

Act  $a$  **weakly dominates** act  $b$  if, for every state  $s$ ,  $a(s) \succeq b(s)$  and, furthermore, there is at least one state  $\hat{s}$  such that  $a(\hat{s}) \succ b(\hat{s})$ .

Using utility,  $U(a(s)) \geq U(b(s))$  for every state  $s$  and there is at least one state  $\hat{s}$  such that  $U(a(\hat{s})) > U(b(\hat{s}))$ .

state	→	$s_1$	$s_2$	$s_3$
act	↓			
$a_1$		1	3	1
$a_2$		0	2	1
$a_3$		1	3	3

- $a_1$  weakly dominates  $a_2$
- $a_3$  weakly dominates  $a_1$
- $a_3$  strictly (and thus also weakly) dominates  $a_2$ .

$a$  and  $b$  are **equivalent**, if, for every state  $s$ ,  $a(s) \sim b(s)$  or, in terms of utility,  $U(a(s)) = U(b(s))$ .

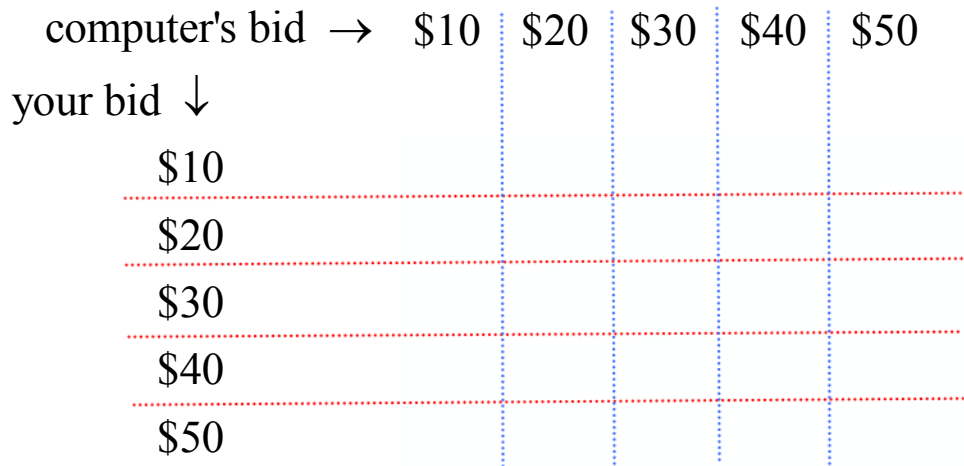
Act  $a$  is **weakly dominant** if, for every other act  $b$ , either  $a$  weakly dominates  $b$  or  $a$  and  $b$  are equivalent.

In the above example, ...

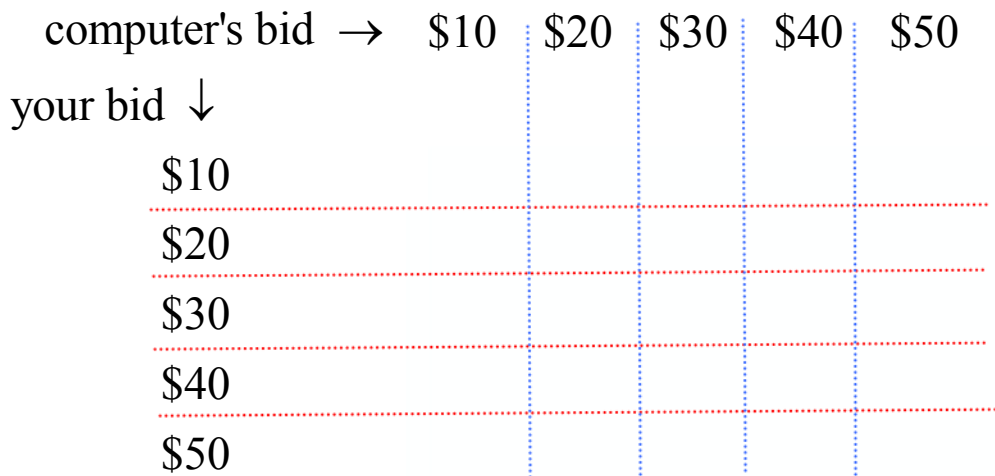
Another example:

state	→	$s_1$	$s_2$	$s_3$	$s_4$
act	↓				
$a_1$		1	3	3	2
$a_2$		0	2	1	2
$a_3$		1	3	3	2

You are bidding against a computer for an item that you **value at \$30**. The allowed bids are \$10, \$20, \$30, \$40 and \$50. The computer will pick one of these bids randomly. Let  $x$  be the bid generated by the computer. If your bid is greater than or equal to  $x$  then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than  $x$  then you get nothing and pay nothing.



