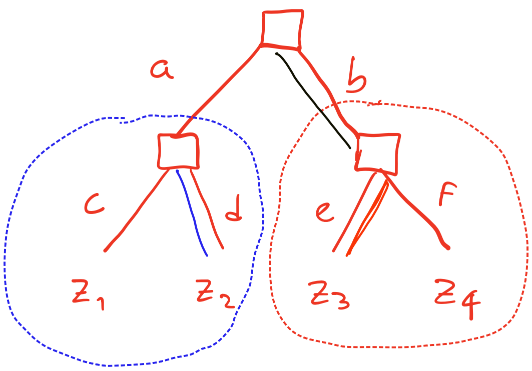
Sequential decisions

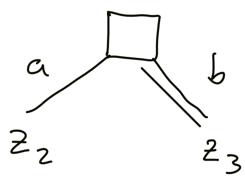
DECISION TREES

□ decision point ○ Nature's choices or external factors



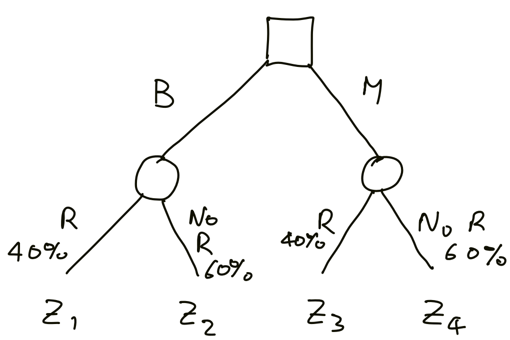
Suppose $z_3 \succ z_4$

Suppose $z_2 \succ z_1$



Suppose $z_3 \succ z_2$

B = go to the beach M = go to the movies



best z_2

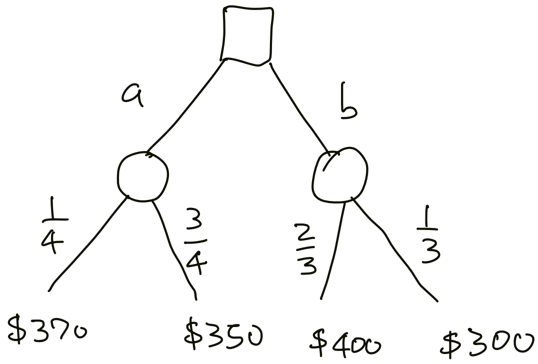
z_3

z_4

worst z_1

$$B = \begin{pmatrix} z_1 & z_2 \\ \frac{4}{10} & \frac{6}{10} \end{pmatrix}$$

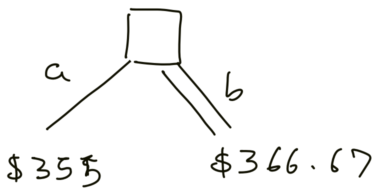
$$M = \begin{pmatrix} z_3 & z_4 \\ \frac{4}{10} & \frac{6}{10} \end{pmatrix}$$



$$a = \begin{pmatrix} \$370 & \$350 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \qquad E[a] = \frac{1}{4}370 + \frac{3}{4}350 = 355$$

$$b = \begin{pmatrix} \$400 & \$300 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \qquad E[b] = \frac{2}{3}400 + \frac{1}{3}300 = 366.67$$

Suppose the agent is risk neutral



Decision to buy a house

- **NEW** (built 2015), costs \$350,000
- **OLD** (built 1980), costs \$300,000

You worry about the **total cost over the next 5 years**.

- **New** houses have a 25% probability of requiring a repair within 5 years and, on average, the repair would cost \$20,000.
- **Old** houses have a 60% probability of requiring a repair within 5 years and, on average, the repair would cost \$100,000.

Your options are: (1) buy house **N**, (2) buy house **O** or (3) pay \$1,000 to an **inspector** to inspect both houses. The inspector will be able to tell you if each house is good or bad.

- A **good new** house has probability 20% of requiring a repair (that costs \$20,000) and probability 80% of requiring no repair.
- A **bad new** house has probability 30% of requiring a repair (that costs \$20,000) and probability 70% of requiring no repair.
- A **good old** house has probability 50% of requiring a repair (that costs \$100,000) and probability 50% of requiring no repair.
- A **bad old** house has probability 70% of requiring a repair (that costs \$100,000) and probability 30% of requiring no repair.

