First question to ask yourself:
what is my ranking of the basic outcomes?

state →	s_1	s_2	s_3
act ↓			
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9

Suppose your answer is:

state →	s_1	s_2	s_3
act ↓			
a_1	z_1	z_2	z_3
~~a_2~~	~~z_4~~	~~z_5~~	~~z_6~~
a_3	z_7	z_8	z_9

best	z_8
	z_3
	z_1, z_9
	z_2, z_6
	z_4, z_5
worst	z_7

$z_1 \succ z_4 \quad z_2 \succ z_5 \quad z_3 \succ z_6$

Note:

- a_1 strictly dominates a_2

Thus …

state →	s_1	s_2	s_3
act ↓			
a_1	z_1	z_2	z_3
a_3	z_7	z_8	z_9

		NORMALIZED Utility
best	z_8	1
	z_3	$\frac{3}{4}$
	z_1, z_9	$\frac{2}{3}$
	z_2	$\frac{2}{5}$
worst	z_7	0

Three questions to ask yourself:

Note that p is the probability of the worst outcome, not the best

(1) What p is such that $\begin{pmatrix} z_3 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$? Suppose the answer is $p = \frac{1}{4}$

(2) What p is such that $\begin{pmatrix} z_1 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$? Suppose the answer is $p = \frac{1}{3}$

(3) What p is such that $\begin{pmatrix} z_2 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$? Suppose the answer is $p = \frac{3}{5}$

		Utility		easier to use ↓
best	z_8	1		60
	z_3	$\frac{3}{4}$	multiply	45
	z_1, z_9	$\frac{2}{3}$	all	40
	z_2	$\frac{2}{5}$	by	24
worst	z_7	0	60	0

In order not to deal with fractions, rescale the utility function by multiplying each number by 60:

with the normalized utility function

	s_1	s_2	s_3
a_1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{3}{4}$
a_3	0	1	$\frac{2}{3}$

		Utility
best	z_8	60
	z_3	45
	z_1, z_9	40
	z_2	24
worst	z_7	0

$$E[U(a_1)] = \frac{1}{5}\,\frac{2}{3} + \frac{3}{5}\,\frac{2}{5} + \frac{1}{5}\,\frac{3}{4} = \frac{31.4}{60}$$

$$E[U(a_3)] = \frac{1}{5}\cdot 0 + \frac{3}{5}\,1 + \frac{1}{5}\cdot\frac{2}{3} = \frac{44}{60}$$

state → act ↓	s_1	s_2	s_3
a_1	40	24	45
a_3	0	60	40

Next step: try to assign probabilities to the states (from objective data or some subjective assessment). Suppose you assess the following:

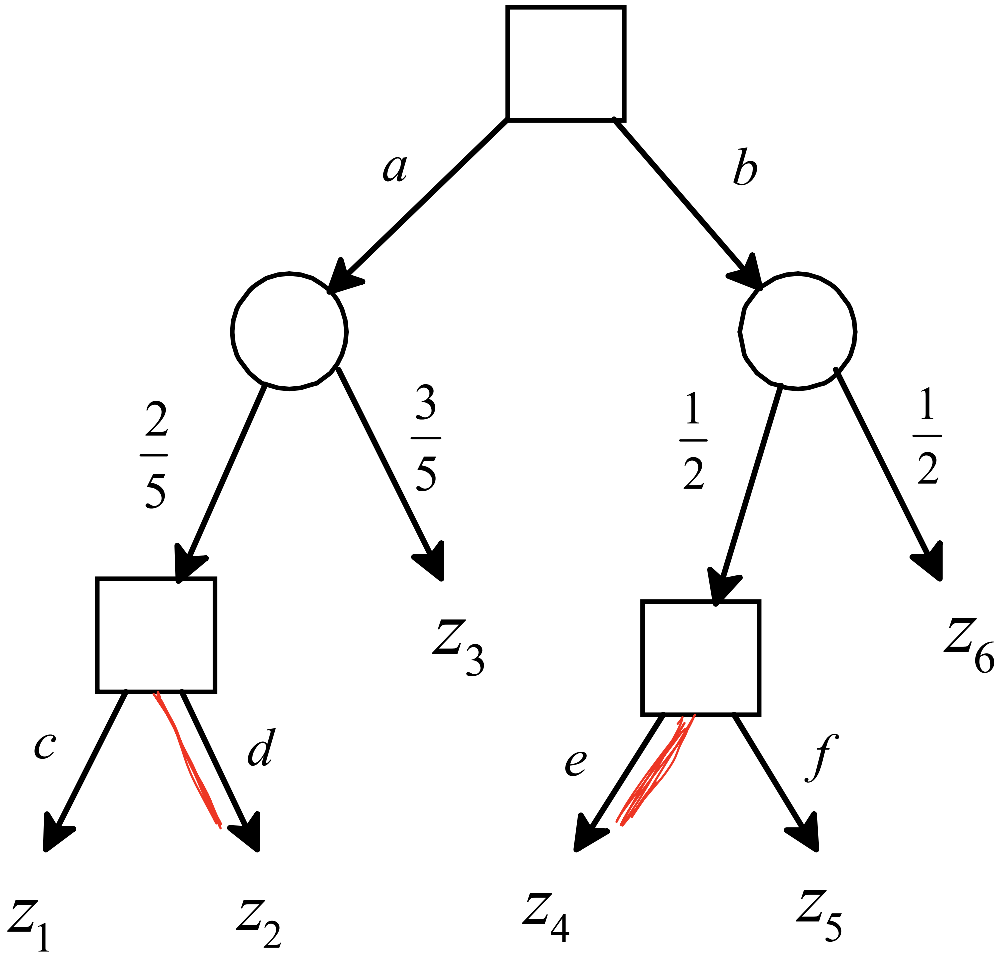
state:	s_1	s_2	s_3
probability:	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$

