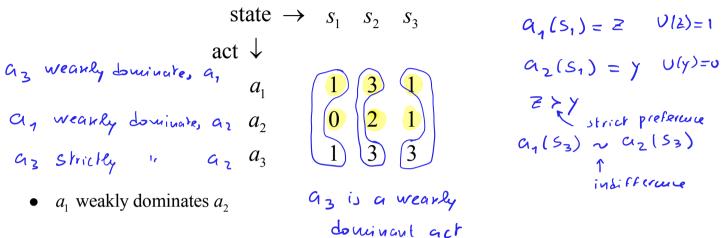Act a **weakly dominates** act b if, for every state s, $a(s) \succsim b(s)$ and, furthermore, there is at least one state $\hat{s}$ such that $a(\hat{s}) \succ b(\hat{s})$. Using utility, $U(a(s)) \geq U(b(s))$ for every state s and there is at least one state $\hat{s}$ such that $U(a(\hat{s})) > U(b(\hat{s}))$.

	state →	s_1	s_2	s_3
	act ↓			
a_3 weakly dominates a_1	a_1	1	3	1
a_1 weakly dominates a_2	a_2	0	2	1
a_3 strictly " a_2	a_3	1	3	3

$a_1(s_1) = z \quad U(z) = 1$

$a_2(s_1) = y \quad U(y) = 0$

$z \succ y$
  ↳ strict preference

$a_1(s_3) \sim a_2(s_3)$
  ↑
  indifference

a_3 is a weakly dominant act

- a_1 weakly dominates a_2

- a_3 weakly dominates a_1

- a_3 strictly (and thus also weakly) dominates a_2.

a and b are **equivalent**, if, for every state s, $a(s) \sim b(s)$ or, in terms of utility, $U(a(s)) = U(b(s))$.

Act a is **weakly dominant** if, for every other act b, either a weakly dominates b or a and b are equivalent.

In the above example, …

Another example:

	state →	s_1	s_2	s_3	s_4
act ↓					
	a_1	1	3	3	2
	a_2	0	2	1	2
	a_3	1	3	3	2

a_1 is weakly dominant

Compare a_1 to a_2 : a_1 weakly dominates a_2

Compare a_1 to a_3 : a_1 and a_3 are equivalent

a_3 is a weakly dominant act

You are bidding against a computer for an item that you **value at $30**. The allowed bids are $10, $20, $30, $40 and $50. The computer will pick one of these bids randomly. Let x be the bid generated by the computer. If your bid is greater than or equal to x then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than x then you get nothing and pay nothing.
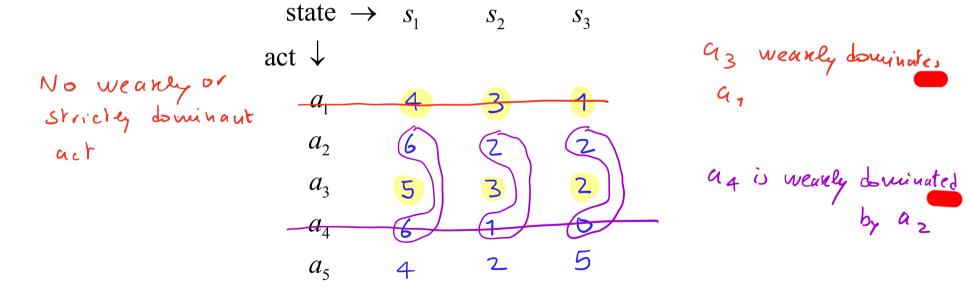
computer's bid →

your bid ↓	$10	$20	$30	$40	$50
$10	20	0	0	0	0
$20	20	10	0	0	0
$30	20	10	0	0	0
$40	20	10	0	-10	0
$50	20	10	0	-10	-20

also weakly dominant → $20 row

weakly dominant → $30 row

FIRST PRICE AUCTION

Now same as above, but if you win the object and **pay your own bid**.

computer's bid →

your bid ↓	$10	$20	$30	$40	$50
$10	20	0	0	0	0
$20	10	10	0	0	0
$30	0	0	0	0	0
$40	-10	-10	-10	-10	0
$50	-20	-20	-20	-20	-20

not true that this is weakly dominant → $30 row

state →	s_1	s_2	s_3			Utility
act ↓						
				best	z_4, z_{10}	6
a_1	z_1	z_2	z_3		z_7, z_{15}	5
a_2	z_4	z_5	z_6		z_1, z_{13}	4
a_3	z_7	z_8	z_9		z_2, z_8	3
a_4	z_{10}	z_{11}	z_{12}		z_5, z_6, z_9, z_{14}	2
a_5	z_{13}	z_{14}	z_{15}		z_3, z_{11}	1
				worst	z_{12}	0

state →	s_1	s_2	s_3
act ↓			
a_1	4	3	1
a_2	6	2	2
a_3	5	3	2
a_4	6	1	0
a_5	4	2	5

No weakly or strictly dominant act

a_3 weakly dominates a_1

a_4 is weakly dominated by a_2

Note: the two sentences "x dominates y" and "y is dominated by x" express the same concept. In one you say "dominates" (active form), in the other "is dominated by" (passive role).