Now same as above, but if you win the object and **pay your own bid**.



state →	$s_1$	$s_2$	$s_3$		Utility
act ↓				best	$z_4, z_{10}$
$a_1$	$z_1$	$z_2$	$z_3$		$z_7, z_{15}$
$a_2$	$z_4$	$z_5$	$z_6$		$z_1$
$a_3$	$z_7$	$z_8$	$z_9$		$z_2, z_8$
$a_4$	$z_{10}$	$z_{11}$	$z_{12}$		$z_5, z_6, z_9, z_{14}$
$a_5$	$z_{13}$	$z_{14}$	$z_{15}$		$z_3, z_{11}$
				worst	$z_{12}$

state →	$s_1$	$s_2$	$s_3$
act ↓			
$a_1$			
$a_2$			
$a_3$			
$a_4$			
$a_5$			

state $\rightarrow$	$s_1$	$s_2$	$s_3$	Dominance:	
act $\downarrow$					
$a_1$	4	3	1		
$a_2$	6	2	2		
$a_3$	5	3	2		
$a_4$	6	1	0		
$a_5$	3	2	5		

So we can simplify

state $\rightarrow$	$s_1$	$s_2$	$s_3$
act $\downarrow$			
$a_2$	6	2	2
$a_3$	5	3	2
$a_5$	3	2	5

What then?

First a different example:

state	→	$s_1$	$s_2$	$s_3$
act	↓			
$a_1$		4	3	1
$a_2$		3	2	2
$a_3$		5	3	2
$a_4$		6	1	0
$a_5$		3	3	4

One criterion that can be used is the **MaxiMin** criterion.

	state	→	$s_1$	$s_2$	$s_3$
	act	↓			
	$a_2$		6	2	2
	$a_3$		5	3	2
	$a_5$		3	2	5

Now back to the previous problem:

MaxiMin =

A refinement is the **LexiMin**

state	→	$s_1$	$s_2$	$s_3$
act	↓			
$a_2$		6	2	2
$a_3$		5	3	2
$a_5$		3	2	5

Here the LexiMin picks

One more example:

state	→	$s_1$	$s_2$	$s_3$	$s_4$	
act	↓					
$a_1$		2	3	1	5	
$a_2$		6	2	2	3	
$a_3$		5	3	2	4	
$a_4$		6	1	0	7	
$a_5$		3	2	5	1	
						MaxiMin =
						LexiMin =

## Special case: outcomes are sums of money

state →	$s_1$	$s_2$	$s_3$	$s_4$
act ↓				
$a_1$	\$12	\$30	\$0	\$18
$a_2$	\$36	\$6	\$24	\$12
$a_3$	\$6	\$42	\$12	\$0

Suppose that we are able to assign probabilities to the states:

state →	$s_1$	$s_2$	$s_3$	$s_4$
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$

$a_1$  is the lottery

$a_2$  is the lottery

$a_3$  is the lottery

The expected values are:

## Definition of attitude to risk ....

Given a money lottery  $L$ , imagine giving the individual a choice between  $L$  and the expected value of  $L$  for sure, that is, the choice

between  $\begin{pmatrix} \mathbb{E}[L] \\ 1 \end{pmatrix}$  and  $L$  or, written more simply, between  $\mathbb{E}[L]$  and  $L$

If she says that

- $\mathbb{E}[L] \succ L$  we say that she is **risk averse** relative to  $L$
- $\mathbb{E}[L] \sim L$  we say that she is **risk neutral** relative to  $L$
- $L \succ \mathbb{E}[L]$  we say that she is **risk seeking** relative to  $L$

So in the above example, if we assume that the agent is risk neutral relative to every lottery and her preferences are transitive, then, since

$$\mathbb{E}[a_1] = 10.5$$

$$\mathbb{E}[a_2] = 24$$

$$\mathbb{E}[a_3] = 14$$



## Can we infer risk attitudes from choices?

Let  $L = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$  Then  $\mathbb{E}[L] =$

Suppose Ann's preferences are transitive, she prefers more money to less and she says that she prefers \$49 to  $L$ .

Suppose Bob's preferences are transitive, he prefers more money to less and he says that he prefers \$51 to  $L$ .