Then: $\mathbb{E}[U(a_1)] = \frac{1}{5}\,40 + \frac{3}{5}\,24 + \frac{1}{5}\,45 = 31.4$

$\mathbb{E}[U(a_3)] = \frac{1}{5}\,0 + \frac{3}{5}\,60 + \frac{1}{5}\,40 = 44$

Hence you should take action a_3

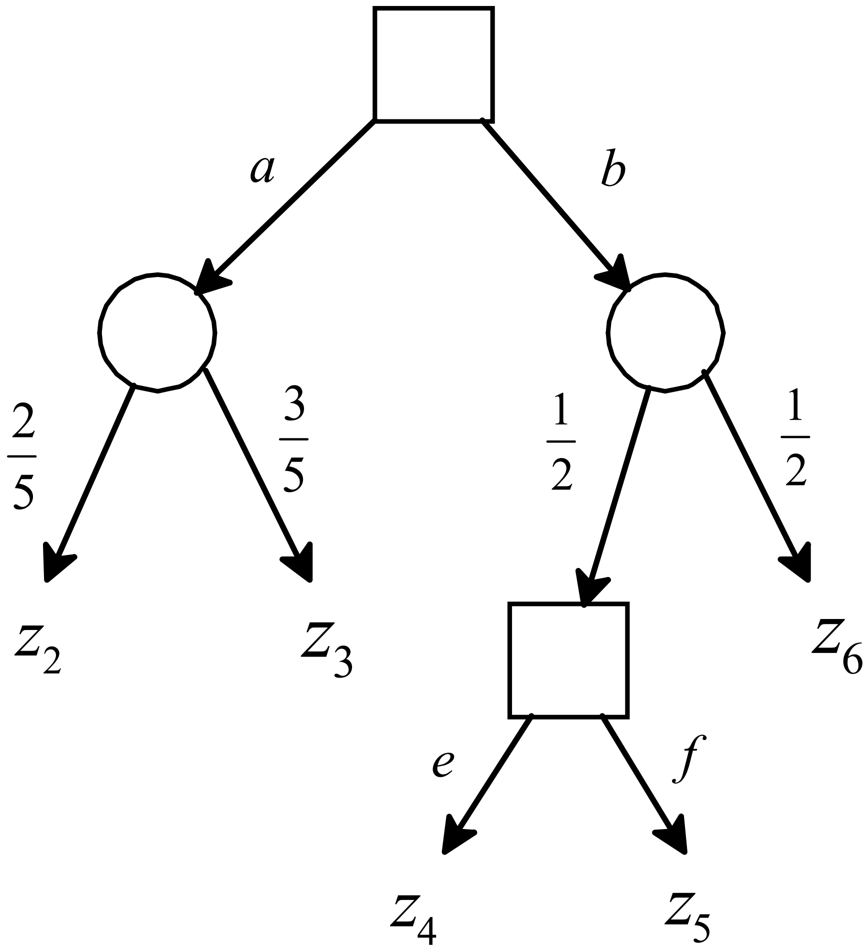
Decision tree



Suppose that for you $z_2 \succ z_1$

Suppose that for you $z_4 > z_3$

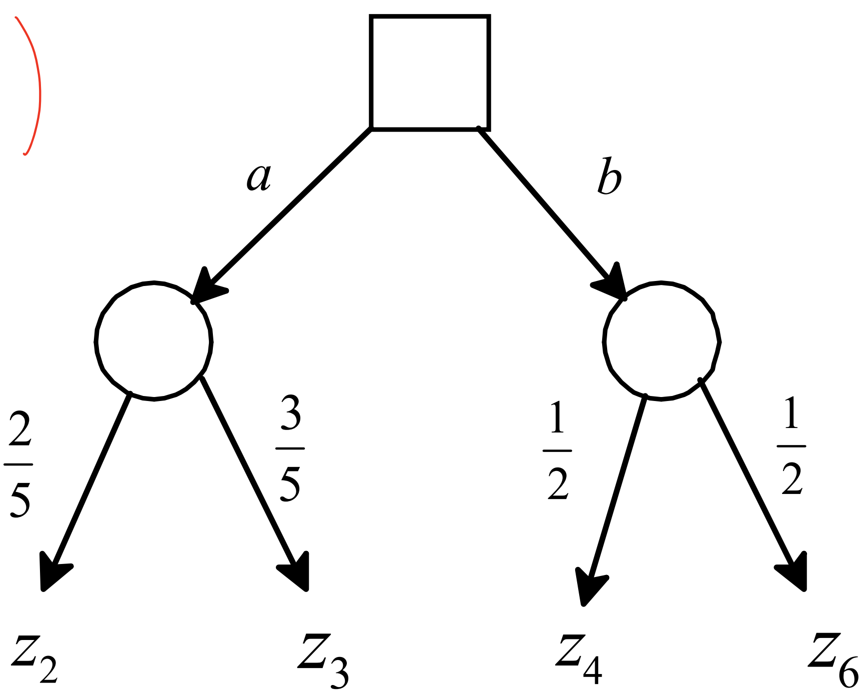
First question to ask yourself: how do I rank z_1 and z_2? Suppose that the answer is $z_2 \succ z_1$.



