CASE 1.2: risk aversion

When a person is risk averse then it is no longer true that the analysis in terms of changes in wealth and the analysis in terms of total wealth are equivalent.

probability	$\frac{4}{5}$	$\frac{1}{5}$
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$18	\$18
b	\$25	\$0

Suppose that the DM's von Neumann-Morgenstern utility-of-money function is: $U(\$x) = \sqrt{x}$ and suppose that the DM's initial wealth is \$600.

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

In terms of total wealth:

probability	$\frac{4}{5}$	$\frac{1}{5}$
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$618	\$618
b	\$625	\$600

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

Thus when we deal with risk aversion or risk love we need to reason in terms of **total wealth**.

Let us go back to the previous example, where the amounts are **changes** in wealth.

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$4	\$36	\$244
b	\$8	\$201	\$18
c	\$124	\$12	\$24

Suppose that the DM's initial wealth is \$140 and her utility function is $U(\$x) = \sqrt{x}$. How much would she be willing to pay for perfect information?

STEP 1. First of all: expected utility is if she does not purchase information.

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$144	\$176	\$384
b	\$148	\$341	\$158
c	\$264	\$152	\$164

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

STEP 2. Calculate her expected utility if she purchases perfect information at price p.

- If I am told that the state is s_1 then I will choose   and get a utility of
- If I am told that the state is s_2 then I will choose   and get a utility of
- If I am told that the state is s_3 then I will choose   and get a utility of

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$144	\$176	\$384
b	\$148	\$341	\$158
c	\$264	\$152	\$164

Expected utility if I purchase information is:

How much should one be prepared to pay for information?

CASE 2: monetary outcomes and IMPERFECT information

CASE 2.1: risk neutrality

The amounts are **changes** in her wealth.

probability	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$64	\$18	\$120
c	\$104	\$12	\$24	\$0

STEP 0. Change the probabilities so that they have the same denominator:

probability				
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

$$\mathbb{E}[a] =$$

$$\mathbb{E}[b] =$$

$$\mathbb{E}[c] =$$

Thus she will choose and expect

Suppose now that Ann is offered, at price *p*, the following imperfect information:

$$\{\{s_1, s_2\}, \{s_3, s_4\}\}$$

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

- If informed that $\{s_1, s_2\}$ then

probability		
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$16	\$36
b	\$10	\$0
c	\$104	\$12

$$\mathbb{E}[a] =$$

$$\mathbb{E}[b] =$$

$$\mathbb{E}[c] =$$

Thus she will choose and expect

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

- If informed that $\{s_3, s_4\}$ then

probability		
state $\rightarrow$	s_3	s_4
act $\downarrow$		
a	\$100	\$12
b	\$18	\$120
c	\$24	\$0

$$\mathbb{E}[a] =$$

$$\mathbb{E}[b] =$$

$$\mathbb{E}[c] =$$

Thus she will choose and expect

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

The probability of $\{s_1, s_2\}$ is and the probability of $\{s_3, s_4\}$ is

Thus the expected change in wealth with perfect information at price p is

Thus as long as

it is worth paying for the information.

CASE 2.2: risk aversion

Smaller example.

Changes in wealth:

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$21	\$0	\$156
b	\$0	\$125	\$0
c	\$96	\$0	\$69

Assume: $U(\$x) = \sqrt{x}$ and initial wealth is \$100. Then

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

STEP 1. If she does **not purchase** information.

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

Thus she will choose with an expected utility of

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

STEP 2. If she purchases information $\{\{s_1, s_2\}, \{s_3\}\}$ at price *p*.

- If informed that $\{s_1, s_2\}$ then the revised decision problem is:

probability		
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$121	\$100
b	\$100	\$225
c	\$196	\$100

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

Thus she will choose with an expected utility of

probability		$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$		s_1	s_2	s_3
act $\downarrow$				
	a	\$121	\$100	\$256
	b	\$100	\$225	\$100
	c	\$196	\$100	\$169

- If informed that $\{s_3\}$ then she will choose with a utility of

Given the initial probabilities: $\begin{array}{lccc} \text{probability} & \frac{2}{9} & \frac{4}{9} & \frac{3}{9} \\ \text{state} \rightarrow & s_1 & s_2 & s_3 \end{array}$ the probability of receiving

information $\{s_1, s_2\}$ is $\frac{6}{9} = \frac{2}{3}$ and the probability of receiving information $\{s_3\}$ is $\frac{1}{3}$.
Thus the expected utility of purchasing information at price p is:

For example, if p = \$30 then

The maximum price the DM is willing to pay for information is given by the solution to:

Which is