Based on past data, the probabilities that the inspector will come up with the various verdicts are:

- Both good: 20%
- Both bad: 30%
- Old house good, new house bad: 20%
- Old house bad, new house good: 30%.

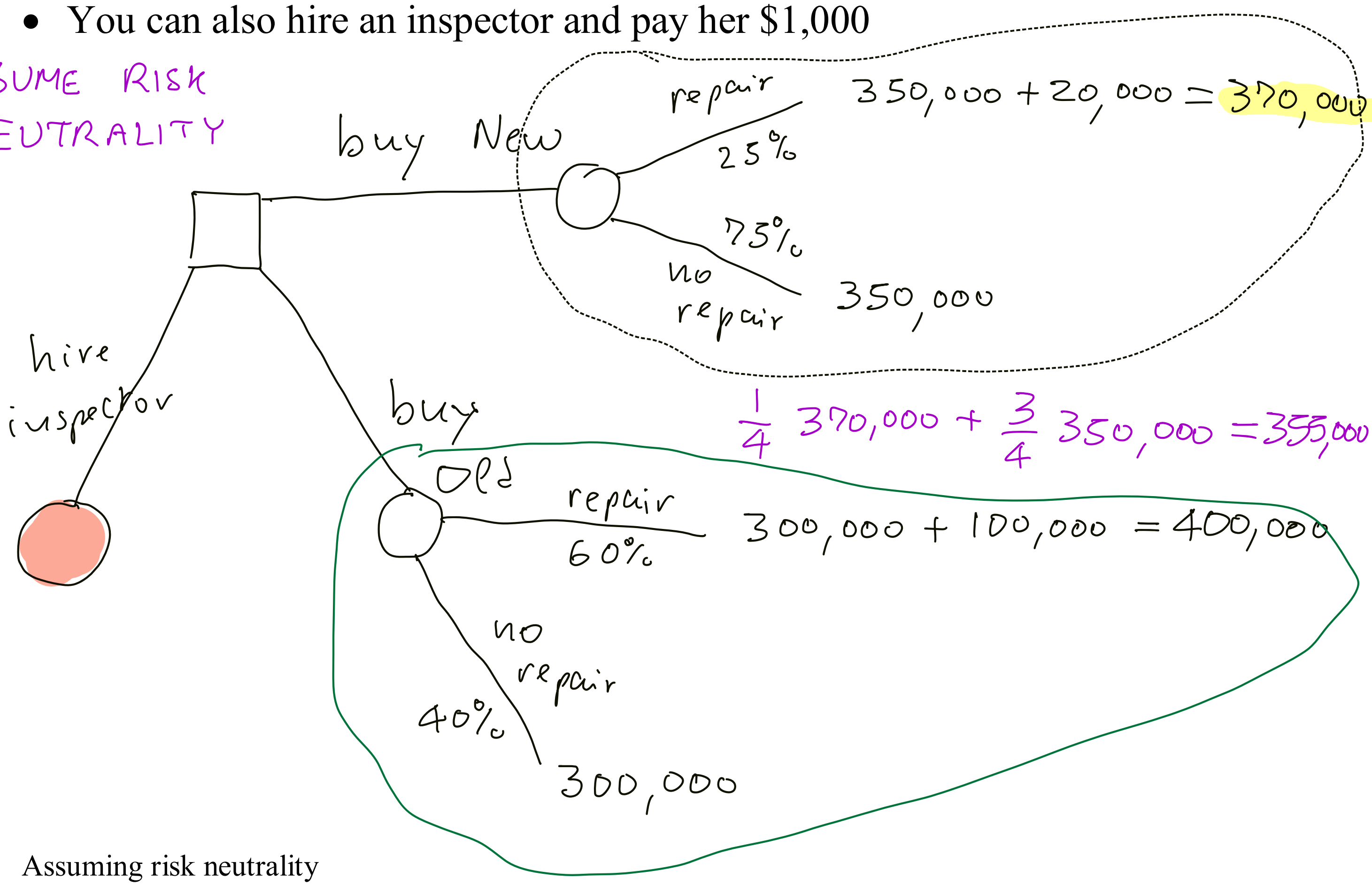
THIS IS A LOT OF INFORMATION!

- **NEW** costs \$350,000. **New** houses have a 25% probability of requiring a repair within 5 years and, on average, the repair would cost \$20,000.

- **OLD** costs \$300,000. **Old** houses have a 60% probability of requiring a repair within 5 years and, on average, the repair would cost \$100,000.