Second question to ask yourself: how do I rank z_4 and z_5? Suppose that the answer is $z_4 \succ z_5$.

$a: \begin{pmatrix} z_2 & z_3 \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}$

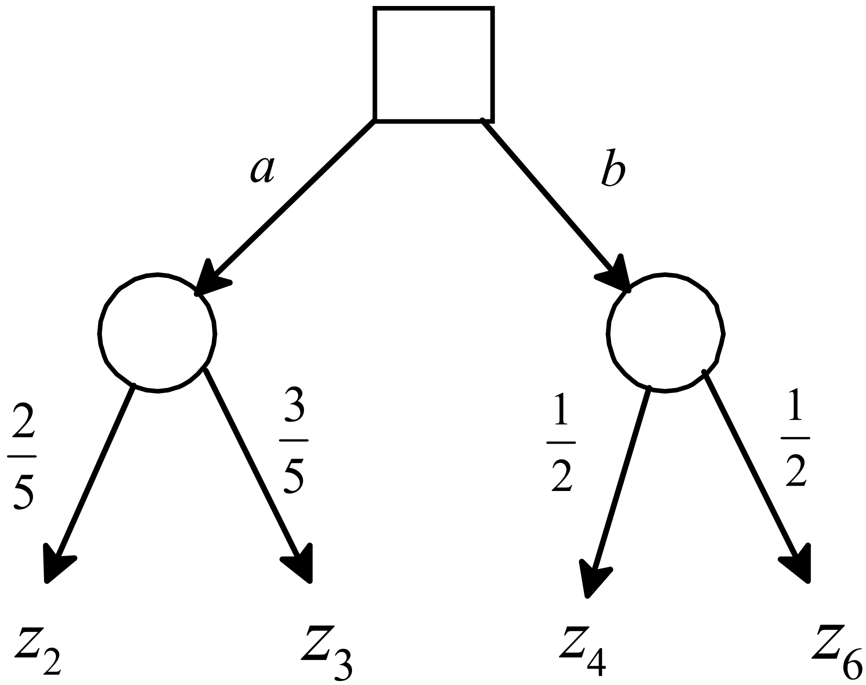
$b: \begin{pmatrix} z_4 & z_6 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$



Assumptions:

$z_2 > z_1$

$z_4 > z_5$



Next question: how do I rank the remaining four outcomes? Suppose:

