PERFECT INFORMATION

Information partition: $\{\{s_1\}, \{s_2\}, \ldots, \{s_n\}\}$

CASE 1.2: risk aversion

When a person is risk averse then it is no longer true that the analysis in terms of changes in wealth and the analysis in terms of total wealth are equivalent.

probability	$\frac{4}{5}$	$\frac{1}{5}$
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$18	\$18
b	\$25	\$0

changes in wealth

Suppose that the DM's von Neumann-Morgenstern utility-of-money function is: $U(\$x) = \sqrt{x}$ and suppose that the DM's initial wealth is \$600.

$$\mathbb{E}[U(a)] = \sqrt{18} = 4.24$$

$$\mathbb{E}[U(b)] = \frac{4}{5}\sqrt{25} + \frac{1}{5}\sqrt{0} = \frac{4}{5}5 = 4$$

In terms of total wealth:

probability	$\frac{4}{5}$	$\frac{1}{5}$
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$618	\$618
b	\$625	\$600

$$\mathbb{E}[U(a)] = \sqrt{618} = 24.86$$

$$\mathbb{E}[U(b)] = \frac{4}{5}\sqrt{625} + \frac{1}{5}\sqrt{600} = 24.9$$

Thus when we deal with risk aversion or risk love we need to reason in terms of **total wealth**.

For a risk neutral person expected value of lottery

$\rightarrow \begin{pmatrix} 124 & 201 & 244 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$. Exp. value of this $- 74 = 95.67$

Let us go back to the previous example, where the amounts are **changes** in wealth.

$E[a] = \frac{1}{2}4 + \frac{1}{3}36 + \frac{1}{6}244 = 54.67$

$\boxed{E[b] = 74}$

$E[c] = 70$

	s_1	s_2	s_3
probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$ / act $\downarrow$			
a	\$4	\$36	\$244
b	\$8	\$201	\$18
c	\$124	\$12	\$24

Suppose that the DM's initial wealth is \$140 and her utility function is $U(\$x) = \sqrt{x}$.
How much would she be willing to pay for perfect information?

STEP 1. First of all: expected utility is if she does not purchase information.

	s_1	s_2	s_3
probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$ / act $\downarrow$			
a	\$144	\$176	\$384
b	\$148	\$341	\$158
c	\$264	\$152	\$164

$$\mathbb{E}[U(a)] = \frac{1}{2}\sqrt{144} + \frac{1}{3}\sqrt{176} + \frac{1}{6}\sqrt{384} = 13.69$$

$$\mathbb{E}[U(b)] = \frac{1}{2}\sqrt{148} + \frac{1}{3}\sqrt{341} + \frac{1}{6}\sqrt{158} = 14.33$$

$$\mathbb{E}[U(c)] = \frac{1}{2}\sqrt{264} + \frac{1}{3}\sqrt{152} + \frac{1}{6}\sqrt{164} = \boxed{14.37}$$

in the absence of information take action c

STEP 2. Calculate her expected utility if she purchases perfect information at price p.

- If I am told that the state is s_1 then I will choose c and get a utility of $\sqrt{264-p}$
- If I am told that the state is s_2 then I will choose b and get a utility of $\sqrt{341-p}$
- If I am told that the state is s_3 then I will choose a and get a utility of $\sqrt{384-p}$

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$144	\$176	\$384
b	\$148	\$341	\$158
c	\$264	\$152	\$164

Expected utility if I purchase information is:

$$f(p) = \frac{1}{2}\sqrt{264-p} + \frac{1}{3}\sqrt{341-p} + \frac{1}{6}\sqrt{384-p}$$

By the theorem about free information expect $f(0) \geq 14.37$

very different from: $\frac{1}{2}\sqrt{264} + \frac{1}{3}\sqrt{341} + \frac{1}{6}\sqrt{384} - p$

WRONG CALCULATION

Suppose $p = 100$

$$f(100) = 14.39 > \underbrace{14.37}_{\text{EU without information}}$$

For a risk neutral person the maximum value of p is 95.67

How much should one be prepared to pay for information?

CASE 2: monetary outcomes and IMPERFECT information

$\{ \{s_1, s_2\}, \{s_3, s_4\} \}$

CASE 2.1: risk neutrality

$U(\$x) = x$

The amounts are **changes** in her wealth.

probability	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	\$16	\$36	\$100	\$12
b	\$10	\$64	\$18	\$120
c	\$104	\$12	\$24	\$0

preliminary step

STEP 0. Change the probabilities so that they have the same denominator:

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

$$\mathbb{E}[a] = \frac{3}{12}16 + \frac{4}{12}36 + \frac{2}{12}100 + \frac{3}{12}12 = 35.67$$

reference utility

$$\mathbb{E}[b] = \frac{3}{12}10 + \frac{2}{12}18 + \frac{3}{12}120 = 35.5$$

$$\mathbb{E}[c] = \frac{3}{12}104 + \frac{4}{12}12 + \frac{2}{12}24 = 34$$

Thus she will choose a and expect 35.67

Suppose now that Ann is offered, at price p, the following imperfect information:

$$\{\{s_1, s_2\}, \{s_3, s_4\}\}$$

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

$3+4=7$

- If informed that $\{s_1, s_2\}$ then

probability	$\frac{3}{7}$	$\frac{4}{7}$
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$16	\$36
b	\$10	\$0
c	\$104	\$12

$\begin{pmatrix} s_3 & s_4 \\ 0 & 0 \end{pmatrix}$

$$\mathbb{E}[a] = \frac{3}{7}16 + \frac{4}{7}36 = 27.43$$

$$\mathbb{E}[b] = \frac{3}{7}10 = 4.29$$

$$\mathbb{E}[c] = \frac{3}{7}104 + \frac{4}{7}12 = 51.43$$

Thus she will choose c and expect 51.43

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

probability	$\frac{2}{5}$	$\frac{3}{5}$
state →	s_3	s_4
act ↓		
a	\$100	\$12
b	\$18	\$120
c	\$24	\$0

$2+3=5$

- If informed that $\{s_3, s_4\}$ then

$$\mathbb{E}[a] = \frac{2}{5}100 + \frac{3}{5}12 = 47.2$$

$$\boxed{\mathbb{E}[b]} = \frac{2}{5}18 + \frac{3}{5}120 = \boxed{79.2}$$

$$\mathbb{E}[c] = \frac{2}{5}24 = 9.6$$

Thus she will choose b and expect 79.2

$$P(\{s_1, s_2\}) = P(s_1) + P(s_2) = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

$$P(\{s_3, s_4\}) = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$$

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

The probability of $\{s_1, s_2\}$ is $\frac{7}{12}$ and the probability of $\{s_3, s_4\}$ is $\frac{5}{12}$

Exp value of free information is $\frac{7}{12}51.43 + \frac{5}{12}79.2 \Big|_{-p} = 63 - p$

maximum p

$63 - p \geq 35.67$ with no information

$= 63 - 35.67 = \$27.33$