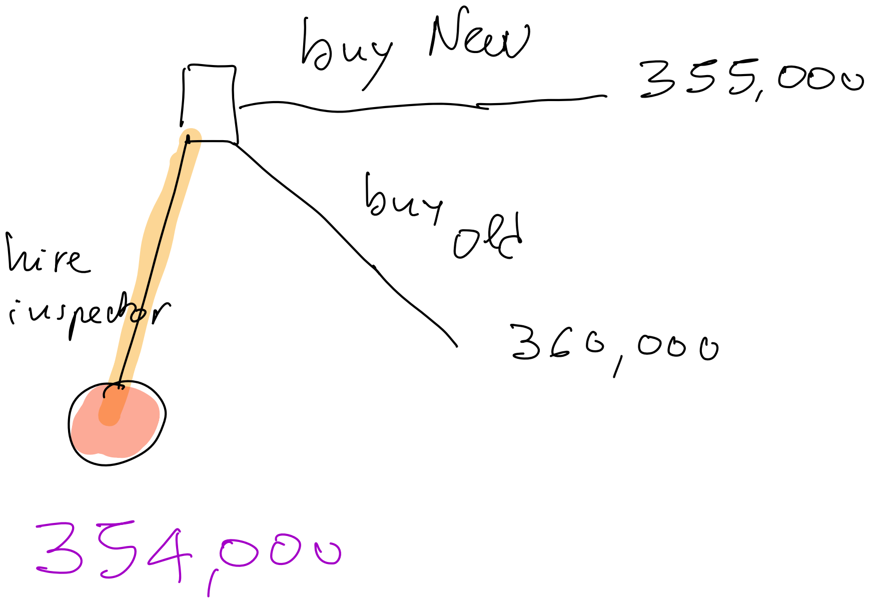
- You can also hire an inspector and pay her \$1,000

