7}{27}$	$\frac{16}{27}$	1	← normalized

6. Expected utility: Part 2.

Decision trees again. MinMax Regret with cardinal utility.

Utility:

	s_1	s_2	s_3
a	9	2	1
b	6	2	2
c	0	5	6

Regret:

	s_1	s_2	s_3
a	0	$5-2=3$	$6-1=5$
b	3	3	4
c	9	0	0

$\leftarrow$ b

	s_1	s_2	s_3
a	9	2	1
b	6	2	2
c	0	5	6

Hurwicz index of pessimism α

↑ weight attached to worst utility

$H_\alpha(a) = 1 \cdot \alpha + 9(1-\alpha)$

$H_\alpha(b) = 2 \cdot \alpha + 6(1-\alpha)$

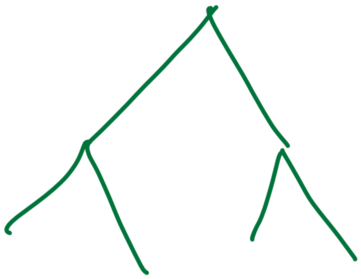
$H_\alpha(c) = 0 \cdot \alpha + 6(1-\alpha)$

if $\alpha = 1$ then back to MaxiMin

For example, if $\alpha = \frac{1}{3}$ then

7. Conditional probability. Bayes' formula: $\boxed{P(E \mid F) = \dfrac{P(F \mid E)\,P(E)}{P(F)}}$.

Bayes' theorem: $\boxed{P(E \mid F) = \dfrac{P(F \mid E)\,P(E)}{P(F \mid E)P(E) + P(F \mid \neg E)P(\neg E)}}$. A simple rule for updating a probability distribution over a finite set.



$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

assuming $P(F) \neq 0$

8. The value of information. Perfect information vs imperfect information. Does information have the potential to change your decision? What information should be chosen?

9. Intertemporal choice: (A) the discounted utility model.

Discounting and present value. Discount factor, discount rate. Time consistency.

$s \geq t$

δ discount factor

$0 < \delta < 1$

$$U_t(x,s) = \delta^{s-t} \underbrace{u_s(x)}_{U_s(x,s)}$$

time consistency

$$= \begin{cases} u_s(x) & s = t \\ \delta^{s-t} u_s(x) & s > t \end{cases}$$

10. Intertemporal choice: (B) hyperbolic discounting.

Conflict between current and future preferences. Time inconsistency. Pre-commitment. Anticipating with time inconsistency: backward induction.

β = present bias

$$U_t(x,s) = \begin{cases} u_s(x) & s = t \\ \beta\delta^{s-t} u_s(x) & s > t \end{cases}$$

possibility of time inconsistency

11. Group decision making: (A) social **preference** functions. Desirable properties (1. Freedom of expression, 2. Rationality, 3. Unanimity, 4. Independence of irrelevant alternatives, 5. Non-dictatorship). Arrow's theorem.

also $\succsim$ for society

12. Group decision making: (B) social choice functions. Desirable properties (1. Unanimity, 2. Non-dictatorship, 3. Nonmanipulability). The Gibbard-Satterthwaite theorem.

one alternative