Theorem 2. Let $\succsim$ be a von Neumann-Morgenstern ranking of the set of basic lotteries $\mathcal{L}$. Then the following are true.

(A) If $U : Z \to \mathbb{R}$ is a von Neumann-Morgenstern utility function that represents $\succsim$, then, for any two real numbers a and b with $a > 0$, the function $V : Z \to \mathbb{R}$ defined by $V(z_i) = aU(z_i) + b$ $(i = 1,2,\ldots,m)$ is also a von Neumann-Morgenstern utility function that represents $\succsim$.

(B) If $U : Z \to \mathbb{R}$ and $V : Z \to \mathbb{R}$ are two von Neumann-Morgenstern utility functions that represent $\succsim$, then there exist two real numbers a and b with $a > 0$ such that $V(z_i) = aU(z_i) + b$ $(i = 1,2,\ldots,m)$.

$$U = \begin{cases} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \\ 10 & 6 & 16 & 8 & 6 & 14 \end{cases}$$

$$\text{operation } O = \begin{pmatrix} \text{cured} & \text{permanent disability} \\ 90\% & 10\% \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \text{cured} & \text{no benefit} & \text{adverse reaction} \\ 75\% & 10\% & 15\% \end{pmatrix}$$

What should you do?

$$\text{operation } O = \begin{pmatrix} \text{cured} & \begin{matrix}\text{permanent} \\ \text{disability}\end{matrix} \\ 90\% & 10\% \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \text{cured} & \begin{matrix}\text{no} \\ \text{benefit}\end{matrix} & \begin{matrix}\text{adverse} \\ \text{reaction}\end{matrix} \\ 75\% & 10\% & 15\% \end{pmatrix}$$

The Allais paradox

(Maurice Allais, 1952)

$$A = \begin{pmatrix} \$1M \\ 100\% \end{pmatrix} \quad \text{versus} \quad B = \begin{pmatrix} \$2.5M & \$1M & 0 \\ 10\% & 89\% & 1\% \end{pmatrix}$$

$$C = \begin{pmatrix} \$1M & 0 \\ 11\% & 89\% \end{pmatrix} \quad \text{versus} \quad D = \begin{pmatrix} \$2.5M & 0 \\ 10\% & 90\% \end{pmatrix}.$$

$$A \succ B \quad \text{if and only if} \quad \mathbb{E}[U(A)] > \mathbb{E}[U(B)]$$