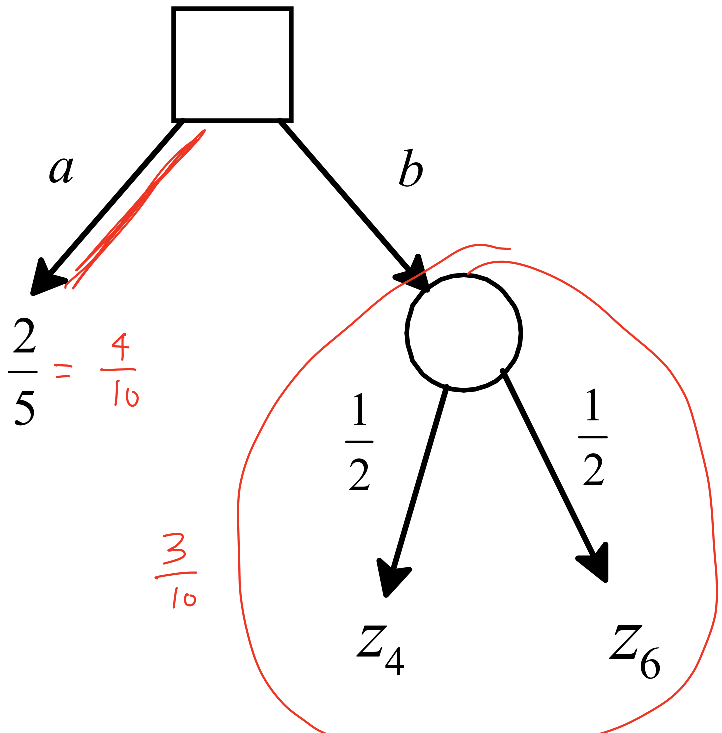
		Utility
best	z_2	1
	z_6	
	z_4	
worst	z_3	0

This is sufficient to eliminate the random event on the left:

$$E[U(a)] = \frac{2}{5} U(z_2) + \frac{3}{5} U(z_3)$$

$$= \frac{2}{5} 1 + \frac{3}{5} 0 = \frac{2}{5}$$

$$a: \begin{pmatrix} z_2 & z_3 \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}$$



z_2 1

z_6 $\frac{1}{2}$

z_4 $\frac{1}{10}$

z_3 0

Two more questions and then you are done!

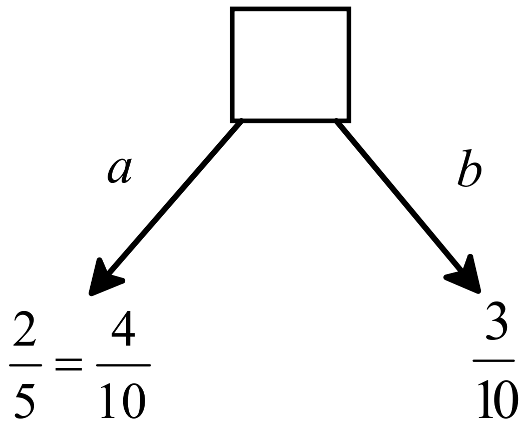
(4) What p is such that $\begin{pmatrix} z_6 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_2 & z_3 \\ p & 1-p \end{pmatrix}$? Suppose the answer is $p = \frac{1}{2}$.

(5) What p is such that $\begin{pmatrix} z_4 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_2 & z_3 \\ p & 1-p \end{pmatrix}$? Suppose the answer is $p = \frac{1}{10}$.