ASSUME RISK NEUTRALITY



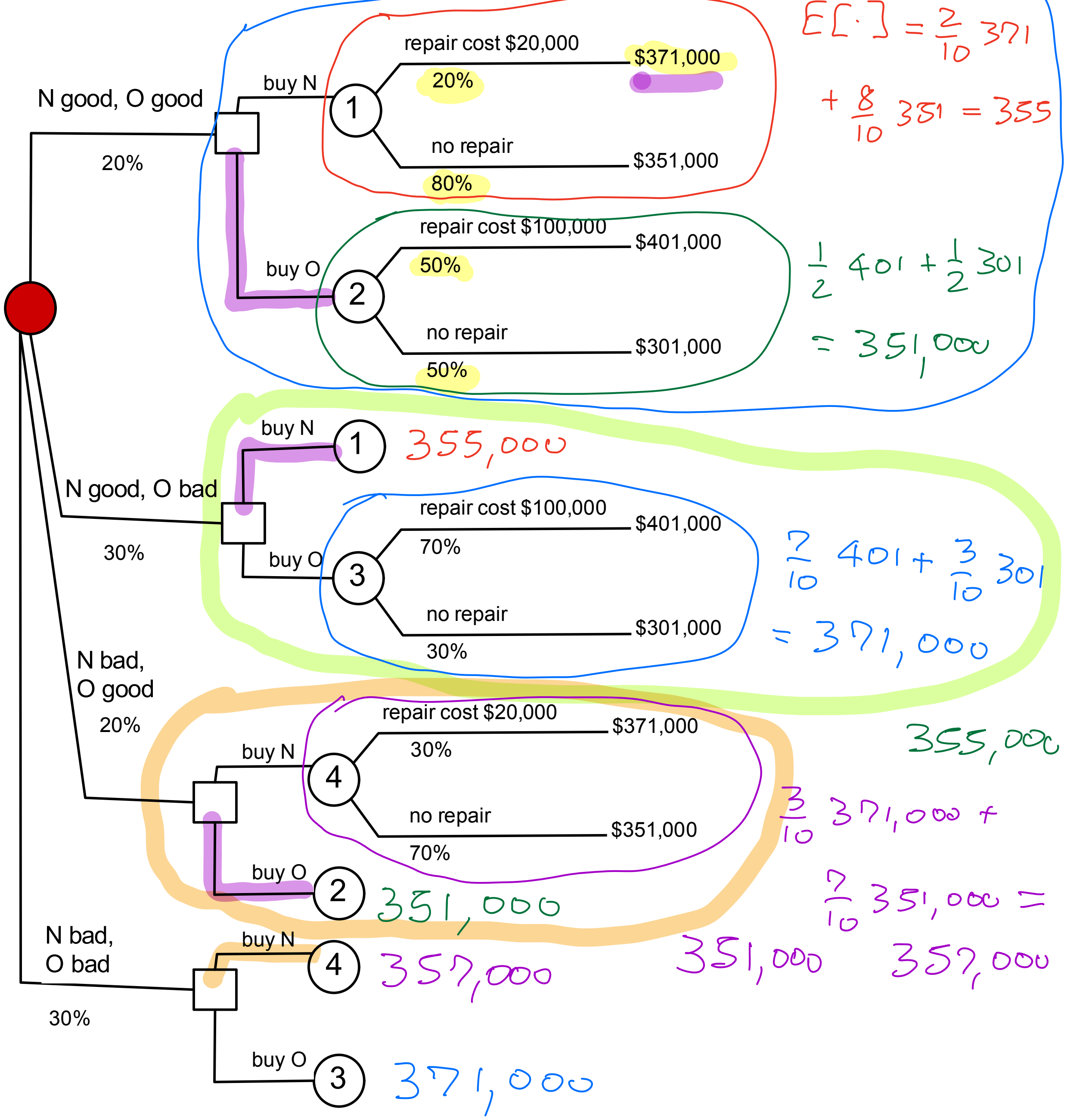
Assuming risk neutrality

$\frac{6}{10}$ 400,000 + $\frac{4}{10}$ 300,000 = 360,000


buy New
355,000
buy old
360,000
hire inspector
354,000

replace with 351,000

The “hire inspector” module is as follows:

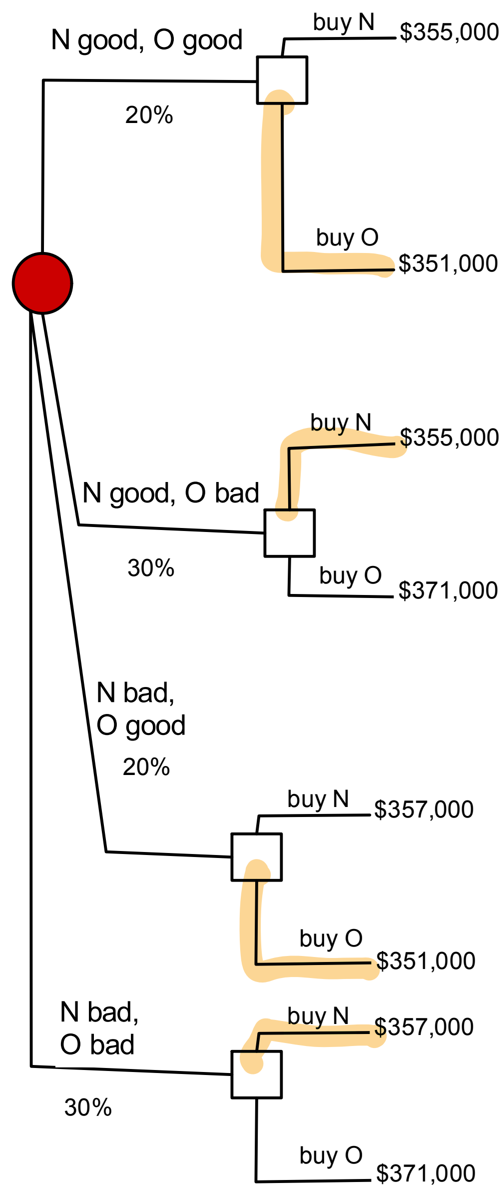


The expected values of the lotteries are:

- For ①:
- For ②:
- For ③:
- For ④:

Thus we can reduce this part of the tree to:

OBJECTIVE: pay the LOWEST 5-year cost



Thus we can reduce the option of hiring the inspector to the following lottery:

$$\begin{pmatrix} 351,000 & 355,000 & 351,000 & 357,000 \\ \frac{2}{10} & \frac{3}{10} & \frac{2}{10} & \frac{3}{10} \end{pmatrix}$$

Whose expected value is

354,000

The optimal decision is:

1. **hire the inspector and then**

2. **(a) if both good, buy** ~~new~~ old

   (b) if N good and O bad, buy

   (c) if N bad and O good, buy

   (d) if both bad, buy