state →	s_1	s_2	s_3
act ↓			
~~a_1~~	~~4~~	~~3~~	~~1~~
$\boxed{a_2}$	6	$\boxed{2}$	2
a_3	5	3	2
~~$\boxed{a_4}$~~	~~6~~	~~$\boxed{1}$~~	~~0~~
a_5	3	2	5

Dominance:

a_3 weakly dominates a_1

or

a_1 is weakly dominated by a_3

a_2 weakly domates a_4

or

a_4 is weakly dominated by a_2

So we can simplify

state →	s_1	s_2	s_3
act ↓			
a_2	6	$\boxed{2}$	$\cancel{2}$
a_3	5	$\boxed{3}$	$\cancel{2}$
a_5	$\boxed{3}$	$\cancel{2}$	5

What then?

MaxiMin = $\{a_2, a_3, a_5\}$

one-step LexiMin = $\{a_3, a_5\}$

two-step " = $\{a_3, a_5\}$

First a different example:

state →	s_1	s_2	s_3
act ↓			
~~a_1~~	4	3	1
~~a_2~~	3	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	3	4

weakly dominated by a_3

weakly dominated by a_5 and also by a3

One criterion that can be used is the **MaxiMin** criterion.

	s_1	s_2	s_3
a_3	5	3	2
a_4	6	1	0
a_5	3	3	4

MaxiMin = a_5

	s_1	s_2
a_1	\$100	\$0
a_2	\$1	\$1

$$\text{MaxiMin} = \{a_2\}$$

A refinement is the **LexiMin**

state →	s_1	s_2	s_3
act ↓			
~~a_2~~	~~6~~	~~2~~	~~2~~
a_3	5	3	2
a_5	3	2	5

MaxiMin = $\{a_2, a_3, a_5\}$

one-step leximin = $\{a_3, a_5\}$

two-step " = $\{a_3, a_5\}$

Here the LexiMin picks

One more example:

state →	s_1	s_2	s_3	s_4
act ↓				
a_1	2	3	1	5
a_2	6	2	2	3
a_3	5	3	2	4
a_4	6	1	0	7
a_5	3	2	5	1

MaxiMin =

LexiMin =

Special case: outcomes are sums of money

state →	s_1	s_2	s_3	s_4
act ↓	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$
a_1	\$12	\$30	\$0	\$18
→ a_2	\$36	\$6	\$24	\$12
a_3	\$6	\$42	\$12	\$0

Suppose that we are able to assign probabilities to the states:

state →	s_1	s_2	s_3	s_4
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$

a_1 is the lottery $\begin{pmatrix} \$12 & \$30 & \$0 & \$18 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix}$ $E[a_1] = \frac{1}{3}12 + \frac{1}{6}30 + \frac{5}{10}0 + \frac{1}{12}18$

$= 10.5$

a_2 is the lottery $\begin{pmatrix} \$36 & \$6 & \$24 & \$12 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix}$ $E[a_2] = 24$

a_3 is the lottery $\begin{pmatrix} \$6 & \$42 & \$12 & \$0 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix}$ $E[a_3] = 14$

The expected values are:

Money lottery $L = \begin{pmatrix} \$x_1 & \$x_2 & \cdots & \$x_n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}$

The expected value of L,

$$E[L] = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

if choice is between

"\$60 for sure" or L

$\begin{pmatrix} \$60 \\ 1 \end{pmatrix}$

$L = \begin{pmatrix} \$0 & \$120 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$E[L] = \frac{1}{2} 0 + \frac{1}{2} 120 = 60$

$\begin{pmatrix} \$60 \\ 1 \end{pmatrix}$

if you prefer \$60 for sure I call you risk averse

" L to \$60 " loving

" indifferent " neutral

Definition of attitude to risk ….

Given a money lottery L, imagine giving the individual a choice between L and the expected value of L for sure, that is, the choice

between $\begin{pmatrix} \mathbb{E}[L] \\ 1 \end{pmatrix}$ and L or, written more simply, between $\mathbb{E}[L]$ and L

If she says that

preference

- $\mathbb{E}[L] \succ L$ we say that she is **risk** averse **relative to** $\boldsymbol{L}$

indifference

- $\mathbb{E}[L] \sim L$ we say that she is **risk** neutral **relative to** $\boldsymbol{L}$
- $L \succ \mathbb{E}[L]$ we say that she is **risk** loving **relative to** $\boldsymbol{L}$

So in the above example, if we assume that the agent is risk neutral relative to every lottery and her preferences are transitive, then, since

$\mathbb{E}[a_1] = 10.5$

$\mathbb{E}[a_2] = 24$

$\mathbb{E}[a_3] = 14$

Can we infer risk attitudes from choices?

Let $L = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Then $\mathbb{E}[L] = \frac{1}{2} \cdot 40 + \frac{1}{2} 60 = 50$

Suppose Ann's preferences are transitive, she prefers more money to less and she says that she prefers \$49 to *L*.

she is indifferent between \$49 and L.

$\$50 \succ \$49 \underset{\sim}{\succ} L$

by transitivity $\$50 \succ L$ Ann is risk averse

Suppose Bob's preferences are transitive, he prefers more money to less and he says that he prefers \$51 to *L*.

Could Bob be risk neutral?

$51 \succ 50 \sim L \Rightarrow 51 \succ L$

Yes

Could he be risk averse?

$51 \succ 50 \succ L \Rightarrow 51 \succ L$ Yes

Could he be risk loving?

$L \succ 50$

$51 \succ 50.50 \sim L$ Yes

$52 \sim L$

$L \succ 51$

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