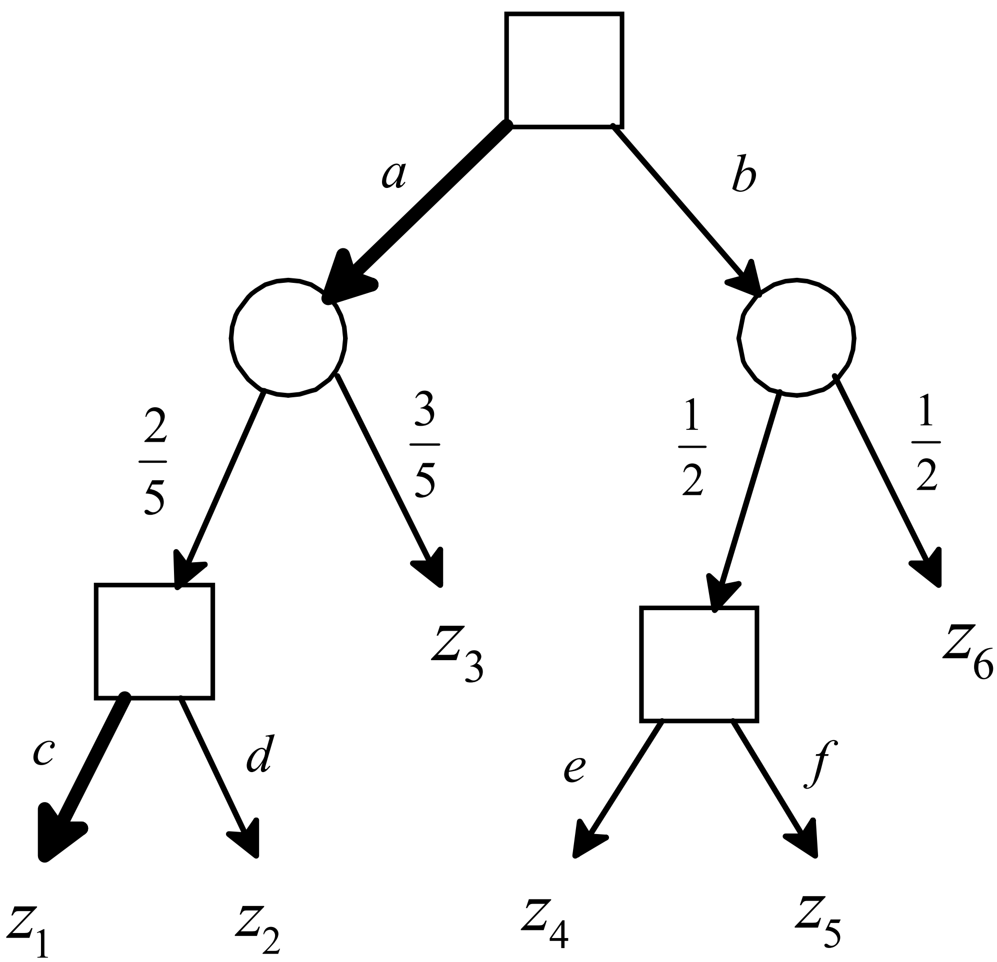
Then the lottery corresponding to the random event on the right has an expected utility of

$$E[U(b)] = \frac{1}{2} U(z_4) + \frac{1}{2} U(z_6) =$$

$$= \frac{1}{2}\frac{1}{10} + \frac{1}{2}\frac{1}{2} = \frac{3}{10}$$



Hence the optimal decision is to first take action *a* and then, if a second choice is required between *c* and *d*, choose *d*:



THE HURWICZ INDEX

- α $(0 \le \alpha \le 1)$ weight attached to the **worst** outcome: ***index of pessimism***
- $(1-\alpha)$ weight attached to the best outcome: index of optimism.

	s_1	s_2	s_3
a_1	8	1	0
a_2	6	2	3
a_3	0	3	4

$\underbrace{\text{MaxiMin}}$ $= a_2$

corresponds to taking $\alpha = 1$

$H_\alpha(a_1) =$ $0 \cdot \alpha + 8 \cdot (1-\alpha) = 8 - 8\alpha$

$H_\alpha(a_2) =$ $2 \cdot \alpha + 6(1-\alpha) = 6 - 4\alpha$ ✓

$H_\alpha(a_3) =$ $0 \cdot \alpha + 4(1-\alpha) = 4 - 4\alpha$

if $\alpha = \frac{1}{8}$

$H_{\frac{1}{8}}(a_1) = 8 - 8\frac{1}{8} = 7$

$H_{\frac{1}{8}}(a_2) = 6 - 4\frac{1}{8} = 5.5$

$H_{\frac{1}{8}}(a_3) = 4 - 4\frac{1}{8} = 3.5$

$\Downarrow$

choose a_1

Then choose the act that gives the highest value. For example, if $\alpha = \frac{3}{4}$ then

$H_{\frac{3}{4}}(a_1) =$ $\qquad H_{\frac{3}{4}}(a_2) =$ $\qquad H_{\frac{3}{4}}(a_3) =$ $\qquad$ thus choose

If $\alpha = 1$ then = MaxiMin criterion.

Suppose I multiply all numbers by 3

	s_1	s_2	s_3
a_1	8	1	0
a_2	6	2	3
a_3	0	3	4

	s_1	s_2	s_3
a_1	24	3	0
a_2	18	6	9
a_3	0	9	12

$$H_\alpha(a_1) = 0\alpha + 8(1-\alpha) = 8 - 8\alpha$$

$$H_\alpha(a_2) = 2\alpha + 6(1-\alpha) = 6 - 4\alpha$$

$$H_\alpha(a_3) = 0\alpha + 4(1-\alpha) = 4 - 4\alpha$$

Is it still true that

$$H_{\frac{1}{8}}(a_1) > H_{\frac{1}{8}}(a_2)$$

$$21 > 16.5$$