Act a ***weakly dominates*** act b if, for every state s, $a(s) \succsim b(s)$ and, furthermore, there is at least one state $\hat{s}$ such that $a(\hat{s}) \succ b(\hat{s})$. Using utility, $U(a(s)) \geq U(b(s))$ for every state s and there is at least one state $\hat{s}$ such that $U(a(\hat{s})) > U(b(\hat{s}))$.

state → act ↓	s_1	s_2	s_3
a_1	1	3	1
a_2	0	2	1
a_3	1	3	3

- a_1 weakly dominates a_2
- a_3 weakly dominates a_1
- a_3 strictly (and thus also weakly) dominates a_2.

a and b are ***equivalent***, if, for every state s, $a(s) \sim b(s)$ or, in terms of utility, $U(a(s)) = U(b(s))$.

Act a is ***weakly dominant*** if, for every other act b, either a weakly dominates b or a and b are equivalent.

In the above example, …

Another example:

state → act ↓	s_1	s_2	s_3	s_4
a_1	1	3	3	2
a_2	0	2	1	2
a_3	1	3	3	2

You are bidding against a computer for an item that you **value at $30**. The allowed bids are $10, $20, $30, $40 and $50. The computer will pick one of these bids randomly. Let *x* be the bid generated by the computer. If your bid is greater than or equal to *x* then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than *x* then you get nothing and pay nothing.

computer's bid → your bid ↓	$10	$20	$30	$40	$50
$10					
$20					
$30					
$40					
$50					

Now same as above, but if you win the object and **pay your own bid**.

computer's bid → your bid ↓	$10	$20	$30	$40	$50
$10					
$20					
$30					
$40					
$50					

state → act ↓	s_1	s_2	s_3
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9
a_4	z_{10}	z_{11}	z_{12}
a_5	z_{13}	z_{14}	z_{15}

	Utility
best	z_4, z_{10}
	z_7, z_{15}
	z_1
	z_2, z_8
	z_5, z_6, z_9, z_{14}
	z_3, z_{11}
worst	z_{12}

state → act ↓	s_1	s_2	s_3
a_1			
a_2			
a_3			
a_4			
a_5			

state → act ↓	s_1	s_2	s_3
a_1	4	3	1
a_2	6	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	2	5

Dominance:

So we can simplify

state → act ↓	s_1	s_2	s_3
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

What then?

First a different example:

state →	s_1	s_2	s_3
act ↓			
a_1	4	3	1
a_2	3	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	3	4

One criterion that can be used is the **MaxiMin** criterion.

Now back to the previous problem:

state →	s_1	s_2	s_3
act ↓			
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

MaxiMin =

A refinement is the **LexiMin**

state →	s_1	s_2	s_3
act ↓			
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

Here the LexiMin picks

One more example:

state →	s_1	s_2	s_3	s_4
act ↓				
a_1	2	3	1	5
a_2	6	2	2	3
a_3	5	3	2	4
a_4	6	1	0	7
a_5	3	2	5	1

MaxiMin =

LexiMin =

Special case: outcomes are sums of money

state $\rightarrow$ act $\downarrow$	s_1	s_2	s_3	s_4
a_1	\$12	\$30	\$0	\$18
a_2	\$36	\$6	\$24	\$12
a_3	\$6	\$42	\$12	\$0

Suppose that we are able to assign probabilities to the states:

state $\rightarrow$	s_1	s_2	s_3	s_4
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$

a_1 is the lottery

a_2 is the lottery

a_3 is the lottery

The expected values are:

Definition of attitude to risk ….

Given a money lottery *L*, imagine giving the individual a choice between *L* and the expected value of *L* for sure, that is, the choice

between $\begin{pmatrix} \mathbb{E}[L] \\ 1 \end{pmatrix}$ and L or, written more simply, between $\mathbb{E}[L]$ and L

If she says that

- $\mathbb{E}[L] \succ L$ we say that she is **risk** **relative to *L***
- $\mathbb{E}[L] \sim L$ we say that she is **risk** **relative to *L***
- $L \succ \mathbb{E}[L]$ we say that she is **risk** **relative to *L***

So in the above example, if we assume that the agent is risk neutral relative to every lottery and her preferences are transitive, then, since

$\mathbb{E}[a_1] = 10.5$

$\mathbb{E}[a_2] = 24$

$\mathbb{E}[a_3] = 14$

Can we infer risk attitudes from choices?

Let $L = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Then $\mathbb{E}[L] =$

Suppose Ann's preferences are transitive, she prefers more money to less and she says that she prefers \$49 to *L*.

Suppose Bob's preferences are transitive, he prefers more money to less and he says that he prefers \$51 to *L*.